The Joint Dynamics of Moving House and Mortgage Refinancing*

Joshua Abel†

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Abstract

Market interest rates affect homeowners' decisions of when to move and when to refinance, but it has been standard to study those two options in isolation. This paper studies their interaction with a dynamic model of a household with both options when interest rates and idiosyncratic housing match quality are stochastic. The options "warp" each other and their coexistence can create an inaction region relative to models in which they are studied in isolation. The nature of their interaction depends on the level of interest rates: when interest rates are low, the option to refinance deters moves, but when interest rates are high, the option to refinance encourages households to move, as they know they can subsequently refinance. An implication of this is that streamlined refinancing opportunities can lessen volatility in mobility stemming from interest rate fluctuations, a concern in the current "lock-in" episode in the housing market, which is explored with a calibrated version of the model.

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†Assistant Professor, College of the Holy Cross. E-mail: jabel@holycross.edu. Address: 1 College Street, Worcester MA, 01610. Phone: 610-937-3164

1 Introduction

1.1 Motivation

Movements in interest rates have important implications for households with fixed-rate mortgages (FRMs), the dominant mortgage form in the United States. When interest rates fall, a household can save money on its mortgage payments by refinancing. When interest rates rise, there is a disincentive against moving house, as doing so would require giving up a low interest rate and accepting a higher one, a phenomenon known as "lock-in."[1] Both effects have been on display in the first half of the 2020s: record-low interest rates during the COVID pandemic spurred a wave of refinancing; then, the sharp rise in interest rates that began in 2022 created a powerful lock-in effect, with sales volume falling by roughly one third. While substantial literatures have studied refinancing and lock-in in isolation, this paper explores them jointly and more generally analyzes how the options to move and refinance interact with each other.

To that end, section 2 extends the tractable but quantitatively relevant dynamic model of the refinancing option in Agarwal *et al.* (2013) and Berger *et al.* (2025) to allow for the additional option to sell the house and move. In particular, I allow a household's match quality to its current home to evolve stochastically, and the household can choose when to move depending on this variable (as well as interest rates). The model has two primary economic insights. First, refinancing and relocating are complements when interest rates are high, and they are substitutes when interest rates are low. When interest rates are low, either action can achieve savings on mortgage payments, so they act as substitutes. In contrast, having the option to refinance *later* can induce households to move (i.e. complements moving) when interest rates are high, since refinancing can be used to subsequently lower the interest rate.

[1] Frictions in the rental market are also necessary to create this disincentive. If there were no effort or search costs associated with renting a home, a homeowner with a favorable interest rate could move to a new home and rent, while simultaneously renting out their initial home. In that way, they could avoid surrendering the favorable interest rate while still relocating. Furthermore, it depends on the institutional features of mortgages that they are not portable or assumable.

This is the logic underpinning practitioners' advice to "marry the house, date the rate" in a high interest rate environment. Second, when interest rates are low – but not very low – and match quality is low – but not very low – the household knows it is likely to take an action soon but is not sure whether moving or refinancing will be optimal. This can create an inaction region that would not exist if either option existed in isolation, as the household waits to see how its state evolves before choosing.

Section 3 shows that a calibrated version of the model can match important moments of the data well, and then Section 4 uses the calibrated model for three quantitative applications of the theoretical insights above. Section 4.1 looks at how streamlining the refinancing process would affect household selling behavior. Specifically, it considers a mortgage that refinance automatically when interest rates fall, what I refer to as a self-refinancing mortgage (SRMs) – the limiting case of streamlined refinancing. On the one hand, the discussion above suggests that by making refinancing easier, SRMs could help to moderate volatility in home sales, encouraging sales when interest rates are high (and refinancing and selling are complements) and discouraging sales when interest rates are low (and refinancing and selling are substitutes). On the other hand, one could imagine that the recent decline in sales would have actually been *worse* if households had SRMs because everyone would have obtained the record-low interest rates available during COVID, creating stronger disincentives against relocating. There are therefore two competing effects: SRMs would have faced households with larger interest rate increases from relocating after 2022, but they would also have made households more willing to relocate *conditional on an interest rate increase*, due to the complementarity between relocating and refinancing when interest rates are high. The model finds that the latter effect is stronger and the probability of sale during this period would have been higher with SRMs.

The other two applications help to explore the importance of the inaction region created by the interaction of the options, discussed above. Section 4.2 looks at the implications for the probability of prepayment (i.e. refinance or sale) and thus prepayment risk for lenders.

Prepayment risk in structural models like Berger *et al.* (2024), Zhang (2024), and Abel (2024) comes only from refinancing behavior, as moves are assumed to be exogenous. However, the endogeneity of moves creates another source of negative correlation between interest rates and prepayment behavior, as the recent lock-in episode demonstrates. I evaluate the importance of this mechanism by comparing the full model to a version where moves are exogenous. While households' ability to time their moves does increase lenders prepayment risk, the option to move also reduces refinancing activity substantially due to the novel inaction region, which reduces prepayment risk. I find that the all-in effect of endogenizing moves is to increase prepayment risk faced by lenders by about 9%. The changed timing of moves themselves causes a roughly 15% increase, but the reduced refinancing pulls this back down. Section 4.3 explores the implications of endogenizing moves for measuring households' inattention to refinancing incentives. In particular, I explore the extent to which refinancing inaction at low interest rates – which the exogenous-move literature ascribes to inattention – can be explained by rational (i.e. attentive) inaction caused by the interaction of the options. I find that endogenizing moves allows the monthly probability of inattention to fall from 98% to 95.5%, a non-trivial but modes effect.

1.2 Related Literature

As discussed above, the paper is very closely related to Agarwal *et al.* (2013) and Berger *et al.* (2025). Agarwal *et al.* (2013) found a closed-form solution for the optimal interest rate decline at which a household should refinance. Berger *et al.* (2025) extended that model to allow inattention to interest rates. While that precluded finding a closed-form solution, they showed the model was still quite tractable and could do a far better job of matching the data on refinancing activity. In both papers, moves are exogenous and their arrival rate only matters because it changes how future payments on the mortgage are discounted. The present paper adds to this framework by endogenizing moves by assuming the household experiences persistent shocks to its idiosyncratic match quality to its current house. This match quality is a reduced-form catchall to encode every factor other than the interest rate that affects a homeowner's willingness-to-pay to move, such as proximity to a new job opportunity, the composition of room types in a home, or the mixture of local amenities, to name a few. Based on the evolution of this match quality and the nominal mortgage interest rate, the household decides when to exercise its two options. From a technical perspective, this model expands the one-dimensional diffusion process in the above papers to a second dimension, as match quality is stochastic in addition to the market mortgage interest rate. In addition, rather than solving for a single scalar refinancing threshold as in those papers, this paper solves for *a pair of functions* that characterize interest rate thresholds for refinancing and moving as functions of match quality. While this complexity precludes a closed-form solution as in Agarwal *et al.* (2013) or a tight characterization of the (scalar) solution as in Berger *et al.* (2025), I show that a number of sharp features of the solution can be established.

This paper contributes to numerous strands of the refinancing literature. A number of empirical analyses (e.g. Keys *et al.* (2016), Agarwal *et al.* (2017), and Andersen *et al.* (2020)) find many households wait too long to refinance, an issue I explore in the context of endogenous moves. Zhang (2024) and Berger *et al.* (2024) show how heterogeneity in refinancing propensities across borrowers leads to equilibrium cross-subsidization of sophisticated borrowers (those with high propensities) by unsophisticated borrowers through the impact of refinancing on *ex-ante* interest rates.[2] The present paper contributes to these structural models of mortgage pricing by considering the novel effect of endogenous moves on prepayment behavior, both directly through moves but also indirectly through changes in refinancing behavior. Those papers also show the salutary effect of automatic refinancing on this regressive transfer. This has added to the already-existing case for streamlined refinancing on macroprudential grounds. Eberly and Krishnamurthy (2014), Campbell *et al.* (2021), and

[2] Fisher *et al.* (2024) show a similar phenomenon in the United Kingdom where adjustable-rate mortgages (ARMs) with teaser rates are the standard mortgage. They show that the failure of many borrowers to refinance promptly at the expiration of the teaser rate creates substantial heterogeneity in realized payments, and they explore the implications for how interests rates are set *ex-ante*.

Guren *et al.* (2021) show that the failure to refinance reduces the ability of monetary policy to deliver payment relief to households in periods when the marginal propensity to consume is high, a problem the streamlined refinancing would alleviate. The existing literature focuses on benefits of streamlined refinancing when interest rates are *low*, but as discussed above the option to refinance is also relevant when interest rates are *high*, as it can impact moving behavior. Analyzing this requires moves to be endogenous, and to my knowledge this is the first paper to study the impact of refinancing in such a setting.

This paper is also related to the growing literature studying the current lock-in episode. On the empirical side, Fonseca and Liu (2024) show that facing an increase in the mortgage interest rate of an additional 1 percentage point (pp) from a move reduces the probability of moving by 9% on average. Liebersohn and Rothstein (2025) also show that mobility is hindered when households would have to give up a low interest rate to relocate, showing that a 1pp increase in the disincentive reduces mobility by roughly 15%. Similarly, Batzer *et al.* (2024) find that a 1pp increase in the disincentive reduces the probability that a home will be sold by roughly 18%. Katz and Minton (2025) focus on the rise in interest rates of roughly 70 basis points (bp) at the end of November 2016 and find that homeowners who bought their homes after this rise were about 25% more likely to sell their homes in the next four years compared to those who bought prior to the interest rate increase.[3]

Turning to structural studies of lock-in, Fonseca *et al.* (2025) develop an equilibrium model with a housing ladder and many housing markets, where prices of different tiers of housing are determined endogenously. They find that the incidence of a tax credit, like that proposed

[3] These papers use large, administrative datasets to show the empirical importance of this friction in recent years. This builds on the work of Quigley (1987) and Ferreira *et al.* (2010), who demonstrated the relevance of the incentive using survey data. On a related topic, Ferreira (2010) and Imrohoroglu *et al.* (2018) showed that property taxes can affect mobility by creating financial disincentives against moving, in particular by looking at caps on taxes paid by incumbent owners in California. Best and Kleven (2018) look at a related policy in the United Kingdom. A number of papers, such as Ferreira *et al.* (2010), Chen (2001), Bernstein and Struyven (2021), and Genesove and Mayer (1997) have also documented another friction that mortgages can cause that reduce mobility, which is low/negative equity: if a homeowner has little or no equity in the home, selling it will not recover a sufficient amount to fund a down payment on a comparable home, which prevents some borrowers from moving. This conceptual point was popularized by Stein (1995).

by the Biden Administration, targeted at the sellers of "starter homes" would in fact mostly accrue to households at the top of the housing ladder via appreciation of their homes. Gerardi *et al.* (2025) develop a life-cycle model with search-and-matching in the housing market and find that lock-in meaningfully reduces welfare due to reduced market thickness, particularly for younger households in lower-income areas, while at the same time increasing prices. Aladangady *et al.* (2025) also model search-and-matching in the housing market and argue that lock-in increases market tightness and consequently prices. Amromin *et al.* (2023) study the dynamics of house prices, arguing that the COVID episode was so different than the Great Financial Crisis in part due to the lock-in effect from the recent rise in interest rates. Katz and Minton (2025) focus on home prices and show how lock-in can increase net housing demand by preventing own-to-rent moves and find that that friction can explain why home prices did not fall in response to rising interest rates, as a standard asset pricing model would predict. These papers assess the impact of lock-in on house prices through reduced supply relative to demand, an equilibrium effect that is beyond the scope of the present paper. In contrast, none of the papers mentioned includes the option to refinance or considers how that affects households' mobility decisions in a dynamic setting, a focus of this paper.

2 Model

This section presents an extension of models in Agarwal *et al.* (2013) and Berger *et al.* (2025) that allows households to endogenously move – a choice that depends on housing match quality as well as interest rates – rather than assuming that moves follow an exogenous Poisson process.[4]

[4] While the setting in those papers – and this one – is quite stylized, they find considerable validation for their models. For instance, Agarwal *et al.* (2013) find close alignment between their results and those of the less tractable setting in Chen and Ling (1989). Berger *et al.* (2025) show that their model can closely match evidence from microdata on refinancing behavior conditional on interest rates; in addition, an extension in Berger *et al.* (2024) that allows for equilibrium determination of mortgage interest rates does a good job of matching observed mortgage pricing. As I will show, when the model is extended to allow for endogenous moves, it is also able to replicate key moments of data on home selling behavior.

2.1 State Variables

Consider a household with a FRM of size M, which is exogenous and without loss of generality is normalized to \$1. The household has a discount rate of ρ and, following Agarwal *et al.* (2013) and Berger *et al.* (2025), assume it is risk-neutral[5] and that the mortgage is non-amortizing (interest-only),[6] which removes M as a state variable. The real mortgage interest rate, m_t, follows a Brownian Motion of the form:

$$dm_t = \sigma_m \cdot dW_t^m, \tag{1}$$

where W_t^m is a Wiener process. For tractability, inflation is assumed to be constant, and for notational convenience, that constant is assumed to be zero. (In the calibrated model shown in Section 3, I allow inflation to be stochastic.) Letting m_{i0} be the market interest rate when household i last moved or refinanced, define $y_{it} \equiv m_t - m_{i0}$ to be the gap between the current market rate and the homeowner's contract rate. This y_{it} represents the change in the interest rate on the household's mortgage if it sells or refinances.

To produce an endogenous margin of moves, assume that a household's willingness-to-pay to move to a new house follows a stochastic process governed by the variable x_{it}. This variable represents the flow of additional housing amenity the household would get if it moved to a new house. Normalize $x_{it} = 0$ in the immediate aftermath of a move and assume x_{it} follows a Brownian Motion with drift of the form:

$$dx_{it} = \mu_x \cdot dt + \sigma_x \cdot dW_t^{x_i}, \tag{2}$$

[5] Assuming risk-neutrality simplifies the model and its solution considerably, as it means that households need only worry about the difference between their current interest rate and the market rate. A risk-averse household's choices would depend not just on the difference but also on the levels, adding a state variable.

[6] Agarwal *et al.* (2013) suggest a way that amortization can be approximated by assuming a higher discount rate. As a result, they do not refer to the mortgage in their model as "interest-only," though it is in a formal sense carried over here.

where $W_t^{x_i}$ is a Wiener process. Assume the shocks to x and m are independent. The household's state at time t is then given by (x_{it}, y_{it}).

2.2 Decision Rules

At each point in time, the household must take exactly one out of three actions:

1. Refinance: This results in a change in the household's mortgage interest rate of y_{it}, generating flow payoff of $-\frac{y_{it}}{\rho} - C^R$, where C^R is the cost of refinancing. In this case, the state jumps to $(x_{it}, 0)$.[7]

2. Move:[8] This results in a change in the household's mortgage interest rate of y_{it} and a change in the flow of housing utility of x_{it}, generating flow payoff of $\frac{x_{it}-y_{it}}{\rho} - C^M$, where C^M is the cost of moving. In this case, the state jumps to $(0, 0)$.

3. Neither: This results in a flow payoff of zero and no jump in x or y.

Following Berger *et al.* (2025), assume that Poisson attention shocks arrive at rate $\lambda > 0$. In the absence of an attention shock, the household takes no action (i.e. choice #3).[9] The household's value function, $V(x, y)$, is characterized by the Hamilton-Jacobi-Bellman equation:

$$\rho \cdot V = \mu_x \cdot V_x + \frac{\sigma_x^2}{2} \cdot V_{xx} + \frac{\sigma_m^2}{2} \cdot V_{yy} + \lambda \cdot \max\{0, R - V, M - V\}, \tag{3}$$

[7]As explained by Agarwal *et al.* (2013), the flow payoff can be modeled as the net present value of the interest rate reduction as a perpetuity, $-\frac{y_{it}}{\rho}$, because future interest rate changes are accounted for by the continuation value, $V(x_{it}, 0)$, which is the value function evaluated at the post-refi state.

[8]As in much of the related literature, I abstract from the process of searching for a home. As shown in Ngai and Sheedy (2020), the main determinant of the sales hazard is the rate at which households list their homes for sale, with the speed of listed homes selling playing only a minor role.

[9]Andersen *et al.* (2020), Zhang (2024), Berger *et al.* (2024), and Berger *et al.* (2025) have shown the importance of inattention for explaining observed refinancing behavior. I assume this attention shock is also required in order to move because it is conceptually difficult to justify a model in which households ever consider moving without considering refinancing. In such instances, a household might be tempted to move primarily in order to lower their interest rate – as in the perverse region A in the No-Refi model in Figure 1 – when in reality any household doing so would clearly just refinance instead. Assuming that attention is required in order to move avoids that awkward conceptual issue.

where:

$$R(x,y) \equiv -y/\rho - C^R + V(x,0), \tag{4}$$

$$M(x,y) \equiv (x-y)/\rho - C^M + V(0,0). \tag{5}$$

The state space, $\mathbb{R}^2$, is partitioned into three regions characterized by which of the three actions listed above is optimal, conditional on an attention shock:

1. **Inaction region**, $\mathcal{I} \equiv \{(x,y) : V(x,y) > \max\{R(x,y), M(x,y)\}\}$;
2. **Refinance region**, $\mathcal{R} \equiv \{(x,y) : V(x,y) = R(x,y) \geq M(x,y)\}$;
3. **Move region**, $\mathcal{M} \equiv \{(x,y) : V(x,y) = M(x,y) \geq R(x,y)\}$.

Note that $M(x,y) - R(x,y) = x/\rho + \big(V(0,0) - V(x,0)\big) - (C^M - C^R)$, which does not depend on y. So, we can define $\tilde{x}$ such that $M(\tilde{x},y) = R(\tilde{x},y)$. In other words, $\tilde{x}$ is the level of x at which the household is indifferent between moving and refinancing (for all y).

Proposition 1 *If $C^M > C^R \geq 0$, $\tilde{x}$ is finite and unique.*

(All Propositions are proven in the appendix.) Proposition 1 implies that $\mathcal{M}$ is empty for $x < \tilde{x}$ and $\mathcal{R}$ is empty for $x > \tilde{x}$. In words, when the willingness-to-pay to move is sufficiently low ($x < \tilde{x}$), refinancing dominates moving and an attentive household will refinance if and only if y is sufficiently low; when that willingness-to-pay is sufficiently high ($x > \tilde{x}$), moving dominates refinancing and an attentive household will move if and only if y is sufficiently low. Define the following thresholds:

$$y_{Refi}(x) = \sup\{y : (x,y) \in \mathcal{R}\} \tag{6}$$

$$y_{Move}(x) = \sup\{y : (x, y) \in \mathcal{M}\}. \tag{7}$$

So for $x \leq \tilde{x}$, $y_{Refi}(x)$ denotes the highest level of y the household will accept as part of a refinance, and for $x \geq \tilde{x}$, $y_{Move}(x)$ denotes the highest level of y the household will accept as part of a move. Note that $y_{Move}(\tilde{x}) = y_{Refi}(\tilde{x})$.

2.3 Characterizing the Solution

This subsection delivers the core theoretical results of the paper and shows how the coexistence of the options to refinance and move warps their thresholds relative to benchmark models with only one option or the other.

2.3.1 The Impact of the Refinancing Option on Moving

To begin, consider the moving region, $\mathcal{M}$, and the moving threshold, $y_{Move}(x)$. A useful benchmark for this analysis is a "No-Refi" (NR) model in which refinancing is disallowed. Define the moving threshold in that case to be $y^{NR}_{Move}(x)$.

Proposition 2 *$y^{NR}_{Move}(x)$ is a line with slope 1.*

Figure 1 shows $y^{NR}_{Move}(x)$ as the upper envelope of the union of regions A, B, and D. The linearity of this threshold is a direct result of risk-neutrality and the persistence of shocks, which ensure a constant marginal rate of substitution between mortgage payments and the housing amenity. The linearity is therefore not a meaningful result in its own right but is a useful benchmark against which to compare the full model with refinancing. The positive slope of the moving threshold *is* a general feature of the problem's solution, even with refinancing. It captures the fact that when the willingness-to-pay to move is greater, the household is willing to accept a higher interest rate increase (y) as part of a move. Alternatively, when the the market interest rate is high, households require a larger willingness-to-pay in order

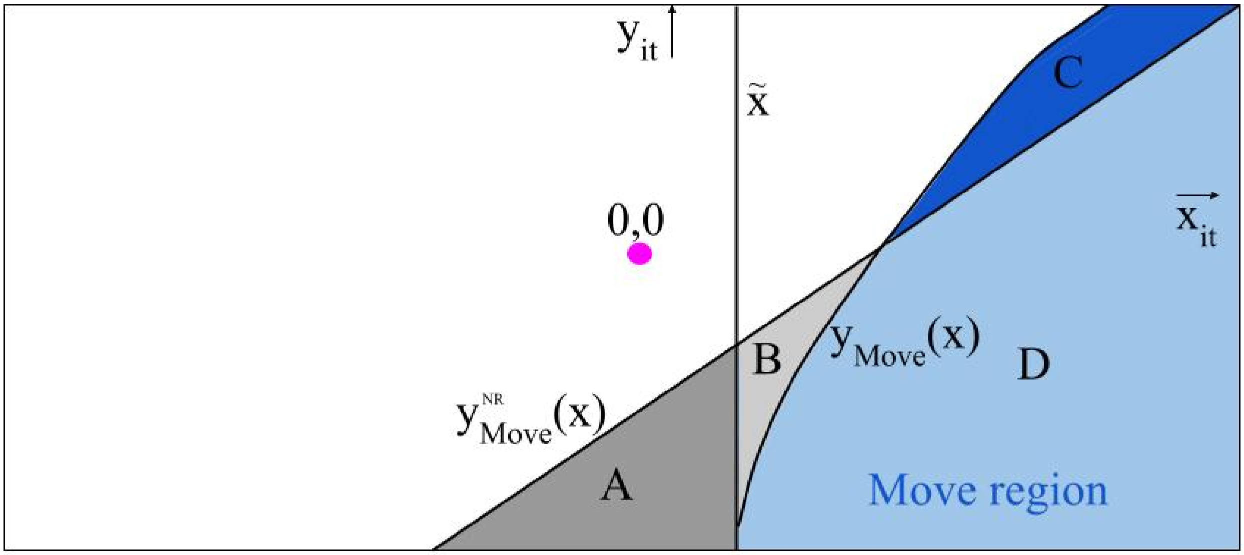


Figure 1: Moving regions, $\mathcal{M}$, under different modeling assumptions. In the No-Refi model, an attentive household moves when in regions A, B, or D; the moving threshold, $y^{NR}_{Move}(x)$, is the line (of slope 1) that serves as the upper envelope of those regions. In the full model with refinancing, the household moves when in regions C or D; the moving threshold, $y_{Move}(x)$, is the concave curve for $x \geq \tilde{x}$ (which converges to a slope of 1) that serves as the upper envelope of those regions. Following a move, the household jumps to (0,0), the point indicated with a pink dot. The existence of region B is not guaranteed and is instead parameter-dependent.

to move. The existence of region A implies perverse behavior in the NR model; households with low willingness-to-pay – even $x < 0$ – will move if y is sufficiently low. In words, the household moves to a less-suitable house in order to lower its mortgage interest rate. This is not reasonable behavior, since refinancing clearly dominates such a move. Nonetheless, this is a useful baseline to help us understand how the option to refinance affects moving behavior.

Introducing refinancing at cost $C^R < C^M$ truncates the moving region at $\tilde{x}$ – as expressed in Proposition 1 – and warps the remaining moving threshold, $y_{Move}(x)$, for $x > \tilde{x}$. This "warping" is summarized by the following Proposition:

Proposition 3 *For $x > \tilde{x}$, $y_{Move}(x)$ is concave with $y'_{Move}(x) \geq (y^{NR}_{Move})'(x) = 1$, and $y'_{Move}(x) \to (y^{NR}_{Move})'(x) = 1$ as $x \to \infty$.*

Proposition 3 states that the option to refinance steepens the moving threshold relative to the

NR model, as seen in Figure 1. In economic terms, the steepening of the moving threshold amounts to the household's moving decision becoming less sensitive to interest rates. This is because refinancing complements moving when y is high and substitutes for it when y is low. Intuitively, low rates no longer make moving as appealing because the household can refinance instead (substitutes); high rates no longer discourage moving as much because the household can refinance them down later (complements). It is useful to compare this steepening to a model in which the mortgage has an adjustable rate that tracks the market, so $y_t = 0$ for all t. In that case, the moving rule would be a vertical line: move if and only $x > x^{ARM}_{Move}$. Refinancing does not replicate that vertical moving rule, but it does move the solution in that direction, weakening the impact of the financing frictions embedded in a FRM on moving behavior and allowing moves to be more reliant simply on housing fundamentals (x).

The economics of the concavity of $y_{Move}(x)$ is as follows. When y is low, the household should expect to take one of the actions soon. At such a y, as the household's x approaches $\tilde{x}$ from above, the refinancing option becomes increasingly appealing and so it becomes less clear that the household's next action should be a move rather than a refinance. As a result, the household should wait to see which action the state will drift toward. Doing so helps it avoid moving when a cheaper refinance would have ended up being preferable, and similarly helps it avoid refinancing and then moving soon after. Compared to a tangent line at any given point, then, that concavity demonstrates the option value of waiting as the household approaches the region in which the actions are of similar value.

Following closely from Proposition 3, we have:

Proposition 4 *There exists $\bar{x} < \infty$ such that $y_{Move}(x) > y^{NR}_{Move}(x)$ for all $x > \bar{x}$.*

With that result in hand, we can discuss the full implications of the refinancing option on moving behavior in Figure 1. Proposition 1 tells us that because of the refinancing option, the household does not move in region A – those perverse moves (discussed above) are now replaced by refinancing (or inaction, as discussed below) – because the willingness-to-pay to move is too low ($x < \tilde{x}$). Proposition 3 tells us that $y_{Move}(x)$, which is the upper envelope of regions C and D, rises quickly for x close to $\tilde{x}$ and then settles into a line as it becomes parallel to $y^{NR}_{Move}(x)$. Proposition 4 guarantees that this creates a region like C, a region of the state space with a high willingness-to-pay where an attentive household moves if and only if they have the option to refinance. These are states in which is y is too high to be acceptable if the household is stuck with that interest rate until the next move but that become acceptable when the possibility of refinancing is introduced. The concavity of $y_{Move}(x)$ may create a region like B in which the reverse happens: the household does *not* move if and only if it has the option to refinance, a result of the option value of waiting discussed above. This is not a general feature of the solution and is instead parameter-dependent.[10] Regions A and B (if it exists) demonstrate refinancing and moving being substitutes at low y, while region C shows them being complements at high y.

2.3.2 The Impact of the Moving Option on Refinancing

It is harder to characterize how the option to move affects the refinancing region, $\mathcal{R}$, and threshold, $y_{Refi}(x)$. The mathematical difficulty stems from the economic fact that whereas moving resets both x and y to zero, refinancing has no effect on x. As a result, the continuation value of moving is a constant, $V(0,0)$, while the continuation value of refinancing, $V(x,0)$, varies in x. This prevents the techniques used to establish results about $y_{Move}(x)$ from carrying over to $y_{Refi}(x)$, as discussed in the appendix. Nonetheless, I present one formal result about the refinancing region and another economically intuitive feature that is borne out by the calibrated model in subsequent sections.

To begin, define an "Exogenous-Move" (EM) variant of the model in which there is no option to move, making x irrelevant. This is the framework used by Agarwal *et al.* (2013)

[10] As discussed in the appendix, $y_{Move}(\tilde{x})$ can be greater than $y^{NR}_{Move}(\tilde{x})$, in which case region B would not exist. Even in that case, however, the concavity of $y_{Move}(x)$ is present, meaning the option value of waiting to see which action is preferable is relevant. Region B exists if that value not only depresses the threshold below a tangent line (guaranteed by concavity) but also below $y^{NR}_{Move}(x)$ for some $x \geq \tilde{x}$.

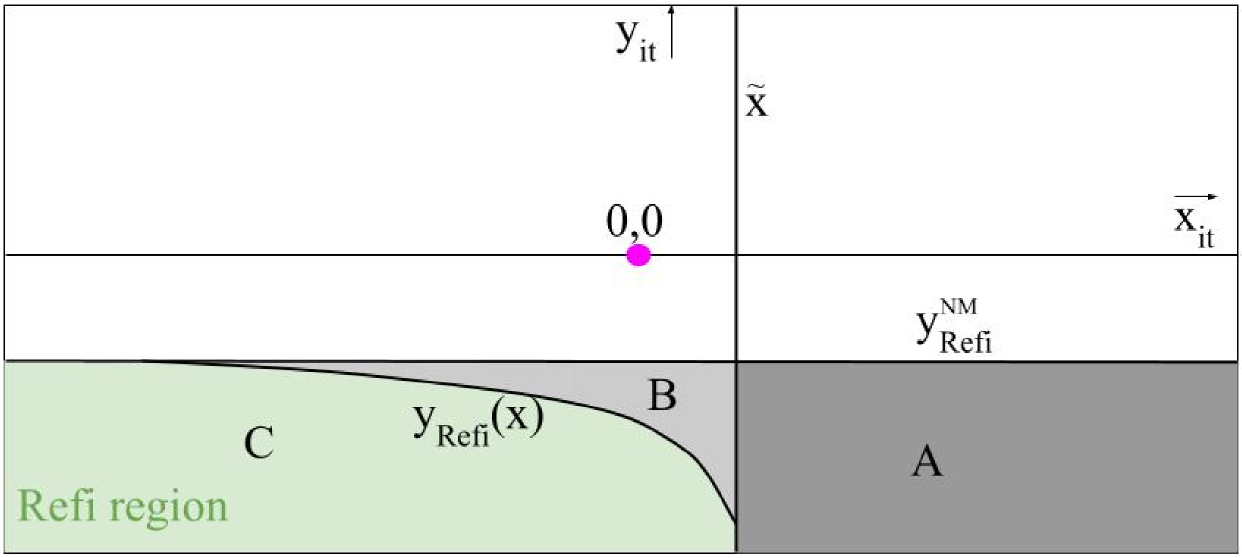


Figure 2: Refinancing regions, $\mathcal{R}$, under different modeling assumptions. In the No-Move model, an attentive household refinances when in regions A, B, or C; the refinancing threshold, $y_{Refi}(x)$, is the horizontal line that serves as the upper envelope of those regions. In the full model with moving, the household refinances when in region C; the refinancing threshold, $y_{Refi}(x)$, is the curve that serves as the upper envelope of that region. After a refinance from (x, y), the household's state jumps to $(x, 0)$ on the horizontal line running through the pink dot. The negative slope of $y_{Refi}(x)$ and the resulting region B are not proven analytically but are consistent with the calibrated model below.

and Berger *et al.* (2025), in which moves follow a Poisson process with intensity, μ. A special case of the EM model is the "No-Move" (NM) model, in which $\mu = 0$.

Proposition 5 *Let $V^{NM}(y)$ be the value function and y^{NM}_{Refi} be the refinancing threshold in the NM model. As $x \to -\infty$, $V(x, y) \to V^{NM}(y)$, and so y^{NM}_{Refi} is the threshold in that limit.*

Intuitively, when the willingness-to-pay to move is arbitrarily low, the next move is so far in the distant future that its impact on the refinancing decision is nil, so the model collapses to that of Berger *et al.* (2025).[11]

Figure 2 shows the impact of the moving option on refinancing behavior. In the absence of the option to move, the refinancing threshold is a scalar cutoff rule: refinance if and

[11]There is an interesting asymmetry here. When x is large, the refinancing option affects the moving rule because once a move occurs, x resets to zero and so refinancing is relevant again. However, when x is small, the moving option has no impact on the refinancing rule because refinancing has no impact on x and so there is no mechanism to speed up x's journey back to a level at which moving is relevant again.

only if $y \leq y^{NM}_{Refi}$ (as in Proposition 5). Introducing moves removes the refinances when the household has a high willingness-to-pay to move (region A). That is obvious: if the household wants the low interest rate but also wants a new house, they should move rather than refinance. More interesting is region B, which is an inaction region created by the moving option. The threshold slopes down[12] because as x increases, the household recognizes that the expected time until its next move decreases, as it is getting nearer to $\tilde{x}$. This decreases the benefit of refinancing, since the new refinanced interest rate will be surrendered upon moving anyway. As a result, the household requires a larger decline in the interest rate to justify the cost of refinancing. Region B in Figure 2 is akin to region B in Figure 1: because y is low but x is near $\tilde{x}$, there is a high likelihood of taking action soon but it is unclear which one will be best, so it is beneficial to wait to see how the state evolves.

2.3.3 *Full Model*

Putting all of the analysis together, Figure 3 shows the full solution, with $\mathcal{M}$, $\mathcal{R}$, and their boundaries on a single graph. An attentive household in the green "Refi region" refinances and its state jumps vertically to the horizontal black line; an attentive household in the blue "Move region" moves and its state jumps to the pink (0,0) dot. Recall that in isolation, each region's threshold has a constant slope (0 for refinancing, 1 for moving); the figure therefore shows the that the options' coexistence warps the thresholds into concavity, as asserted at the beginning of section.

3 A Calibration

This section describes how the model of Section 2 is brought to the data and calibrated. Because housing market data I use is at the monthly and quarterly frequency, I first describe a discrete-time analogue of the model that I use for the quantitative analysis. The only substantive change to the model is that I allow for a mean-reverting inflation process. Once

[12] This is the result that is not proven but is consistent with the calibration presented below.

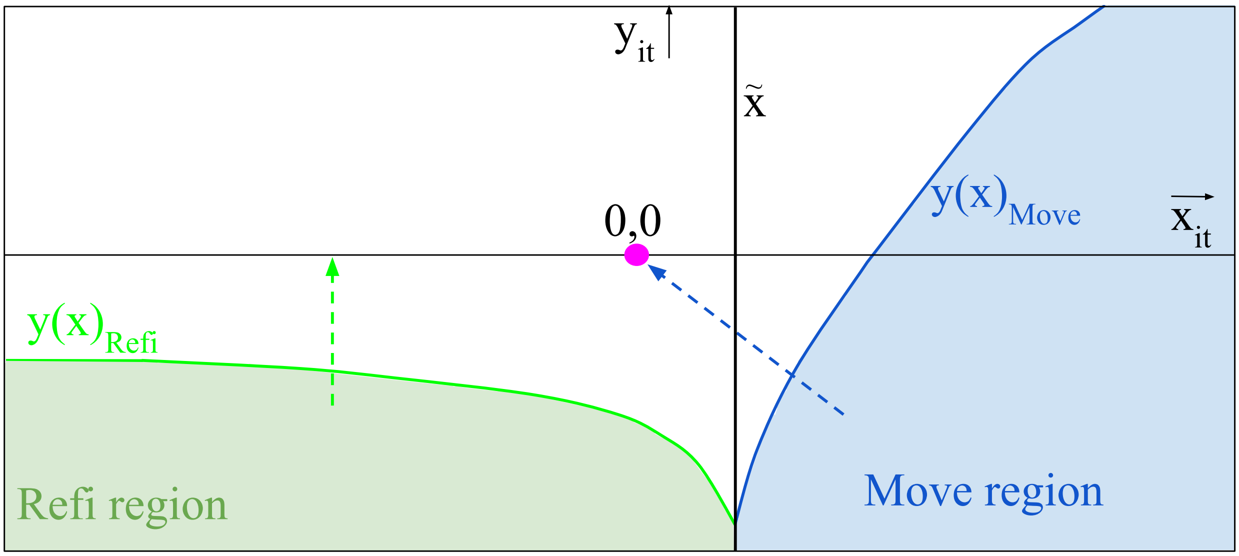


Figure 3: Refinancing and moving regions, $\mathcal{R}$ and $\mathcal{M}$, and associated thresholds when both options are allowed.

the discrete-time model is described, I discuss the relevant data, the calibration procedure, and the model's ability to fit the data during the sample period.

3.1 Discrete-Time Model

The economic setting in the discrete-time model differs little from the continuous-time model of Section 2. The only substantive change is that I allow for inflation to vary and be mean-reverting. I do this because variation in inflation was an important driver of interest rates in the post-COVID period when the lock-in phenomenon suppressed home sales, and while this feature would have undermined the tractability that supported the theoretical results in Section 2, the subsequent analysis will use numerical solutions and so can accommodate this complication. Specifically, inflation will follow a Markov process with two states: High and Low. In the High state, inflation is π_H and the probability of remaining in the High state the following period is p_π^H; in the Low state, inflation is π_L, and the probability of remaining in the Low state the following period is p_π^L. The nominal mortgage interest rate is assumed to be $m_t = r_t + \pi_t$,[13] where r is the real mortgage interest rate and follows a Random Walk:

[13]This model does not solve for equilibrium interest rates but rather takes them as given. Solving for equilibrium interest rates would require lenders to forecast refinancing and selling behavior, which depend

$$r_t - r_{t-1} = \epsilon^r_{i,t} \sim N(0, \sigma_r^2). \tag{8}$$

The household's willingness-to-pay to move follows a discrete-time Random Walk with drift:

$$x_{it} - x_{i,t-1} = \epsilon^x_{i,t} \sim N(\mu_x, \sigma_x^2). \tag{9}$$

An attention shock arrives with probability p. None of the shocks to attention, x, the real mortgage interest rate, or inflation are correlated with each other or autocorrelated. The state is given by (x_{it}, y_{it}, π_t), and the value function is:

$$\begin{aligned} V(x,y,\pi) = (1-p) \cdot \frac{1}{1+\rho} E_{x,y,\pi}\big[V(x_+, y_+, \pi_+)\big] \\ + p \cdot \max\Big\{ \underbrace{\frac{1}{1+\rho} E_{x,y,\pi}\big[V(x_+, y_+, \pi_+)\big]}_{\text{Value of no action}}, \\ \underbrace{-q_\pi \cdot y - C^R + \frac{1}{1+\rho} E_{x,0,\pi}\big[V(x_+, y_+, \pi_+)\big]}_{\text{Value of refinancing}}, \\ \underbrace{\frac{1+\rho}{\rho} \cdot x - q_\pi \cdot y - C^M + \frac{1}{1+\rho} E_{0,0,\pi}\big[V(x_+, y_+, \pi_+)\big]}_{\text{Value of moving}} \Big\}, \end{aligned} \tag{10}$$

where $q_\pi \equiv 1 + E[\sum_{t=1}^{\infty} \left(\frac{1}{(1+\rho)^t \cdot \Pi_{\tau=1}^t (1+\pi_\tau)}\right) | \pi_0 = \pi]$. Row 1 of Equation 10 accounts for the probability that the household does not receive an attention shock, in which case it takes no action. The remaining lines allow for the household to choose among no action, refinancing, and moving when an attention shock has arrived. The flow values of refinancing and moving are analogous to the continuous-time model, with the only deviation being that x and y are discounted differently due to inflation, since y is nominal and x is real. As such, x is

on the evolution of the joint distribution of households' interest rates and willingness-to-pay to move. Given the computational difficulty of that problem and this paper's focus on household behavior, I assume this simple exogenous process for mortgage interest rates based on the Fisher Equation.

discounted simply by $\frac{1}{1+\rho}$, whereas the discounting of y includes an adjustment for inflation, as shown in the equation for q_π. For a set of parameters, Equation 10 is solved through value function iteration.

3.2 Data

The analysis relies on monthly data on mortgage interest rates[14] and inflation[15] from April 1991 through September 2025 as shown in Figure 4. The inflation measure is lagged by 12 months, as the lagged inflation series has a much stronger correlation with nominal mortgage rates and so seems to be a better measure of expected inflation that markets used to set mortgage rates.[16]

Home sales activity is measured using the quarterly sales rate as reported in Batzer *et al.* (2024) from 2005Q1-2024Q1.[17,18] Using microdata on mortgages guaranteed by Fannie Mae and Freddie Mac (the government-sponsored enterprises, or GSEs), these authors linked mortgage prepayments to deeds records to identify which are sales. Their series displays a very strong correlation with aggregate home sales data as reported by the National Association of Realtors (NAR), as shown in Figure 5a.[19]

To measure refinancing activity, I use public, loan-level origination and performance data from Fannie Mae's "Single-Family Fixed Rate Mortgage" dataset.[20] The loan-level data is

[14]In particular, the average interest rate on a 30-year FRM as reported by Freddie Mac's Primary Mortgage Market Survey is used. This is available from the Federal Reserve Economic Database (FRED) under the name "MORTGAGE30US."

[15]In particular, the average Consumer Price Index year-over-year inflation for urban consumers, less food and energy, as reported by the Bureau of Labor Statistics is used. This is available from FRED under the name "CPILFESL."

[16]In particular, $\text{corr}(m_t, \pi_t) = 0.36$ and $\text{corr}(m_t, \pi_{t-12}) = 0.57$

[17]I am grateful to the authors of Batzer *et al.* (2024) for making auxiliary materials from their analysis available. These can be found at `https://www.fhfa.gov/research/papers/wp2403`.

[18]In the model, moving and selling are equivalent.

[19]The data from Batzer *et al.* (2024) is a preferable measure for this calibration for two reasons. First, it reports the probability of sale, which is a more natural analog to the model than is the aggregate number reported by the NAR. And as described in the next paragraph, the present paper uses microdata from Fannie Mae to infer the time series of refinancing activity during this time period. This makes it appealing to have a sales measure based on mortgages guaranteed by the GSEs, as the Batzer *et al.* (2024) measure provides.

[20]This dataset track 30-year FRM Fannie Mae mortgages that are fully-amortizing and fully-documented. Fannie Mae makes additional restrictions that remove high-risk mortgages, such as removing loans with

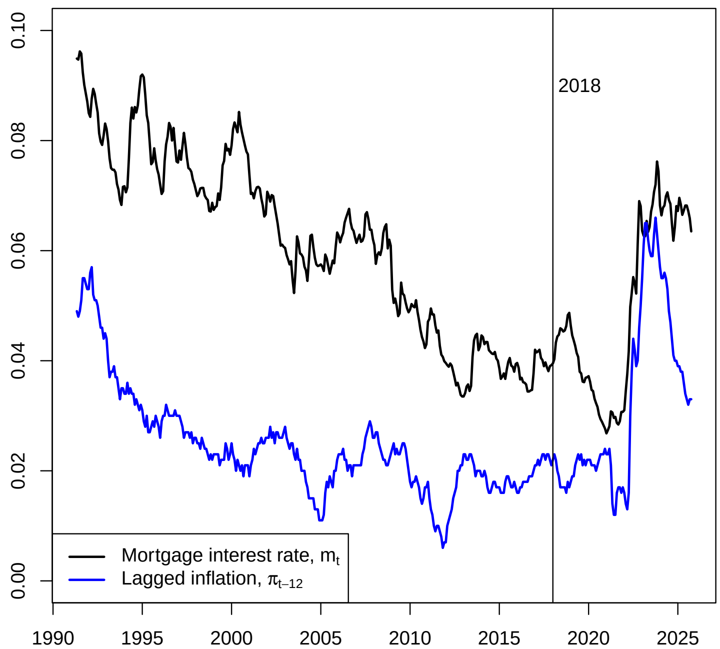
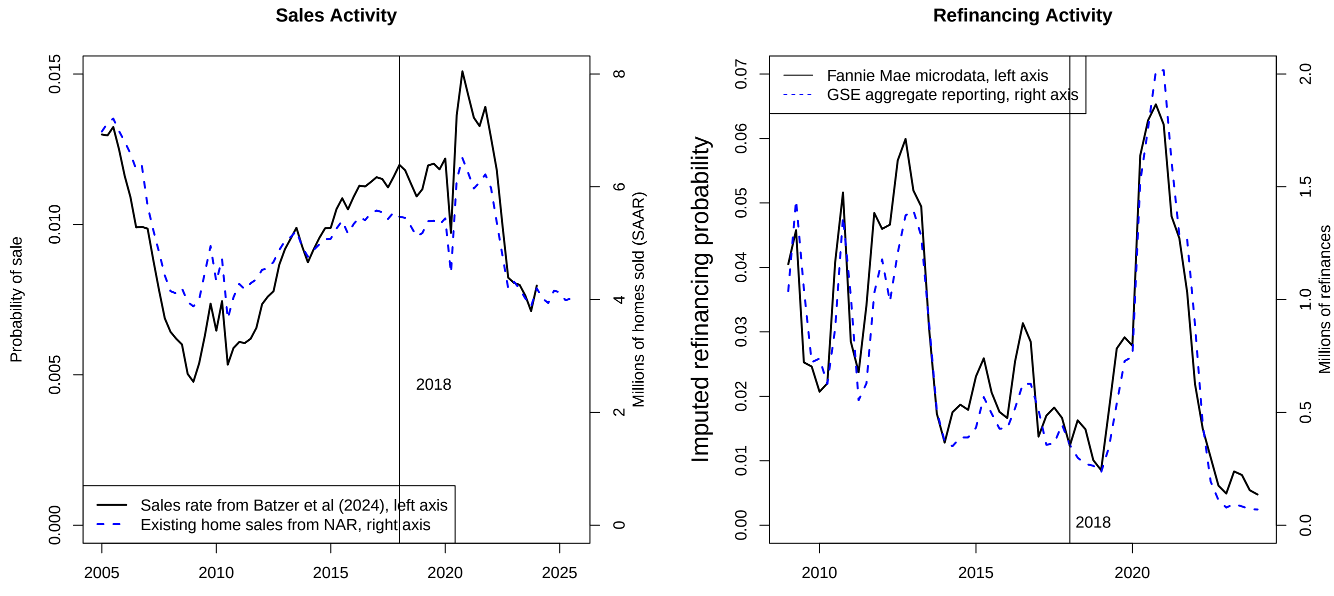


Figure 4: Nominal mortgage interest rate and inflation rate. The inflation rate shown here is lagged by 12 months.

used to construct a quarterly time series of mortgage prepayments. From there, the quarterly refinancing probability is calculated as the share that prepaid their mortgage (from the microdata) minus the share that sold their home (from Batzer *et al.* (2024)). This makes the strong assumption that all mortgage prepayments result either from sales or refinances, which is not true. Nonetheless, Figure 5b shows that the series of refinancing probabilities constructed in this way has a strong correlation with aggregate refinancing activity of GSE mortgages as reported by the Federal Housing Finance Agency (FHFA).[21]

origination loan-to-value ratios greater than 97%. While some of these sample selection criteria are more restrictive than is ideal for this paper, note that the data is used here to measure refinancing activity, rather than mortgage default, making the loss of higher-risk loans less troubling.

[21] In particular, two sets of reports produced by FHFA can be used to construct a time series of refinance volume of GSE mortgages: Refinance Reports, which provide the relevant data from 2009Q1 through 2019Q1; and Foreclosure Prevention, Refinance, and FPM Reports, which took the place of the Refinance Reports after March 2019. They are all available at https://www.fhfa.gov/reports?topic=fannie-mae-and-freddie-mac-reports.



(a) Home sales series.

(b) Refinancing series.

Figure 5: Sales and refinancing data. Figure 5a is a validation of similarity between the probability of sale calculated in Batzer *et al.* (2024) and aggregate data reported by the National Association of Realtors. Figure 5b is a validation of similarity between the probability of refinancing – calculated by combining microdata from Fannie Mae with the probability of sale from Batzer *et al.* (2024) – and aggregate refinancing data of GSE mortgages reported by the FHFA. "SAAR" stands for "seasonally-adjusted annual rate."

Finally, the American Housing Survey (AHS) is used to to evaluate the model's prediction of households' baseline hazard function, the probability of selling t years after purchase, conditional on not selling previously.

3.3 Calibration Procedure and Validation

The model is parameterized at the monthly frequency in order to capture richness in interest rate variation across households.[22] Table 1 shows the calibrated values of the model's parameters that are used in the analysis in subsequent sections. The first panel shows parameters whose values are set based on other papers in the literature. The second panel lists the inflation-related parameters, which are calibrated based on empirical analogs from April

[22]Results, however, will be shown at the quarterly frequency to facilitate comparison with the quarterly data series described above.

Parameter	Meaning	Value	Source
ρ	Discount rate (annualized)	0.05	Agarwal *et al.* (2013)
C^R	Refinancing cost	0.018	Agarwal *et al.* (2013) (for $250k mortgage)
C^M	Moving cost	0.09	Fonseca and Liu (2024)
π_L	Low inflation rate (annualized)	0.022	Empirical averages in the US from April 1991 - September 2025, using 4% as the cutoff value between low and high inflation
π_H	High inflation rate (annualized)	0.052	
p_π^L	Probability of remaining in low-inflation state	0.994	
p_π^H	Probability of remaining in high-inflation state	0.941	
σ_r	Real mortgage rate volatility (annualized)	0.006	Matches nominal mortgage rate volatility from April 1991 - September 2025 (0.017)
μ_x	Drift of x	$8{\cdot}10^{-5}$	Matches sales data from 2018Q1-2022Q1 as described in text
σ_x	Volatility of x	0.0036	
p	Probability of attention	0.055	

Table 1: Calibrated parameters. The top three panels show parameters that are taken from the literature or calibrated to match moments of macroeconomic data. The bottom panel shows three parameters jointly calibrated to best match aggregate sales data from 2018Q1-2022Q1.

1991 through September 2025. Given those values, σ_r is calibrated to match the empirical volatility of the nominal interest rate, assuming the macroeconomic processes described in Section 3.1. The three parameters in the final panel of Table 1 (μ_x, σ_x, p) are calibrated via simulation. For each triple in a grid of those three parameters, the following steps are performed for 100,000 households:

1. Start in time $t = 0$ with $x_{it} = y_{it} = 0$;
2. Simulate 1,000 months of behavior with shocks to x drawn from $N(\mu_x, \sigma_x^2)$, assuming y remains constant at 0 and inflation is in its low state;
3. Store $x_{i,1000}$, the household's willingness-to-pay to move at the end of the 1,000 months;
4. Simulate 372 additional months, where shocks to x continue to be drawn from $N(\mu_x, \sigma_x^2)$ and the nominal mortgage interest rate and (lagged) inflation follow the paths shown in Figure 4 from April 1991 to March 2022 (372 months).

The parameter values shown in Table 1 are the ones that best matched the moments of the sale probability from 2018Q1-2022Q1 shown in Table 2. The moments being matched draw from the period starting in 2018 because the model is not designed to capture some important dynamics of the housing market for the decade or so prior, which was heavily influenced by a series of destabilizing developments.[23] As shown in Figure 5a, home sales activity seemed to stabilize after these fluctuations in 2018, and so that is where I begin evaluating the model. Nonetheless, I cannot simply start the simulations in 2018 because, while the distribution of y (the household's interest rate gap) could be observed, the analysis requires the joint distribution of y with x, and x is not observable. The 1,000 months of simulation prior to introducing observed macroeconomic data approximates the ergodic distribution of x in a steady state where interest rates are constant. Then, by introducing a long "burn-in" period of observed data (April 1991 through December 2017), the procedure approximates what the joint distribution of y and x looked like prior to the evaluation period of 2018Q1-2022Q1.[24] I cut off in 2022Q1 to leave data to evaluate the model out-of-sample.[25] So, the model is calibrated to match the level of sales prior to the COVID-era boom as well as the surge in sales associated with the sharp decline in interest rates during the pandemic. The collapse in the probability of sale associated with the ensuing rise in interest rates is then left as an out-of-sample episode to evaluate the model.

Table 2 and Figure 6 are useful to consider together. Figure 6 shows that the model is able to match the key movements in the probability of sale from 2018-2025. One evident

[23]The developments include a notable relaxation and then tightening of lending standards, large gyrations in expectations and realizations of home price growth, a large rise and decline in the use of housing as a method of speculation among investors (i.e. non-owner occupants), the Great Recession, and the long recovery and stabilization of the market from all of these factors. The literature on this episode of the housing market is far too large to cite adequately. Two papers that are particularly germane are Berger *et al.* (2020), who study the role of investors in driving the joint dynamics of price and sales volume in the housing market, and Kaplan *et al.* (2020), who quantify the importance of home price expectations and credit conditions in driving the housing cycle of the 2000s.

[24]Given the extremely unusual circumstances surrounding the onset of the COVID pandemic, 2020Q2 is not used in the calculation of any moments used in the model's calibration.

[25]While the sales data from Batzer *et al.* (2024) ends in 2024Q1, I extend the sale probability series through 2025Q3 by applying the growth rate of NAR-based home sales series since 2024Q1.

Sale Probability			
Period	Data	Model	Model, lagged
2018Q1 - 2020Q1	0.0117	0.0122	0.0120
2020Q3 - 2022Q1	0.0138	0.0137	0.0141
2022Q2 - 2025Q3	0.0082	0.0080	0.0082

Table 2: Comparison of key sale probability moments in data and model. The final column shows results from model lagged one quarter (ignoring 2020Q2 due to the onset of the COVID pandemic). The parameters μ_x, σ_x, and p were calibrated to have the lagged model best match the observed sale probabilities in 2018Q1-2020Q1 and 2020Q3-2022Q1. The final row compares the model's behvaior to the data for 2022Q2-2025Q3, which was not targeted in the calibration.

discrepancy is that the probability seems to respond to interest rate movements somewhat faster in the model than in the data, as there appears to be a closer correspondence when the model's output is lagged by one quarter (solid line) than when it is not (dashed line). For this reason, I choose (μ_x, σ_x, p) to minimize the sum of squared discrepancies between the sale probabilities in the lagged model and the data in the two sub-periods in the top panel of Table 2. That issue aside, the model is able to essentially match the roughly 20% increase in the probability of sale that occurred after the immediate onset of the COVID-19 pandemic, when interest rates declined steeply. The bottom row of Table 2 shows that the model performs well out-of-sample. While it was calibrated to match the rise in the sale probability when interest rates fell, it does a good job of matching the ensuing decline of roughly 35% in the sale probability when interest rates rose, which is useful validation of the model.

Figure 7 shows the model's performance on another untargeted moment, which is the probability of refinancing. Overall, the model performs well, but with some subtleties. In periods of rising interest rates, such as 2018, the model predicts essentially zero refinancing, whereas there is some refinancing in the data. In the model, there is no reason to refinance when market interest rates have risen, but in reality some borrowers might refinance in such environments for reasons not considered by the model, such as equity withdrawal, term change, or an improved credit profile that could allow a household to lower its interest rate even if

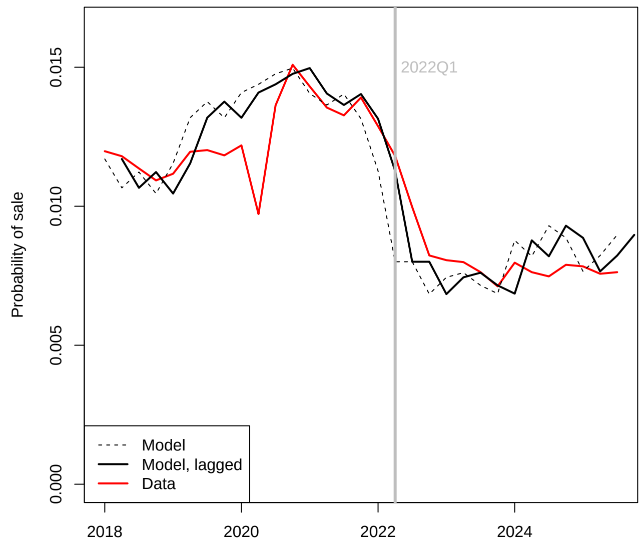


Figure 6: Comparison between data and model of sale probability from 2018Q1-2025Q3. Because the Batzer *et al.* (2024) sales data are only available through 2024Q1, the sales data series is extended through 2025Q3 by applying the growth rate of sales as observed in the NAR series. As described in the text, the parameters μ_x, σ_x, and p are calibrated to match moments of the sales data from 2018Q1-2022Q1. The sales data to the right of vertical gray line were not targeted in the calibration.

market rates have gone up since their mortgage origination. As a result, it is not surprising that the model's probability is too low in 2018 and the post-2022 period, when those would have been the predominant motivations for refinancing. Notably, Gerardi *et al.* (2023) find that cash-out refinancing is not very correlated with interest rates and so is orthogonal to the issues explored in this paper.

Once interest rates fell in 2019 and then hit record lows in 2020, refinancing in the model spikes, as in the data, and it crashes back down in 2022 following interest rate hikes. While the model does not track the data perfectly, the resemblance is close. For 2019-2022, the model's average quarterly refinancing probability is 3.34%, compared to 3.38% in the data. They arrive at this average in somewhat different ways, with the data being more gradual in its rise and fall, whereas refinancing by households in the model is more highly concentrated

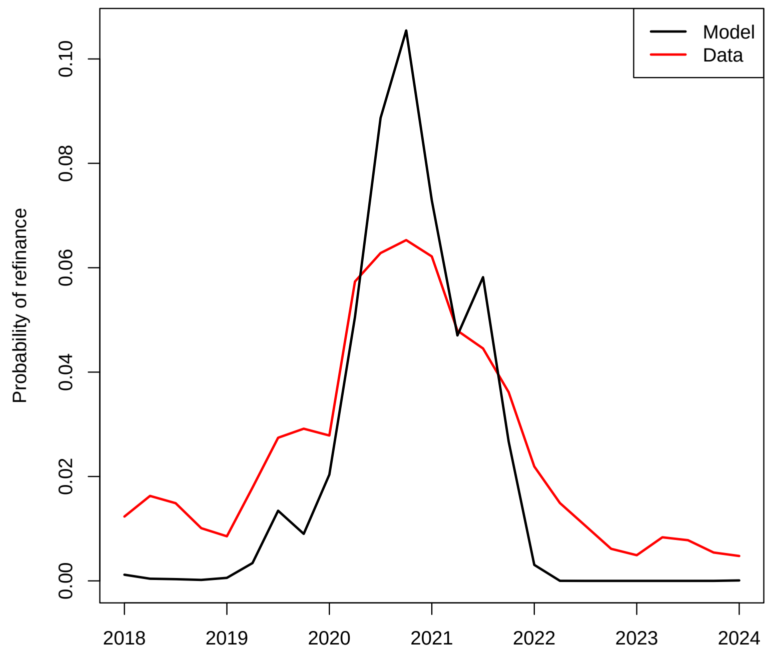


Figure 7: Comparison between data and model of sale probability from 2018Q1-2024Q1.

in the period from mid-2020 to mid-2021. One possible explanation for the less dramatic spike and more gradual decline in the data is that capacity constraints of mortgage lenders may have prevented them from processing the volume of refinancing shown in the model, causing the actual refinancing activity to spread out over time a little more evenly.

The model is also able to match quite well the empirical distribution of y over time. Figure 8 uses the Fannie Mae data described in Section 3.2 to construct the empirical distribution for January 2018 through September 2025 and compares it to the results generated by the calibrated model. The Fannie Mae data shows that while the distribution of y crept up during 2018 as interest rates were rising, even in this period it was very mild, with only roughly 20% of households facing interest rate increases of more than 1pp if they were to move. These gaps eroded over the course of 2019 as interest rates fell and then when interest rates hit historic lows in 2020 during COVID, no household had seen interest rates increase

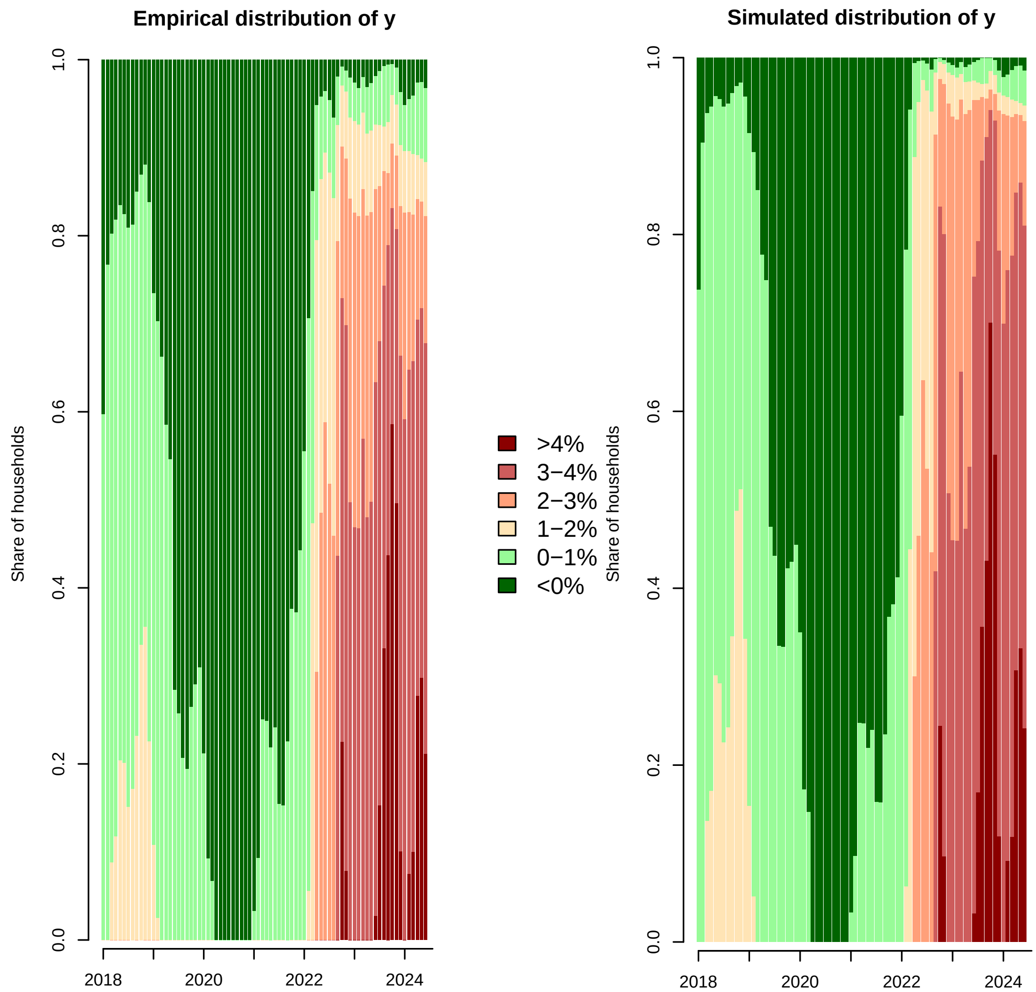


Figure 8: Distributions of y from January 2018 through September 2025. The empirical distribution is based on the Fannie Mae loan-level data described in Section 3.2. The simulated distribution is based on the calibrated model.

since their most recent mortgage origination (i.e. $y \leq 0$ for all households). The increase in interest rates that began in 2022 caused the distribution of y to spike up sharply, with over 90% of households facing interest rate increases of over 2pp, and many households above 3pp or even 4pp. The simulated distribution matches these patterns both qualitatively and quantitatively, though not exactly.

As a final untargeted moment, Figure 9 shows the relationship between probability of sale and the amount of time since the household purchased the house – the baseline sale hazard. The figure compares the model to the empirical analogue computed from the AHS. Following Ngai and Sheedy (2020), the empirical hazard function is calculated for each cohort (i.e. year of prior purchase) as the decline in the number of homes from that cohort in successive surveys. The baseline hazard function is then a weighted average across cohorts.[26] Both hazard functions peak relatively early in an ownership spell: 2 years in the model, 3-4 years in the data.[27] Following that early peak, the hazard declines quite steadily. The model's interpretation of the non-monotonic pattern is that households are well-matched to their homes immediately after purchasing them and it takes some time for the willingness-to-pay to move to build up. The probability of sale rises while that happens, but the relationship then reverses because the households who remain in their homes that long are disproportionately well-matched and so unlikely to move. That selection effect gets stronger over time, driving down the sale hazard. That declining sale hazard has a direct parallel to the concave cumulative probability of migration documented by Howard and Shao (2026). They show that the data is well fit by a model with persistence in spatial preferences, a close analog to the persistence in x embedded in the assumed Random Walk process in the present paper.

4 Applications

4.1 Streamlined Refinancing

A growing literature – including Campbell (2013), Zhang (2024), and Berger *et al.* (2024) – has pointed out that heterogeneous refinancing of FRMs leads to worse outcomes for borrowers with less financial sophistication. Eberly and Krishnamurthy (2014), Campbell *et al.* (2021), and Guren *et al.* (2021) point out that the failure of borrowers to refinance FRMs also has adverse macroprudential consequences. One oft-discussed solution to both

[26] As the AHS is conducted every two years, the hazard rate for cohort c in year t is calculated as $1 - \sqrt{N_{ct}/N_{c,t-1}}$, where N_{ct} is the number of homes in cohort c that remain in the survey in year t.

[27] The AHS is conducted every other year, which is why the red function is sparser than the black one.

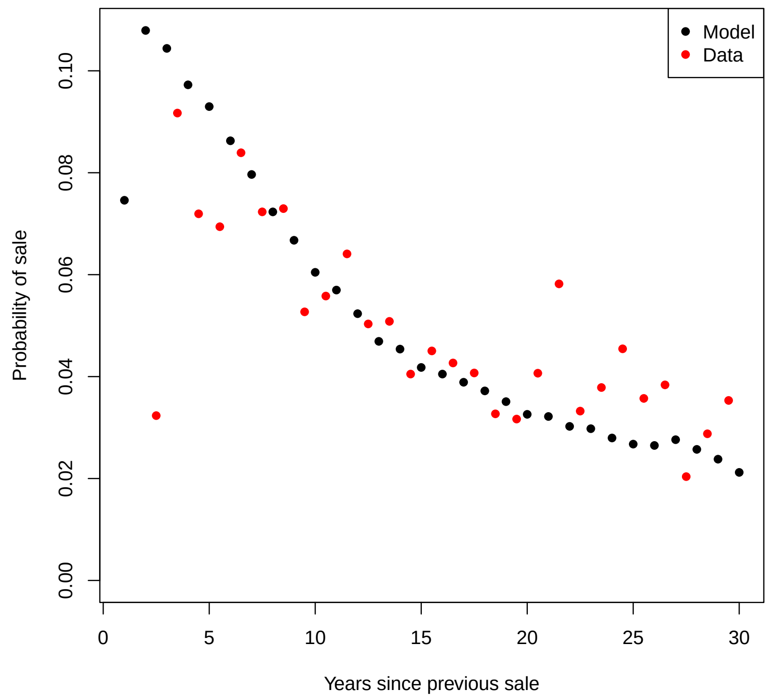


Figure 9: Baseline sale hazard (i.e. annualized probability of sale as a function of time since purchase, conditional on not selling earlier) from the data and the model. The "Data" series is constructed using the AHS following the approach used by Ngai and Sheedy (2020). As the AHS is conducted every two years, the sale hazard at durations less than two years cannot be calculated.

of these problems is to make refinancing easier. An extreme case of that that has been advocated in the literatrue is a mortgage that refinances automatically and costlessly when interest rates fall – what I will call a "Self-Refinancing Mortgage," or SRM. Advocates of mortgages like this have made clear the equity, efficiency, and macroprudential benefits of of the SRM over the standard FRM when interest rates decline, as they typically do during recessions. But what about when interests rates come back up? This literature has not considered the possibility of lock-in, and it seems plausible that in fact the SRM would have *adverse* consequences when interest rates rise. After all, if all borrowers got the record-low interest rates available during COVID, would the lock-in problem have been more severe once interest rates rose, with home sales reduced even further? More generally, given the theoretical findings of this paper's model, streamlining the refinancing process would affect moving behavior, an issue not addressed by the refinancing literature's exogenous-move models.

Because a SRM makes refinancing costless and automatic, the discussion in Section 2 suggests that it could serve to mitigate volatility in the probability of sale in general – and the sharp decline from the recent lock-in episode in particular – by allowing households to worry less about the interest rate when they consider moving: they are more willing to accept a higher interest rate with the SRM because it will be easier to refinance down later. That is the precisely the logic underpinning the moving threshold's steepening, which expands $\mathcal{M}$ at high y, as established by Proposition 4. So in terms of the recent decline in the probability of sale, there is a tradeoff when considering a FRM relative to a SRM: the SRM would create a worse (i.e. higher) distribution of y – since everyone would have refinanced into the record-low interest rates available during COVID – but it would make households more willing to move conditional on those high values of y.[28] To explore this quantitatively, I rerun the model under the counterfactual assumption that households have SRMs rather than FRMs. Specifically, I keep all of the parameters in Table 1 the same with the following exceptions:

- I set $C^R = 0$ to capture that refinancing a SRM is costless;
- I set $p = 100\%$, allowing the mortgage to be refinanced in any month in which $y_{it} < 0$;
- Even though households can refinance in any month, I continue to assume that households only consider moving in 5.5% of months, the same level as in the FRM baseline.

A caveat is warranted before discussing the results of this analysis. This analysis continues to focus on household behavior taking as given the interest rate process. However, in equilibrium

[28]There is also a subtle impact that switching from the FRM to the SRM has on the distribution of x. As discussed below, the SRM scenario experiences less of a boom in sales in the pre-2022 period since borrowers are less sensitive to the low interest rates. As a result, the willingness-to-pay to move will tend to be somewhat higher in that scenario since fewer households have recently moved, which puts some upward pressure on the sale probability in the ensuing periods after interest rates rise.

SRMs would have higher interest rates than FRMs due to their higher prepayment risk. In this model, which has risk-neutral households and no payment-to-income constraints, the level of the interest rate is immaterial – all that matters is how much the market interest rate has changed since the last purchase or refinance. As a result, the relevant feature of the interest rate process is its *volatility* rather than its *level*. So, there is no need to explicitly model the increase in the level of interest rates when analyzing the SRM. The counterfactual SRM simulation assumes the same interest rate volatility as the FRM baseline. While this need not necessarily be the case, Abel (2024) presents a model where interest rates are determined in equilibrium and shows that, under a number of assumptions maintained in this paper – risk-neutrality, interest-only mortgages, and Brownian Motion for interest rates – FRM and SRM interest rates would have the same volatility.

Figure 10 shows the distribution of y in a given month, September 2025 (the end of the simulation), in the FRM baseline (black) and SRM counterfactual (orange). As asserted above, the SRM generates a much worse distribution of y during this time period because all households who have not moved since December 2020 would be paying the record-low interest rate of that month. As a result, over 80% of households would face an increase in their interest rate of over 3.5pp if they were to move.

Figure 11a shows the paths of the sale probability from 2018Q1-2025Q3 implied by the model for the FRM and SRM scenarios. Notice first that the SRM eliminates the boom in sale probability coming from the COVID-era decline in interest rates, as households would have gotten the interest rate reduction automatically. This is the idea that refinancing substitutes for moving when interest rates are low. More interestingly, after the rise in interest rates in 2022, sales would actually have been higher with the SRM than the FRM. As suggested above, this surprising result occurs because, even though the interest rate gaps would have been higher with the SRM, households would be more willing to move with a high interest rate gap because the SRM would strengthen their ability to get savings subsequently; or, said differently, refinancing complements moving when interest rates are high.

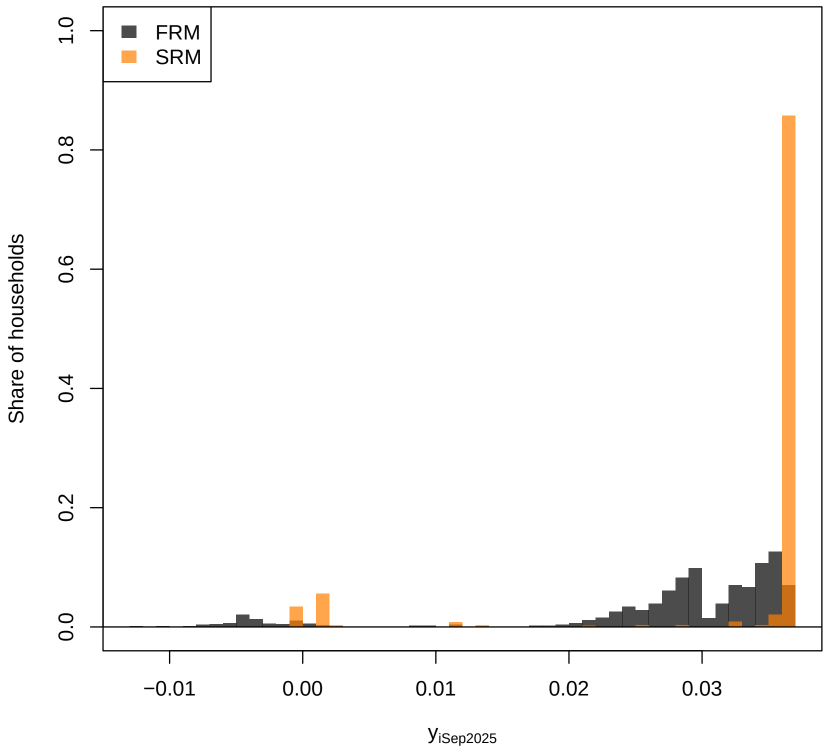
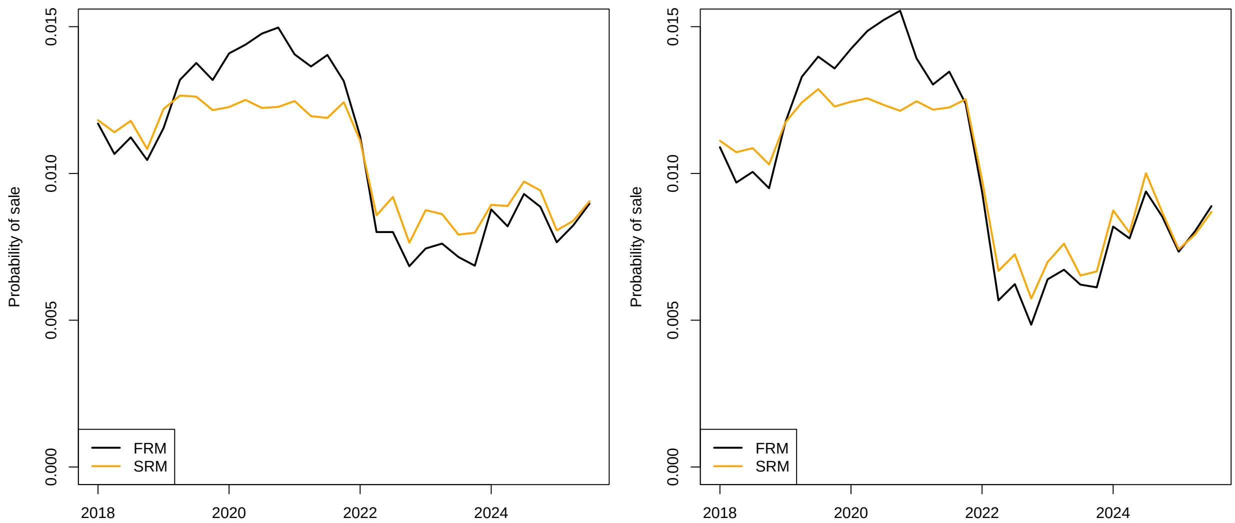


Figure 10: Distribution of y in September 2025 in the baseline FRM and counterfactual SRM simulations.

One feature of the model that biases it toward this favorable finding regarding the SRM is that it assumes households only care about the NPV of costs and not their timing. But if they are disproportionately concerned with initial payments, perhaps due to a credit constraint, that could lessen their willingness to accept a higher interest rate, even if they are confident it will be refinanced downward later. A full treatment of that concern requires a very different model, for instance with risk aversion. To shed light on this issue with the current model, however, I rerun the analysis using a discount rate of $\rho = 0.10$ instead of $\rho = 0.05$ to mimic the reduced value of the benefits of future refinancing that a household nearing a credit constraint might exhibit.[29] Figure 11b shows that assuming *all* households have that high discount rate does dampen the SRM's benefit of buffering the fall in the probability of sale,

[29] This change also necessitated recalibrating the parameters in the bottom panel of Table 1, which resulted in $\mu_x = 1.2 \cdot 10^{-4}$, $\sigma_x = 0.0065$, and $p = 0.135$.



(a) Baseline: $\rho = 0.05$.

(b) High discounting: $\rho = 0.10$.

Figure 11: Probability of sale under the FRM baseline and SRM counterfactual scenarios. Figure 11a uses the baseline calibration which includes a discount rate of $\rho = 0.05$. Figure 11b assumes $\rho = 0.10$. Changing ρ also necessitated recalibrating the parameters in the bottom panel of Table 1, which explains why the results for the FRM model are different. The recalibration resulted in $\mu_x = 1.2 \cdot 10^{-4}$, $\sigma_x = 0.0065$, and $p = 0.135$.

but it still does so to some extent. Interestingly, the intuition that SRMs would *worsen* the decline of sales due to their impact on the distribution of y is still not borne out.

This analysis shows that streamlined refinancing – in addition to the other benefits already identified in the literature – can help to moderate home sales volume volatility stemming from movements in interest rates. Because the moving threshold steepens, akin to the key Proposition 3, the SRM decreases the relevance of the interest rate gap in households' moving decisions, inducing them to act more like ARM households, whose moving decisions are based entirely on housing fundamentals. Because SRM households would be less tempted to move by low interest rates and less deterred from moving by high interest rates relative to FRM households, home sales volatility would be reduced.

4.2 Prepayment Patterns

This section looks at how endogenizing moves affects the behavior of overall prepayments, which include both refinances and sales. In structural models of the mortgage market, it is common to assume that lenders only face prepayment risk from refinancing (see, e.g., Berger *et al.* (2024), Abel (2024), and Zhang (2024)), but the possibility of mortgage lock-in – and the broader analysis in this paper of how the sale probability depends on the interest rate – suggests another source of prepayment risk coming from moves/sales. This subsection takes a closer look at the joint behavior of the mortgage interest rate and selling and refinancing probabilities, which will give a deeper understanding of how the two types of prepayments interact and allow for analysis of the implications of endogenous moves for prepayment risk to lenders.

Before doing that analysis, it is useful to consider the role of μ, the exogenous moving parameter, in the EM model. With exogenous moves, higher μ means a household's low interest rate from a refinance is not expected to last as long, since they will give it up at the next move. Therefore, as Proposition 6 states, households require a larger reduction in interest rates to justify a refinance in the EM model relative to NM:

Proposition 6 *If $\mu > 0$, $y^{EM}_{Refi} < y^{NM}_{Refi}$.*

Figure 12 compares the prepayment behavior of the full model to the EM model. The EM model has identical parameters as the full model, except rather than the option to move, moves occur (in discrete time) with probability $p^{EM}_m = 0.003$, which is the unconditional probability of moves from simulations of the calibrated full model. The thresholds partition the state space into six regions, and the figure shows how the probability of a prepayment changes in each region when moving from the EM model to the full model (i.e. endogenizing moves). For instance, region 1 is an inaction region for both models, but since moves occur exogenously with probability p^{EM}_m in the the EM model, endogenizing moves lowers the

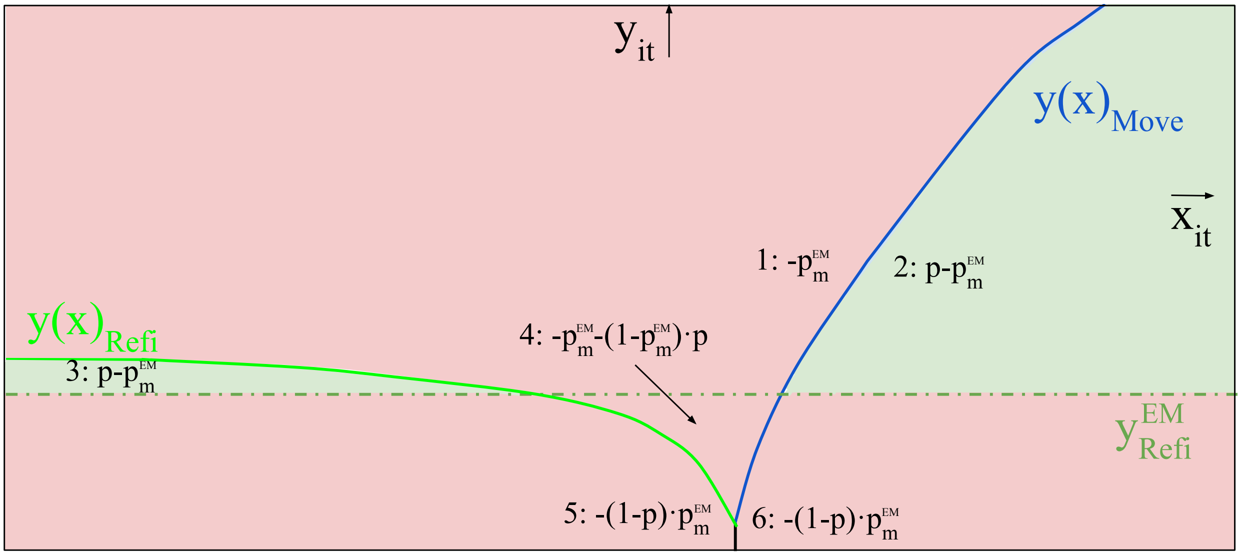


Figure 12: Comparison of prepayment behavior in the full model (with thresholds $y_{Refi}(x)$ and $y_{Move}(x)$) and the EM model (with refinancing threshold y^{EM}_{Refi}). For each of six regions created by the thresholds, the change in the probability of prepayment (refinance or move) when switching from EM to the full model is shown, where p represents the probability of paying attention (the same for both models) and p^{EM}_m is the probability of moving in the EM model. In the calibration, $p > p^{EM}_m$. Regions 2 and 3 are green because the probability of a prepayment is higher in the full model, while the other regions are red because the probability is lower.

probability of a prepayment by p^{EM}_m, which is why region 1 is colored red. That effect is active in region 2 also, but since region 2 is in $\mathcal{M}$ for the full model, a prepayment occurs conditional on attention arriving, which occurs with with probability $p > p^{EM}_m$, which is why that region is green.

The figure shows that endogenizing moves affects not only moves but also refinances. Begin by considering moves. When y is greater than y^{NM}_{Refi} (the limit of $y_{Refi}(x)$ as $x \to -\infty$), the green region shrinks relative to the red region as y increases further. This is the model's manifestation of lock-in, as households avoid moving at high interest rates when they have the option to. This is an obvious source of prepayment risk to lenders from endogenous moves. But Figure 12 shows that there are also effects on refinancing behavior at low levels of y. The most interesting region in that regard is region 4, an inaction region created by the option to move, where y is low and x is near $\tilde{x}$ (not shown), where the household waits to see which action will be optimal. In this region, endogenizing moves actually reduces prepayment risk for lenders by preventing households from refinancing while they wait for uncertainty to resolve.[30] The reverse happens in region 3, as households in the full model refinance because they know they will not be moving soon. As we will see below, this is swamped by the negative impact on refinancing in regions 4, 5, and 6.

To quantify these effects, I define prepayment risk as the reduction in the NPV of mortgage payments relative to a perpetuity that pays the initial interest rate forever, and then I track behavior from 10,000 simulated paths of inflation, interest rates, and household-level shocks for 1,000 months, assuming inflation starts in its Low state. Household selling and refinancing behavior is then tracked for the full and EM models. Finally, I look at a "Hybrid" model in which households follow the moving threshold of the full model but the refinancing threshold of the EM model. This allows us to disentangle the effects of endogenizing moves described in the previous paragraph: the direct effect of being able to time moves and the indirect effect of altering refinancing behavior. Table 3 shows the results.

	Full	EM	Hybrid
NPV of payment reduction	16.0%	14.7%	16.9%

Table 3: Reduction in lender revenue from prepayments. The Hybrid model uses the moving rule from the full model and the refinancing rule from the EM model. Revenue reduction is measured relative to a baseline in which the initial interest rate is paid forever.

Comparing columns 1 and 2, we can see that endogenizing moves has a modest but non-trivial effect on prepayment risk, increasing lost revenue by about 9% (from 14.7pp of the mortgage balance to 16.0pp). As the Hybrid model shows, this understates the importance of moves themselves, since removing the impact on refinancing allows the prepayment risk to rise up to 16.9%, 15% above the EM model. But in the full model, there is 40% less refinancing activity,[31] which is what pulls the full model's prepayment risk back down to

[30]Regions 5 and 6 also have lower probabilities of prepayment in the full model. Unlike region 4, regions 5 and 6 are action regions in both models where a prepayment will occur if attention arrives, and they differ only because the EM model has the additional possibility of an exogenous moving shock.

[31]While this is a very large reduction in refinancing, recall that the refinances that are not occurring are

16.0%.

I do not endeavor to solve for equilibrium interest rates. However, we can use a back-of-the-envelope calculation to recast these magnitudes in terms of the change in the initial mortgage rate that lenders would charge in response to endogenizing moves. Specifically, given a discount rate of 5% and average inflation of around 3%, a 2.3pp increase in the NPV reduction would translate to a 2.3pp $\cdot (1 - \frac{1}{1+0.05+0.03}) =$ 17bp increase in the initial interest rate to compensate lenders for the additional prepayment risk caused by endogenous moves. If we neutralize the impact on refinancing by focusing on the Hybrid model, there would be a 24bp increase in the initial interest rate. For context, this is about half of the monthly standard deviation of the nominal mortgage interest rate.

4.3 Endogenous Moves and Inattention

Recent papers – including this one – have used a substantial degree of inattention to rationalize observed refinancing behavior. For instance, Berger *et al.* (2025) find that their model of refinancing with Poisson attention shocks best match data from 1999-2023 when households pay attention to interest rates in 1-2% of months. Stated simply, households rarely when they "should," so a model with inattention infers that households are rarely aware of the refinancing incentive. The present paper's model with endogenous moves adds some subtlety to this. As region B of Figure 2 shows, endogenizing moves creates an inaction region in which even attentive household chooses not to refinance at low levels of y because they suspect they will be moving soon. In other words, some inaction at low levels of y can be the result of a rational decision to wait rather than inattention. This means that households may be more attentive than found by exogenous-move models and that some of those "failures" to refinance are in fact rational.

To explore this, I look at refinancing behavior in the full model and compare it to versions

precisely those that have the least impact on lenders because they are the ones that would not have long-lived effects, due to an impending move.

of the EM model with different levels of the attention parameter, p. In particular, in the full model with endogenous moves and $p = 0.055$, the average quarterly refinance share from 2018-2022 was 2.51%. An EM model with the same $p = 0.055$ generates far more refinancing over that time period: an average quarterly share of 3.25%. In other words, $p = 0.055$ generates roughly 30% more refinancing from 2018-2022 in the EM model relative to the full model. Alternatively, an EM model with p cut all the way down to 2.0% – similar to that of Berger *et al.* (2025) – does the best job of matching the full model, with a quarterly refinancing share of 2.46% during that time period. In other words, allowing for endogenous moves allows for calibrated attention to be more than two times as high (0.055 relative to 0.020), though substantial inattention is still required by the full model with endogenous moves.

5 Conclusion

This paper has studied the dynamic interaction between refinancing a FRM and moving house by extending the models of Agarwal *et al.* (2013) and Berger *et al.* (2025) to allow for an endogenous moving margin. By adding a second option (moving) and a second state variable (willingness-to-pay to move), the solution to this model requires two functions (rather than a single scalar) to characterize action thresholds. In spite of this challenge, a series of Propositions in Section 2 sharply characterized how the introduction of a refinancing option influences moving behavior. Those theoretical findings are most tidily summarized as follows: the option to refinance steepens the moving threshold because refinancing and relocating are complements when interest rates are high and substitutes when interest rates are low. From a policy perspective, this points to an additional rationale for easing the process of refinancing. While attention has previously focused on payment savings for households and the transmission of monetary policy resulting from the direct impact of refinancing, enabling easier access to refinancing can also help to improve the allocation of housing across households by weakening the dependence of home-selling behavior on the level of interest rates. When refinancing is easier, there is less incentive for a household to move early in order to secure a favorable interest rate and there is less incentive to delay moving to avoid an unfavorable one. Taken to its limit in the form of a SRM, while easier refinancing cannot eliminate the relevance of interest rates for relocation decisions as an ARM would, the quantitative evaluation in Section 4.1 suggests that the SRM would have meaningfully attenuated the effect of lock-in currently afflicting the housing market.

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Appendix

A Proofs

A.1 Proposition 1: If $C^M > C^R \geq 0$, $\tilde{x}$ is finite and unique.

Define $h(x) \equiv V(x,0) - V(0,0)$. I will show there is a unique $\tilde{x} \in (0,\infty)$ such that:

$$\tilde{x}/\rho - h(\tilde{x}) = C^M - C^R. \tag{11}$$

For existence, first define $f(x) \equiv x/\rho - h(x)$. f has the following features:

1. f is continuous in x. *Consider two households A and B with different initial values of x but the same initial value of y: A starts at (x,y) and B starts at $(x+\Delta,y)$. Assume they receive the same Brownian increments to x and y and the same attention shocks. Suppose that household B mimics household A's actions, i.e. moves, refinances, or does neither exactly as A does. Defining V^{Mimic} as the value that household B gets from mimicking A's actions, we must have $V^{Mimic}(x+\Delta,y) \leq V(x+\Delta,y)$. Furthermore, the gap can be no larger than $\frac{|\Delta|}{\rho}$, the (undiscounted) difference in flow value at the the time of A's first move: since this move will bring them to the same state, (0,0), no further deviations will occur, and refinances will give them the same flow values since their values of y are identical. As a result, $\frac{|\Delta|}{\rho} > |V(x+\Delta,y) - V^{Mimic}(x+\Delta,y)|$. By the same logic, we get $\frac{|\Delta|}{\rho} > |V(x,y) - V^{Mimic}(x+\Delta,y)|$. Therefore, by the triangle inequality we have $|V(x,y) - V(x+\Delta,y)| \leq |V(x,y) - V^{Mimic}(x+\Delta,y)| + |V^{Mimic}(x+\Delta,y) - V(x+\Delta,y)| \leq 2 \cdot \frac{|\Delta|}{\rho}$. Since this holds for all $\Delta > 0$, V is continuous in x, and therefore so is f.*

2. $f(0) = 0 < C^M - C^R$.

3. x exists such that $f(x) > C^M - C^R$. *Assume to contradiction that $f(x) \leq C^M - C^R$*

for all x. In this case, $M(x,y) \leq R(x,y)$ in the entire state space, so the household never moves, meaning the problem reduces to the no-move (NM) model, with $V(x,y) = V^{NM}(y)$. This would imply $h(x) = 0$ for all x, meaning $f(x) = x/\rho$, which exceeds $C^M - C^R$ for $x > \rho \cdot (C^M - C^R)$, contradicting the assumption that $f(x) \leq C^M - C^R$ for all x.

By the Intermediate Value Theorem, these three features imply the existence of $\tilde{x} > 0$ such that $f(\tilde{x}) = C^M - C^R$, immediately implying the existence of a solution to Equation 11.

For uniqueness, consider separately the cases of $C^R > 0$ and $C^R = 0$.

The argument for $C^R > 0$ proceeds in three parts:

1. Any solution $\tilde{x}$ of $f(\tilde{x}) = C^M - C^R$ must be such that $(\tilde{x}, 0)$ is on the interior of the inaction region, $\mathcal{I}$. *At that point, $R(\tilde{x},0) = V(\tilde{x},0) - C^R$, implying $V(\tilde{x},0) - R(\tilde{x},0) = C^R > 0$. This means that $V(\tilde{x},0) > R(\tilde{x},0)$, and by the definition of $\tilde{x}$, we also have $V(\tilde{x},0) > M(\tilde{x},0)$. Since $V(\tilde{x},0) > max\{R(\tilde{x},0), M(\tilde{x},0)\}$, $(\tilde{x},0)$ is on the interior of $\mathcal{I}$.*

2. f is strictly monotone at $\tilde{x}$. *We must have $V_x \leq 1/\rho$ everywhere. To see why, consider a household at state (x,y) and a different one at $(x+\Delta, y)$ who receives the same Brownian increments and attention shocks. If the latter mimicked the former's behavior, the difference in value can be no larger than Δ/ρ, the undiscounted flow payoff at the time of first move. So $V(x+\Delta,y) - V(x,y) \leq \Delta/\rho$. Taking $\Delta \to 0$, we get $V_x \leq 1/\rho$. Furthermore, smooth pasting gives $V_x = 1/\rho$ on the shared boundary of the inaction and moving regions. So V_x attains its maximum on the boundary, and the "strong maximum principle" states that if it also reached that maximum on the interior of the inaction region, then $V_x = 1/\rho$ throughout the region – see Theorem 3.5 of Gilbarg and Trudinger (2001). But since $V \to 0$ as $y \to \infty$ and V_x is bounded (by $1/\rho$), $V_x \to 0$ as $y \to \infty$. Therefore, V_x is not constant, implying that $V_x(\tilde{x},0) < 1/\rho$, since $(\tilde{x},0)$ is on*

the interior of $\mathcal{I}$. Therefore, $f'(\tilde{x}) = 1/\rho - V_x(\tilde{x}, 0) > 0$.

3. There is only one $\tilde{x}$ such that $f(\tilde{x}) = C^M - C^R$. *Because $f'(x) = 1/\rho - V_x(x, 0)$ and $V_x \leq 1/\rho$, $f'(x) \geq 0$. Since we have shown that the inequality is strict at $\tilde{x}$, then $f(x) < C^M - C^R$ for all $x < \tilde{x}$ and $f(x) > C^M - C^R$ for all $x > \tilde{x}$.*

So if $C^R > 0$, $\tilde{x}$ is unique. **The argument for $C^R = 0$** proceeds in two parts:

1. Any solution $\tilde{x}$ of $f(\tilde{x}) = C^M$ must be such that $(\tilde{x}, 0)$ is on the boundary of the move region, $\mathcal{M}$. *To begin, note that when $C^R = 0$, $R(x, 0) = V(x, 0)$ for all x, meaning the $y = 0$ axis is the refinancing boundary, y_{Refi}. This means $(\tilde{x}, 0)$ is on the refinancing boundary, so $V(\tilde{x}, 0) = R(\tilde{x}, 0)$. By definition of $\tilde{x}$, we therefore have $V(\tilde{x}, 0) = M(\tilde{x}, 0)$, putting $(\tilde{x}, 0)$ on the move boundary.*

2. Proposition 3 shows that the boundary, $y_{Move}(x)$ is strictly increasing for all $x \in \mathcal{M}$, with $y'_{Move}(x) \geq 1$. The threshold therefore crosses $y = 0$ at exactly one point, the unique $(\tilde{x}, 0)$.

Note that the uniqueness of $\tilde{x}$ is not used to prove Proposition 3, so the logic is not circular.

A.2 Proposition 2: $y^{NR}(x)$ is a line with slope 1.

Define $z \equiv x - y$. The move payoff is therefore $M(x, y) = \frac{1}{\rho} \cdot z - C^M + V^{NR}(0, 0)$, meaning M depends on x and y only through z. The dynamics of z are given by:

$$dz_t = \mu_x \cdot dt + \sigma_x \cdot W_t^x - \sigma_m \cdot W^m, \tag{12}$$

which is a one-dimensional Brownian Motion with drift μ_x and variance $\sigma_z^2 = \sigma_x^2 + \sigma_m^2$. Given this setup and defining $\tilde{V}(z) = V(x, y)$, the Hamilton-Jacobi-Bellman equation is:

$$\rho \cdot \tilde{V}^{NR}(z) = \mu_x \cdot \frac{d\tilde{V}^{NR}(z)}{dz} + \frac{\sigma_z^2}{2} \cdot \frac{d^2\tilde{V}^{NR}(z)}{dz^2} + \lambda \cdot \max\{0, z/\rho - C^M + \tilde{V}(0) - \tilde{V}(z)\}. \quad (13)$$

This has the same form as the problems studied by Agarwal *et al.* (2013) and Berger *et al.* (2025) and so is solved by a scalar stopping rule z^* such that an attentive household moves if and only if $z \geq z^*$, or equivalently $y \leq -z^* + x$. Therefore, $y^{NR}(x) = -z^* + 1 \cdot x$.

A.3 Proposition 3: For $x > \tilde{x}$, $y_{Move}(x)$ is concave with $y'_{Move}(x) \geq (y^{NR}_{Move})'(x) = 1$, and $y'_{Move}(x) \to (y^{NR}_{Move})'(x) = 1$ as $x \to \infty$.

This proposition has a number of distinct statements and a long proof, so I organize it into three parts: (i) concavity of the threshold based on convexity of V; (ii) comparison of slopes of full and No-Refi models based on an augmented version of the No-Refi value function; (iii) limiting slope of the threshold.

A.3.1 $y_{Move}(x)$ is concave.

The main work is showing that V is the fixed point of a convexity-preserving Bellman Operator. To begin, let $\mathcal{B}$ denote the space of continuous functions $u : \mathbb{R}^2 \to \mathbb{R}$, and let $\mathcal{B}_c \subset \mathcal{B}$ be the subset of such functions that are jointly convex in (x, y). Define a norm with some $\beta > 0$ (to be bounded later) as:

$$||u|| \equiv \sup_{(x,y)}|u(x,y)| \cdot e^{-\beta\cdot(|x|+|y|)}. \quad (14)$$

- Claim: In the Banach space defined by $(\mathcal{B}, ||\cdot||)$, $\mathcal{B}_c$ is a closed cone. *The cone property is immediate from the convexity of sums of convex functions. To see why it is closed, consider a sequence, $\{u_n\}$, such that $u_n \in \mathcal{B}_c$ and $||u_n - u|| \to 0$. For any fixed (x, y), the definition of the norm gives $|u_n(x,y) - u(x,y)| \leq ||u_n - u|| \cdot e^{\beta\cdot(|x|+|y|)}$. Because $||u_n - u|| \to 0$, so must $|u_n(x,y) - u(x,y)|$, meaning $u_n \to u$ pointwise. Pointwise* *convergence preserves convexity, so u is convex, meaning $u \in \mathcal{B}_c$.*

Now define a Bellman operator, T, on $u \in \mathcal{B}$:

$$Tu(x,y) \equiv E\Big[e^{-\rho\cdot\tau} \cdot A(x_\tau, y_\tau; u)\Big], \tag{15}$$

where $\tau \sim Exp(\lambda)$ is the time of the first attention shock, x and y follow Brownian motions as described in the main text, and $A(x,y;u) \equiv \max\big\{u(x,y), (x-y)/\rho - C^M + u(0,0), -y/\rho - C^R + u(x,0)\big\}$. T is defined to mimic the flow payoffs and recursive structure of the economics problem in the paper: wait until the first attention event and then take optimal action at that time, inclusive of continuation value.

I will now show that T is a contraction mapping, which implies a unique fixed point $u^* \in \mathcal{B}$. The contraction result is shown in three steps.

1. Construct a pointwise bound for $|Tu_1 - Tu_2|(x,y)$. *Since $Tu_1(x,y) - Tu_2(x,y) = E\Big[e^{-\rho\cdot\tau} \cdot \big(A(x_\tau, y_\tau; u_1) - A(x_\tau, y_\tau; u_2)\big)\Big]$, the positivity of $e^{-\rho\cdot\tau}$ and Jensen's Inequality ($|E[Z]| \leq E[|Z|]$) give:*

$$|Tu_1 - Tu_2|(x,y) \leq E\Big[e^{-\rho\cdot\tau} \cdot |A(x_\tau, y_\tau; u_1) - A(x_\tau, y_\tau; u_2)|\Big]. \tag{16}$$

2. Rewrite the bound in terms of $||\cdot||$. *Define a, b, and c as the three arguments of the max function in A: $a_i \equiv u_i(x', y')$, $b_i \equiv (x'-y')/\rho - C^M + u_i(0,0)$, $c_i \equiv -y'/\rho - C^R + u_i(x', 0)$. Therefore, $|a_1 - a_2| = |u_1(x', y') - u_2(x', y')|$, $|b_1 - b_2| = |u_1(0,0) - u_2(0,0)|$, and $|c_1 - c_2| = |u_1(x', 0) - u_2(x', 0)|$. From the inequality that $|max\{a_1, b_1, c_1\} - max\{a_2, b_2, c_2\}| < max\{|a_1 - a_2|, |b_1 - b_2|, |c_1 - c_2|\}$, we get:*

$$|A(x_\tau, y_\tau; u_1) - A(x_\tau, y_\tau; u_2)| \leq \max\big\{|u_1 - u_2|(x', y'), |u_1 - u_2|(0,0), |u_1 - u_2|(x', 0)\big\}. \tag{17}$$

By the definition of the norm, we have $|u_1 - u_2|(x', y') \leq ||u_1 - u_2|| \cdot e^{\beta\cdot(|x|+|y|)}$*,* $|u_1 - u_2|(0,0) \leq ||u_1 - u_2||$*, and* $|u_1 - u_2|(x', 0) \leq ||u_1 - u_2|| \cdot e^{\beta\cdot|x|}$*. Because* β*,* $|x|$*, and* $|y|$ *are all non-negative, the inequality in 17 reduces to* $|A(x_\tau, y_\tau; u_1) - A(x_\tau, y_\tau; u_2)| \leq ||u_1 - u_2|| \cdot e^{\beta\cdot(|x|+|y|)}$*. Combining with the inequality in 16, we get:*

$$|Tu_1 - Tu_2|(x, y) \leq E\Big[e^{-\rho\cdot\tau} \cdot ||u_1 - u_2|| \cdot e^{\beta\cdot(|x|+|y|)}\Big]. \tag{18}$$

3. Bound $E\Big[e^{-\rho\cdot\tau} \cdot ||u_1 - u_2|| \cdot e^{\beta\cdot(|x|+|y|)}\Big]$ by iterating the expectation over Brownian paths (given $\tau = t$) and then over τ. *Given the Brownian motion paths described in the main text and initial condition* (x_0, y_0)*, the triangle inequality gives* $|x_\tau| \leq |x_0| + |\mu_x| \cdot \tau + \sigma_x \cdot |W_\tau^x|$ *and* $|y_\tau| \leq |y_0| + \sigma_m \cdot |W_\tau^m|$*, meaning:*

$$e^{\beta\cdot(|x_\tau|+|y_\tau|)} \leq e^{\beta\cdot(|x_0|+|y_0|)} \cdot e^{\beta\cdot|\mu_x|\cdot\tau} \cdot e^{\beta\cdot\sigma_x\cdot|W_\tau^x|} \cdot e^{\beta\cdot\sigma_m\cdot|W_\tau^m|}. \tag{19}$$

Because W^x *and* W^m *are independent, we get:*

$$E\Big[e^{\beta\cdot(|x_\tau|+|y_\tau|)}\Big] \leq e^{\beta\cdot(|x_0|+|y_0|)} \cdot e^{\beta\cdot|\mu_x|\cdot\tau} \cdot E\Big[e^{\beta\cdot\sigma_x\cdot|W_\tau^x|}\Big] \cdot E\Big[e^{\beta\cdot\sigma_m\cdot|W_\tau^m|}\Big]. \tag{20}$$

Because $|W_\tau|$ *has the same distribution as* $\sqrt{\tau} \cdot |Z|$*, where* $Z \sim N(0,1)$*,* $E\Big[e^{\beta\cdot\sigma\cdot|W_\tau|}\Big]$ *is the moment-generating function of a normal distribution with variance* σ^2 *(and* $\sqrt{\tau}$ *instead of the standard notation of* t*). Therefore,* $E\Big[e^{\beta\cdot\sigma\cdot|W_\tau|}\Big] = 2\cdot e^{\beta^2\cdot\sigma^2\cdot\tau/2}\cdot\Phi(\beta\cdot\sigma\cdot\sqrt{\tau})$*, where* $\Phi()$ *is the CDF of the standard Normal. This allows us to rewrite the inequality in 20 as:*

$$E\Big[e^{\beta\cdot(|x_\tau|+|y_\tau|)}\Big] \leq 4\cdot\Phi(\beta\cdot\sigma_x\cdot\sqrt{\tau})\cdot\Phi(\beta\cdot\sigma_m\cdot\sqrt{\tau})\cdot e^{\beta\cdot(|x_0|+|y_0|)}\cdot e^{K(\beta)\cdot\tau}, \tag{21}$$

where $K(\beta) \equiv \beta\cdot|\mu_x| + \frac{\beta^2}{2}\cdot(\sigma_x^2+\sigma_m^2)$. *We can now multiply* $e^{-\rho\cdot\tau}$ *and take the expectation over* τ*:*

$$E\Big[e^{-\rho\cdot\tau+\beta\cdot(|x_\tau|+|y_\tau|)}\Big] \leq D(\beta)\cdot e^{\beta\cdot(|x_0|+|y_0|)}, \tag{22}$$

where $D(\beta) \equiv 4\cdot E[\Phi(\beta\cdot\sigma_x\cdot\sqrt{\tau})\cdot\Phi(\beta\cdot\sigma_m\cdot\sqrt{\tau})\cdot e^{-(\rho-K(\beta))\cdot\tau}]$. *As* $\beta\to 0$, $D(\beta)\to 4\cdot\Phi(0)^2\cdot e^{-\rho\cdot\tau} = e^{-\rho\cdot\tau} = \frac{\lambda}{\lambda+\rho}$, *where the last equality comes from the moment-generating function of the Exponential distribution. So, there exists some* $\beta^*>0$ *such that for any* $\beta\in(0,\beta^*)$, $D(\beta)<1$. *Take such a* β *and define* $\delta\equiv D(\beta)<1$. *Combining Equation 18 and inequality 22, we have:*

$$|Tu_1 - Tu_2|(x_0,y_0) \leq \delta\cdot e^{\beta\cdot(|x_0|+|y_0|)}\cdot||u_1-u_2||. \tag{23}$$

Multiplying both sides by $e^{-\beta\cdot(|x_0|+|y_0|)}$ *and taking the supremum over* (x,y) *yields* $||Tu_1 - Tu_2|| \leq \delta\cdot||u_1-u_2||$. *So* T *is a contraction mapping.*

Because T is a contraction mapping, it has a unique fixed point. By a standard dynamic programming principle – see e.g. Oksendal and Sulem (2019), Theorem 11.1 – that unique fixed point of T is the value function V of the associated dynamic programming problem. **I will now show that T preserves convexity.** Take $u\in\mathcal{B}_c$. In three steps, I will show that $Tu\in\mathcal{B}_c$, too.

1. The three candidates, the maximum of which is chosen by $A()$, are convex. *One candidate is* $u(x,y)$, *which is convex by assumption that* $u\in\mathcal{B}^c$. *The second,* $u(0,0)+(x-y)/\rho-C^M$ *is affine in* x *and* y *and so therefore is also convex. The third candidate is* $-y/\rho-C^R+u(x,0)$. *The last term is the slice of the convex function* $u(x,y)$ *on*

which $y = 0$, which is convex because $u \in \mathcal{B}^c$, and the other terms form an affine function of y. Adding this to the convex $u(x, 0)$ function preserves convexity.

2. As $A()$ is the maximum of three convex functions, it is also convex.

3. Multiplying by $e^{-\rho \cdot \tau}$ and taking the expectation with respect to τ over Brownian paths maintains convexity. *First, note that x is affine in x_0 and y is affine in y_0, since the initial conditions simply serve as level shifts for the state, given the assumed Wiener processes. This means that for any given path, A is a convex function of functions that are affine in x_0 and y_0. That composition is therefore convex – see section 3.2.2 of Boyd and Vandenberghe (2004). Furthermore, multiplying by the scalar $e^{-\rho \cdot \tau}$ preserves that convexity. So we have convexity over every path, and because expectations are linear and monotone, the inequality that gives convexity for each path is preserved in the expectation over the paths (see section 3.2.1 of Boyd and Vandenberghe (2004)).*

Now we can wrap up the proof of the concavity of $y_{Move}(x)$. Take $u_0 \in \mathcal{B}_c$, e.g. $u_0 = 0$. Defining $u_n \equiv T^n u_0$, I have shown that u_n is convex and that it converges to the value function, V. Since $\mathcal{B}_c$ is closed, $V \in \mathcal{B}_c$ is also convex. Since $V(x, y)$ is convex in (x, y) and $M(x, y)$ is affine, $V - M$ is also convex. This means that the move region, $\mathcal{M} \equiv \{(x, y) : V - M \leq 0 \text{ and } x \geq \tilde{x}\}$, is convex. On each x slice of the set, the set is $\{y : y \leq y_{Move}(x)\}$. A two-dimensional set whose slices are half lines of the form "$y \leq f(x)$" is convex if and only if f is concave – see Theorem 4.1 of Rockafellar (1970). Therefore, $y_{Move}(x)$ is concave.

A.3.2 $y'_{Move}(x) \to (y^{NR}_{Move})'(x) = 1$ as $x \to \infty$

The key to this argument is to introduce an "augmented No-Refi value function" in which the continuation value of moving is equal to $V(0, 0)$ from the full model. Given that construction, the difference $D^{aug} \equiv V - V^{NR-aug}$ comes exclusively from refinances prior to the first move.

To that end, consider an augmented version of the NR model's value function in which the continuation value from moving is $V(0, 0)$ rather than $V^{NR}(0, 0)$. This value function solves the Hamilton-Jacobi-Bellman equation:

$$\rho \cdot V^{NR-aug} = \mu_x \cdot V_x^{NR-aug} + \frac{\sigma_x^2}{2} \cdot V_{xx}^{NR-aug} + \frac{\sigma_m^2}{2} \cdot V_{yy}^{NR-aug} + \lambda \cdot \max\{0, M - V^{NR-aug}\}, \quad (24)$$

where M incorporates $V(0,0)$ as the continuation value of moving rather than $V^{NR}(0,0)$. Note that $D^{aug}(x,y) \equiv V(x,y) - V^{NR-aug}(x,y) \geq 0$ since the household in the full model could replicate the augmented-NR model's payoffs by never refinancing and so any refinancing activity it does do must be value-enhancing.

Now, define $\tau^* \equiv \inf\{t \geq 0 : x_t \leq \tilde{x}\}$. The household will not refinance before τ^*, by definition of $\tilde{x}$, so the difference between the full and the augmented-NR models does not come from $t < \tau^*$. Therefore, we have $D^{aug}(x,y) \leq E\left[e^{-\rho \cdot \tau^*} \cdot D^{aug}(x_{\tau^*}, y_{\tau^*})\right]$, since the first refinance must occur at τ^* or later, and refinances are the only source of difference between the value functions. Note that for the remainder of this argument, I will only focus on the case where $x > \tilde{x}$ since the claim being proved is about the limit as $x \to \infty$. Given that, we have $x_{\tau^*} = \tilde{x}$. Combining with the non-negativity of D^{aug}, we have:

$$0 \leq D^{aug}(x,y) \leq K \cdot E[e^{-\rho \cdot \tau^*}], \quad (25)$$

where $K \equiv \sup_{y \in \mathbb{R}} D^{aug}(\tilde{x}, y)$. I will now show that this implies $D^{aug}(x,y) \leq K \cdot e^{-\theta \cdot (x - \tilde{x})}$ for finite K and θ, with $\theta > 0$. This proceeds in two steps:

1. K is finite. *D^{aug} inherits the continuity of V and V^{NR-aug}, so it suffices to show that K is finite in both limits, $y \to \infty$ and $y \to -\infty$. The first is straightforward: as $y \to \infty$ both the moving and refinancing options lose their value, meaning both $V(\tilde{x}, y)$ and $V^{NR-aug}(\tilde{x}, y)$ both go to zero, as does D^{aug}. For $y \to -\infty$, the household will want to act at the first attention shock. Defining the time of that first attention shock as $\tau_\lambda \sim$*

$Exp(\lambda)$, we get $V(\tilde{x},y) \to -y/\rho \cdot \frac{\lambda}{\lambda+\rho} + E\Big[e^{-\rho\cdot\tau_\lambda} \cdot max\{V(0,0) + x_{\tau_\lambda}/\rho - C^M, V(x_{\tau_\lambda},0) - C^R\}\Big]$ and $V^{NR-aug}(\tilde{x},y) \to -y/\rho \cdot \frac{\lambda}{\lambda+\rho} + E\Big[e^{-\rho\cdot\tau_\lambda} \cdot \big(V(0,0) + x_{\tau_\lambda}/\rho - C^M\big)\Big]$. The expectation terms in both expressions do not depend on y, so as $y \to -\infty$, the leading terms dominate. Given that they are the same in both models ($-y/\rho \cdot \frac{\lambda}{\lambda+\rho}$), the difference goes to zero, so D^{aug} is bounded as $y \to -\infty$.

2. By the classic result for the first passage time of a Brownian motion, $E[e^{-\rho\cdot\tau^*}] = e^{-\theta\cdot(x-\tilde{x})}$, where θ is the positive root of $\frac{\sigma_x^2}{2} \cdot \theta^2 - \mu_x \cdot \theta - \rho = 0$.

As K and the θ have no dependence on y, this shows that $V(x,y) \to V^{NR-aug}(x,y)$ uniformly in y as $x \to \infty$, implying that they share a moving threshold: in the limit as $x \to \infty$, $y_{Move}(x) = y_{Move}^{NR-aug}(x)$.

Note that, by the same dimension-reduction argument as used for $y_{Move}^{NR}(x)$ in Proposition 2, it can be shown that $y_{Move}^{NR-aug}(x)$ is also linear with slope 1. So now define $l(x) = y_{Move}(x) - y_{Move}^{NR-aug}(x)$. Because the first term is concave and the second term is affine, $l(x)$ is concave. Since $l(x)$ is bounded as $x \to \infty$ (as just shown), that concavity requires $l'(x) \to 0$. Therefore, $y'_{Move}(x) \to (y_{Move}^{NR-aug})'(x) = 1 = (y_{Move}^{NR})'(x)$ as $x \to \infty$.

A.3.3 $y'_{Move}(x) \geq (y_{Move}^{NR}(x))' = 1$

Defining $g(x) = y_{Move}(x) - x$, we know g is concave (difference between concave and linear function) and $g'(x) \to 0$, as just shown at the end of the last subsection. Assume to a contradiction that there exists $\hat{x} > \tilde{x}$ such that $y'_{Move}(\hat{x}) < 1$. This would imply $g'(\hat{x}) < 0$, and concavity would imply $g'(x) < 0$ for all $x > \hat{x}$. But this would violate the convergence of $g'(x)$ to zero as $x \to \infty$. Therefore, $y'_{Move}(x) \geq 1$ for all $x > \tilde{x}$.

A.4 Proposition 4: There exists $\bar{x} < \infty$ such that $y_{Move}(x) > y_{Move}^{NR}(x)$ for all $x > \bar{x}$.

Return to the No-Refi (NR) and augmented NR value functions discussed in earlier proofs. Each of these models can be collapsed to one dimension, $z \equiv x - y$, as done in the proof of Proposition 2. In the inaction region, each of these models solves the Hamilton-Jacobi-Bellman equation:

$$\rho \cdot V^i = \mu_x \cdot V_z^i + \frac{\sigma_x^2 + \sigma_m^2}{2} \cdot V_{zz}^i, \tag{26}$$

where $i \in \{NR, NR - aug\}$. Such one-dimensional problems are studied in Agarwal *et al.* (2013) and Berger *et al.* (2025) and produce value functions of the form $V^i(z^{*i}) = C^i \cdot e^{\alpha^i \cdot z^{*i}}$. Note that α^i is the positive root of $\frac{\sigma_x^2+\sigma_m^2}{2} \cdot \alpha^{i2} + \mu_x \cdot \alpha^i - \rho = 0$. Since ρ, σ_x, and σ_m are shared by the two models, they share the value of α. Furthermore, smooth pasting implies that at the moving boundary, each value function's derivative with respect to z is equal to $1/\rho$. So for each model, we get:

$$1/\rho = C^i \cdot \alpha \cdot e^{\alpha \cdot z^{*i}} \Rightarrow \frac{1}{\rho \cdot \alpha} = V^i(z^{*i}). \tag{27}$$

In words, the two value functions are equal at their respective boundaries. I will invoke this below.

Value matching implies:

$$V^{NR-aug}(x, y_{Move}^{NR-aug}(x)) - V^{NR}(x, y_{Move}^{NR}(x)) = \frac{y_{Move}^{NR}(x) - y_{Move}^{NR-aug}(x)}{\rho} + D(0,0), \tag{28}$$

where $D(0,0) \equiv V(0,0) - V^{NR}(0,0)$. Note that $D(0,0) > 0$ because the full model has the option to refinance, which is valuable for finite y. Equation 27 implies that the lefthand side of Equation 28 is equal to zero, since the the two value functions are equal at their respective boundaries. As a result, that equation becomes: $y_{Move}^{NR-aug}(x) - y_{Move}^{NR}(x) = \rho \cdot D(0,0)$.

The proof of Proposition 3 already showed that $y_{Move}(x) - y_{Move}^{NR-aug}(x) \to 0$ as $x \to \infty$. So for any $\varepsilon > 0$, there is some $\hat{x}$ such that $|y_{Move}(x) - y_{Move}^{NR-aug}(x)| < \varepsilon$ for all $x > \hat{x}$. Since

$\rho \cdot D(0,0) > 0$, we can set $\epsilon = \rho \cdot D(0,0)$ and get:

$$y_{Move}(x) > y_{Move}^{NR-aug}(x) - \rho \cdot D(0,0) = y_{Move}^{NR}(x), \tag{29}$$

for all x above some $\bar{x}$.

A.5 Proposition 5: Let $V^{NM}(y)$ be the value function and y_{Refi}^{NM} be the refinancing threshold in the NM model. As $x \to -\infty$, $V(x,y) \to V^{NM}(y)$, and so y_{Refi}^{NM} is the threshold in that limit.

This argument proceeds in four steps:

1. Defining $V_\infty(y) \equiv \lim_{x \to -\infty} V(x,y)$, $V(x,y)$ converges to $V_\infty(y)$ uniformly on compact sets as $x \to -\infty$. *As argued in the proof of Proposition 1, $V_x \geq 0$. Furthermore, V is bounded below by zero since the household could simply take no action ever, guaranteeing a payoff of 0. Those two results imply that the limit $V_\infty(y)$ exists for all y. Now, note that $V_y \in [-1/\rho, 0]$, since the flow value of getting a new mortgage falls by $1/\rho$ per unit of y, and this bounds the impact on the change of overall value. So for any y, x, and any $\Delta > 0$, we have $-\frac{\Delta}{\rho} \leq V(x, y+\Delta) - V(x,y) \leq 0$. Taking the limit as $x \to -\infty$, both $V_\infty(y)$ and $V_\infty(y+\Delta)$ exist, as just shown, and $|V_\infty(y+\Delta) - V_\infty(y)| \leq \frac{\Delta}{\rho}$, meaning $V_\infty(y)$ is continuous in y. Finally, consider a sequence of functions of y, $V(x_n, y)$, where $x_n \to -\infty$. $V_\infty(y)$ and each $V(x_n,y)$ is continuous in y (by the slope bounds above) and $V_x \geq 0$ implies that $V(x_n, y)$ is non-increasing in n for each y. Therefore, Dini's Theorem (see Theorem 7.13 of Rudin (1976)) implies that the convergence of $V(x_n, y)$ to $V_\infty(y)$ is uniform on any compact set.*

2. $V_\infty(y)$ solves the No-Move Hamilton-Jacobi-Bellman equation. *The Hamilton-Jacobi-*

Bellman equation for $x < \tilde{x}$ is:

$$\rho \cdot V(x,y) = \mu_x \cdot V_x + \frac{\sigma_x^2}{2} \cdot V_{xx} + \frac{\sigma_m^2}{2} \cdot V_{yy} + \lambda \cdot \max\{0, -y/\rho - C^R + V(x,0) - V(x,y)\}, \quad (30)$$

where M has been removed from the max function because of the definition of $\tilde{x}$. Taking $x \to -\infty$, the uniform convergence result from the previous step allows us to pass the limit through the value function (see Lemma 6.2 of Fleming and Soner (2006)), yielding:

$$\rho \cdot V_\infty(y) = \frac{\sigma_m^2}{2} \cdot (V_\infty)_{yy} + \lambda \cdot \max\{0, -y/\rho - C^R + V_\infty(0) - V_\infty(y)\}, \quad (31)$$

which describes the refinancing problem without an option to move as in Berger et al. (2025). (The terms with V_x and V_{xx} drop because $V_\infty(y)$ does not depend on x.)

3. $V_\infty(y) = V^{NM}(y)$. *Because $V_\infty(y)$ is a solution of the same No-Move Hamilton-Jacobi-Bellman equation used to define $V^{NM}(y)$, we have $V_\infty(y) = V^{NM}(y)$ – see Theorem 12.4 of Oksendal and Sulem (2019).*

4. Because $V_\infty(y) = V^{NM}(y)$, they share a refinancing threshold.

A.6 Proposition 6: If $\mu > 0$, $y^{EM}_{Refi} < y^{NM}_{Refi}$.

The Exogenous-Move model is the the refinancing problem studied by Berger *et al.* (2025). In that model, the (scalar) optimal refinancing threshold, y^*, is characterized by the following three equations (see Appendix A of Abel (2024) for additional details on the derivation):

$$J_\psi = J_\eta + \frac{\lambda}{\rho + \mu + \lambda} \cdot \Big(J_\psi \cdot e^{\psi \cdot y^*} + C^{Refi} + \frac{y^*}{\rho + \mu} \Big), \quad (32)$$

$$-\psi \cdot J_\psi = \eta \cdot J_\eta + \frac{\lambda}{(\rho + \mu) \cdot (\rho + \mu + \lambda)}, \quad (33)$$

$$J_\psi = J_\psi \cdot e^{\psi \cdot y^*} + C^{Refi} + \frac{y^*}{\rho + \mu}, \tag{34}$$

where $\phi \equiv \frac{\sqrt{2 \cdot (\rho+\mu)}}{\sigma_m}$, $\eta \equiv \frac{\sqrt{2 \cdot (\rho+\mu+\lambda)}}{\sigma_m}$, and J_ψ and J_η are constants. Equation 32 is a value matching condition, Equation 33 is a smooth pasting condition, and Equation 34 ensures that an attentive household at y^* is indifferent between refinancing and waiting.

The proof of Proposition 6 proceeds in the three stages: (i) combine Equations 32-34 into a parsimonious single equation implicitly defining y^*; (ii) implicitly differentiate with respect to μ and show that $\frac{dy^*}{d\mu} < 0$ is equivalent to $F(s, u) < 0$, all of which terms will be defined subsequently; (iii) confirm $F(s, u) < 0$ in the relevant region.

To simplify the system of Equations 32-34, note that 32 and 34 imply $J_\eta = \frac{\rho+\mu}{\rho+\mu+\lambda} \cdot J_\psi$. Combining this with Equation 33 yields:

$$J_\psi = -\frac{\lambda}{(\rho + \mu) \cdot \big(\psi \cdot (\rho + \mu + \lambda) + \eta \cdot (\rho + \mu)\big)} \tag{35}$$

Now note that $\psi^2 \cdot \sigma_m^2 = 2 \cdot (\rho + \mu)$, $\eta^2 \cdot \sigma_m^2 = 2 \cdot (\rho + \mu + \lambda)$, and as a result $\lambda = \sigma_m^2 \cdot (\eta^2 - \psi^2)/2$. These results imply that $J_\psi = -\frac{2 \cdot \lambda}{(\rho+\mu) \cdot \sigma_m^2 \cdot \psi \cdot \eta \cdot (\eta + \psi)}$ and $\frac{\lambda}{\psi \cdot (\rho+\mu+\lambda) + \eta \cdot (\rho+\mu)} = \frac{1}{\psi} - \frac{1}{\eta}$. Therefore:

$$y^* = \left(\frac{1}{\eta} - \frac{1}{\psi}\right) \cdot \left(1 - e^{\psi \cdot y^*}\right) - (\rho + \mu) \cdot C^R. \tag{36}$$

It will be convenient to define $u \equiv \psi \cdot y^*$ and $s \equiv \psi/\eta \in (0, 1)$, in which case Equation 36 becomes:

$$u = (s - 1) \cdot (1 - e^u) - \psi \cdot (\rho + \mu) \cdot C^R. \tag{37}$$

Now, I begin work with the object of interest, $\frac{dy^*}{d\mu}$:

$$\frac{dy^*}{d\mu} = \frac{1}{\psi} \cdot \Big(\frac{du}{d\mu} - \frac{u}{2 \cdot (\rho + \mu)}\Big). \tag{38}$$

Since $\psi > 0$, I need to show that $\frac{du}{d\mu} < \frac{u}{2 \cdot (\rho+\mu)}$. I will now implicitly differentiate Equation 37 with respect to μ, but before doing so, note the following relationships:

1. $\frac{d\psi \cdot (\rho+\mu) \cdot C^R}{d\mu} = C^R \cdot \psi \cdot 3/2$;
2. $\frac{d(1-s)}{d\mu} = \frac{\eta^2 - \psi^2}{\sigma_m^2 \cdot \psi \cdot \eta^3}$.

With these in hand, differentiating Equation 37 with respect to μ yields:

$$\frac{du}{d\mu} \cdot \Big(1 - (1-s) \cdot e^u\Big) = \frac{(\eta^2 - \psi^2) \cdot (1 - e^u)}{\sigma_m^2 \cdot \psi \cdot \eta^3} - C^R \cdot \psi \cdot 3/2. \tag{39}$$

This means that $\frac{dy^*}{d\mu} < 0$ is equivalent to:

$$\frac{(\eta^2 - \psi^2) \cdot (1 - e^u)}{\sigma_m^2 \cdot \psi \cdot \eta^3} - C^R \cdot \psi \cdot 3/2 < \frac{u \cdot \Big(1 - (1-s) \cdot e^u\Big)}{2 \cdot (\rho + \mu)}, \tag{40}$$

since $1 - (1-s) \cdot e^u > 0$. Now, note that $\frac{\lambda \cdot \psi}{\eta \cdot (\rho+\mu+\lambda)} = s - s^3$. With that, condition 40 can be rewritten as:

$$(s - s^3) \cdot (1 - e^u) + 3 \cdot \big(u + (1-s) \cdot (1 - e^u)\big) < u \cdot \big(1 - (1-s) \cdot e^u\big), \tag{41}$$

implying that I must show:

$$2 \cdot u + (1 - e^u) \cdot (3 - 2 \cdot s - s^3) + u \cdot (1-s) \cdot e^u < 0. \tag{42}$$

So now define $F(u, s) \equiv 2 \cdot u + (1 - e^u) \cdot (3 - 2 \cdot s - s^3) + u \cdot (1-s) \cdot e^u$. I will now show that the inequality in expression 42 holds for all $u < 0$ and $s \in (0, 1)$. This is the relevant region, since $u = \psi \cdot y^*$ and $\psi > 0$ and $y^* < 0$, and $s = \frac{\rho+\mu}{\rho+\mu+\lambda}$ with ρ, μ, all λ strictly positive. To see this, first note that the first two derivatives of F with respect to u are:

1. $F_u(u, s) = 2 + e^u \cdot \Big((1 - s) \cdot (1 + u) - 3 + 2 \cdot s + s^3\Big)$
2. $F_{uu}(u, s) = e^u \cdot \Big((1 - s) \cdot (2 + u) - 3 + 2 \cdot s + s^3\Big)$

As a result, $F(0, s) = 0$ and $F_u(0, s) = s^3 + s > 0$. Finally, note that $3 - 2 \cdot s - s^3 > 2 \cdot (1 - s) > (2 + u) \cdot (1 - s)$, where the first inequality follows from $s \in (0, 1)$ and the second comes from that fact and $u < 0$. As a result, we have $F_{uu} < 0$ because $e^u > 0$.

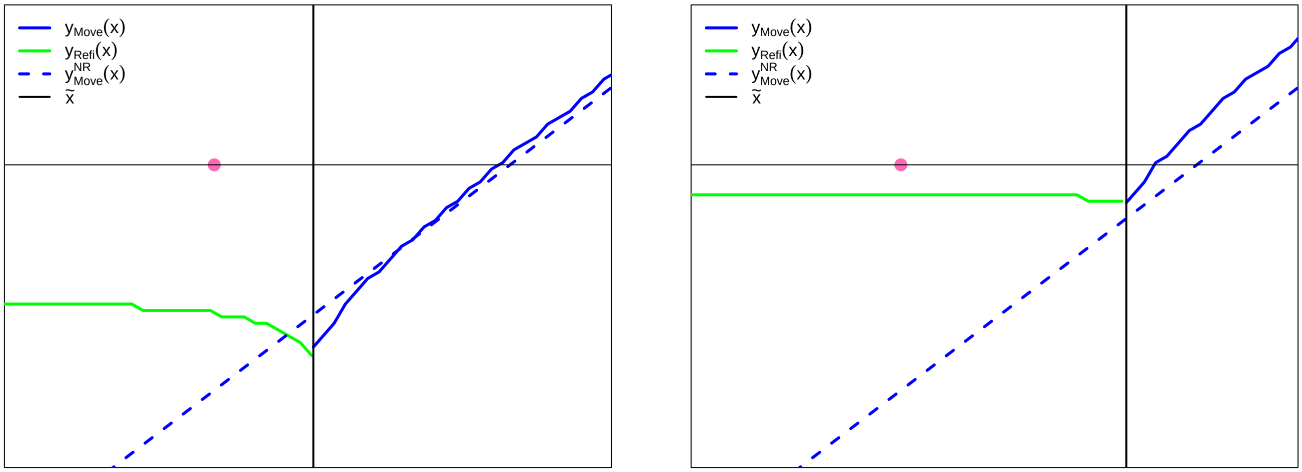
The combination of $F(0, s) = 0$, $F_u(0, s) > 0$, and $F_{uu}(u, s) < 0$ for all $u < 0$ means that F is increasing for all $u < 0$ until it reaches zero at $u = 0$, which means $F < 0$ for all $u < 0$. That confirms that expression 42 holds, meaning $\frac{dy^*}{d\mu} < 0$, proving the proposition.

A.7 Region B of Figure 1

As discussed in Section 2.3, there is the potential for the introduction of the refinancing option to create an inaction region for some $x > \tilde{x}$ – in other words, an interval of x above $\tilde{x}$ where $y_{Move}(x) < y^{NR}_{Move}(x)$, creating a region where an attentive household moves in the NR model but not in the full model. This is shown as region B in Figure 1. Section 2.3 asserts that such a region may or may not exist, depending on parameters. This is demonstrated by Figure 13. Figure 13a shows a parameterization in which $y^{NR}_{Move}(\tilde{x}) > y_{Move}(\tilde{x})$, which creates such a region B. On the other hand, in Figure 13b there is no region B because $y^{NR}_{Move}(\tilde{x}) < y_{Move}(\tilde{x})$ and, as established in Proposition 3, the latter function has a (weakly) larger slope, implying that the two curves never intersect.

A.8 On the Difficulty of Characterizing $y_{Refi}(x)$

Section 2.3 is able to characterize more features of $y_{Move}(x)$ – and how it is affected by the option to refinance – compared $y_{Refi}(x)$ and how it is affect by the option to move. Most



(a) $C^R = 0.05$: region B exists.

(b) $C^R = 0.05$: region B does not exist.

Figure 13: Action thresholds for the full model and the NR model under different parameters. Both models share the following parameters: $\rho = 0.05$, $\lambda = 0.5$, $C^M = 0.09$, $\mu_x = 0$, $\sigma_x = \sigma_m = 0.0017$. In the full model, an attentive household moves when below the solid blue line and refinances when below the green line; in the NR model, an attentive household moves when below the dashed blue line. The vertical black line indicates $\tilde{x}$, which separates the values of x for which moving is preferred to refinancing from the values of x for which the reverse is true. The pink dot represents (0,0), the state that a household jumps to after a move. A household jumps vertically to the horizontal black line after a refinance. The rules were solved for via value function iteration.

concretely, the only theoretical result proven for $y_{Refi}(x)$ concerns its level in the limit as $x \to -\infty$. In contrast, Proposition 3 establishes the concavity of $y_{Move}(x)$ and Proposition 4 ensures that it lies above $y^{NR}_{Move}(x)$ for large but finite x.

The source of this disparity is that the continuation value of moving, $V(0,0)$ is a constant that does not depend on the current state, whereas the continuation value of refinancing is $V(x,0)$ and so does. To see why this is problematic, recall from Section A.3 that the proof of the concavity of $y_{Move}(x)$ used the fact that $V - M$ was convex, since V is convex and $M \equiv \frac{x-y}{\rho} - C^M + V(0,0)$ is affine. However, $V - R$ is not necessarily convex because $R(x,y) = -\frac{y}{\rho} - C^R + V(x,0)$ is in fact convex in x, and the difference in convex functions is not generally convex or concave.

B Calibration Procedure

This appendix provides additional detail on how the calibration of the parameters μ_x, σ_x, and p was performed. As mentioned in Section 3.3, this was done via grid search over triples of those parameters. The process of running simulations and how that output was evaluated was discussed in the main text, so here I discuss the grid that was searched. I used an iterative process in which a relatively large portion of the parameter space was searched in a coarse manner, and then a smaller subsection around the best-performing triple was searched more finely. I did two iterations.

Parameter values for the first iteration were as follows:

- μ_x: from $2 \cdot 10^{-5}$ to $1 \cdot 10^{-4}$ in even increments of $2 \cdot 10^{-5}$;
- σ_x (annualized): from 0.005 to 0.025 in even increments of 0.005;
- p: from 3.5% to 7.5% in even increments of 1pp.

For each of the 125 triples available for those parameter values, I solved Equation 10 and then simulated behavior as described in Section 3.3. The combination that best matched the moments in Table 2 was ($\mu_x = 8 \cdot 10^{-5}$, $\sigma_x = 0.0125$, $p = 0.065$). I then explored a finer grid around this set of parameter values:

- μ_x: from $6 \cdot 10^{-5}$ to $1 \cdot 10^{-4}$ in even increments of $1 \cdot 10^{-5}$;
- σ_x: from 0.01 to 0.02 in even increments of 0.0025;
- p: from 5.5% to 7.5% in even increments of 0.5pp.

The combination that best matched the moments in Table 2 is the one shown in Table 1: ($\mu_x = 8 \cdot 10^{-5}$, $\sigma_x = 0.0125$, $p = 0.055$), though Table 1 shows the volatility of x at the monthly level ($0.0125/\sqrt{12}$).

An identical procedure was used to calibrate the model with a high discount rate in Section 4.1. For the this model, the range for p in the initial coarse grid had to be extended up to 0.155 because the initial approach resulted in a corner solution.

For the entirety of the analysis, y was discretized as into 81 equally spaced points between (and including) -0.1 and 0.1. x was discretized into 81 points as well, though its grid was not equally spaced and also depended on σ_x. Specifically, 41 points equally spaced between (and including) $-3 \cdot \sigma_x$ and $3 \cdot \sigma_x$; then 20 points were more coarsely placed above $3 \cdot \sigma_x$ at evenly-spaced intervals until $35 \cdot \sigma_x$; and similarly, 20 points were placed below $-3 \cdot \sigma_x$ at evenly-spaced intervals until $-35 \cdot \sigma_x$.