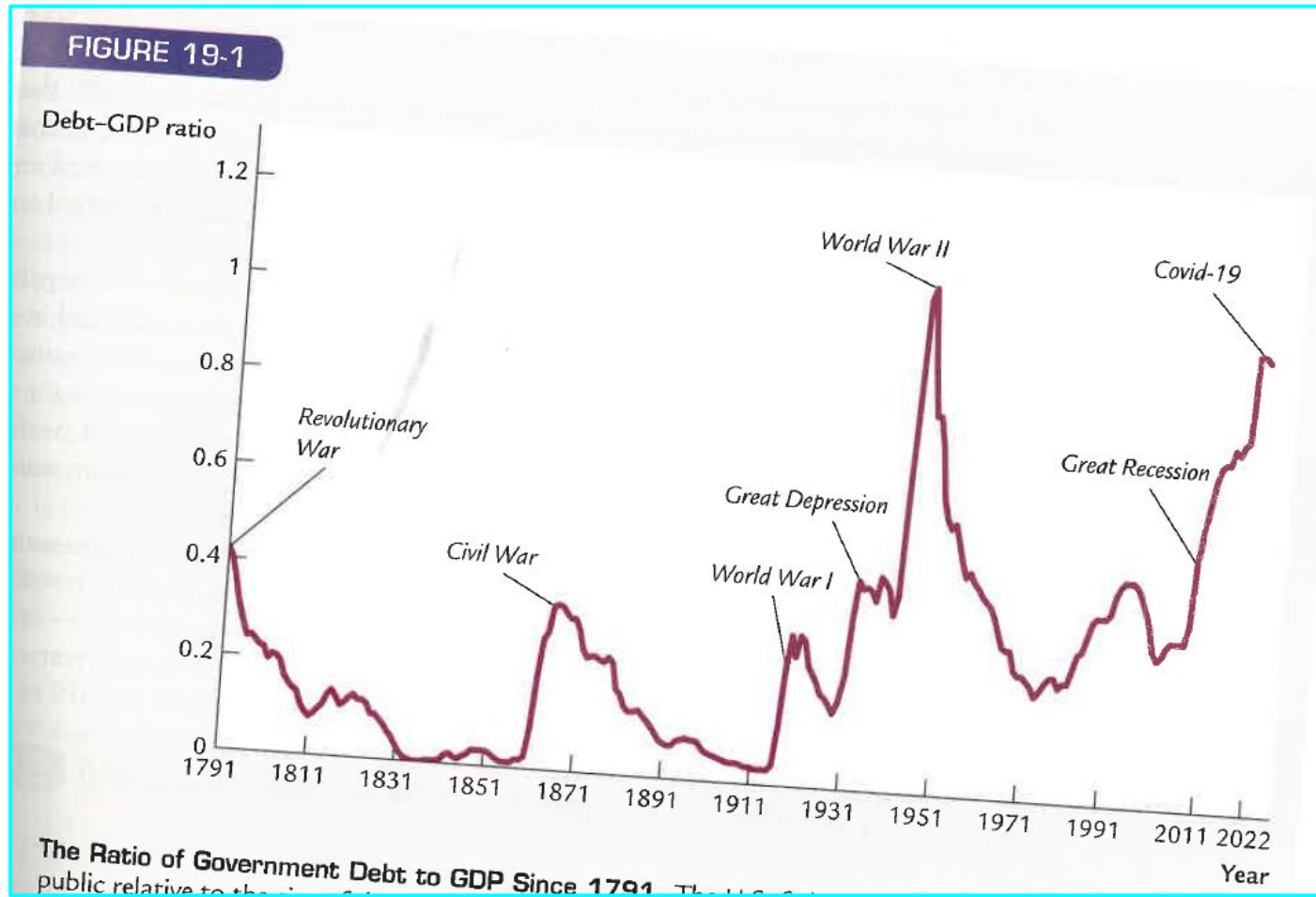


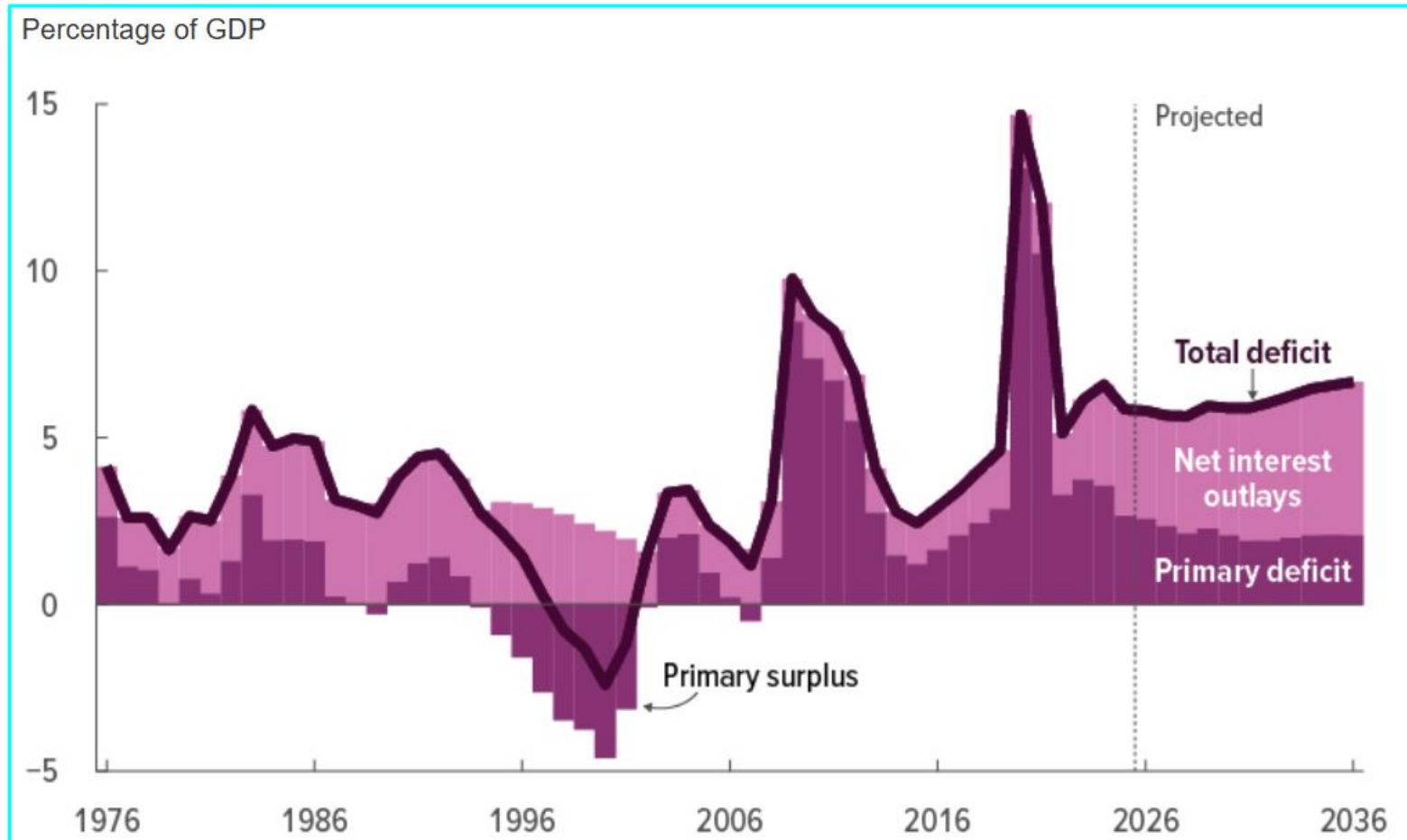
Government Debt

Spring 2026
Econ 256, College of the Holy Cross
Prof. Josh Abel

US Federal Government Debt Relative to GDP



US Federal Government Deficit Relative to GDP



Debt's Law of Motion

- Define:
 - D_t : debt at end of period t
 - $d_t = G_t - T_t$: primary deficit in period t
 - i_t^f : interest rate on government debt in period t

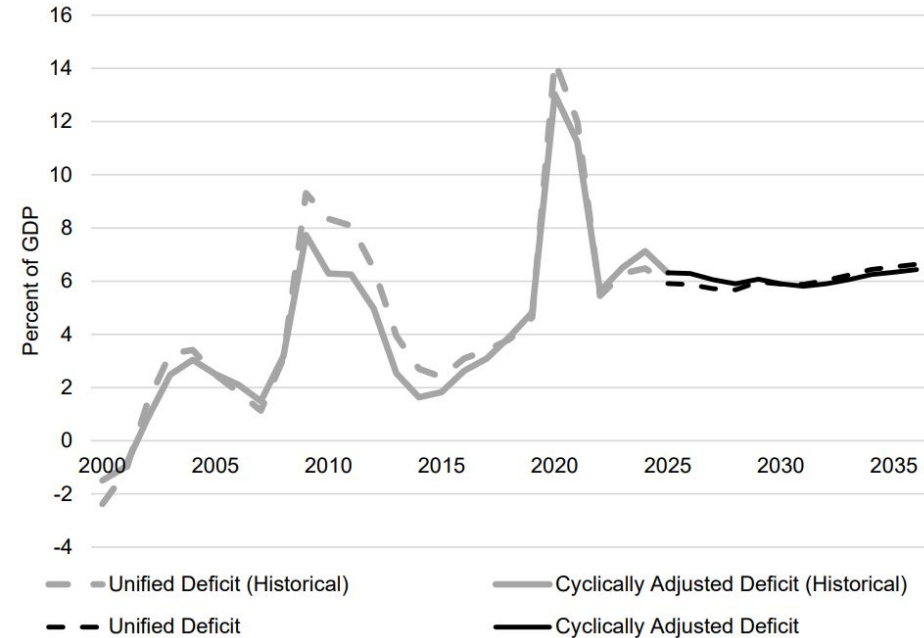
$$D_t - D_{t-1} = d_t + i_t^f \cdot D_{t-1}$$

- Keep in mind:
 - Debts are stocks, deficits are flows
 - Total deficit, $d_t + i_t^f \cdot D_{t-1}$, includes interest payments on the debt
 - Primary deficit excludes interest payments

Adjusting the Deficit for the Business Cycle

FIGURE 12

Cyclically Adjusted and Unified Deficit, 2000 – 2036



- Deficits are worsened by recessions
 1. Automatic stabilizers
 2. Discretionary policy
- Common to remove impact of automatic stabilizers → “cyclically-adjusted deficit”
- But even so, no adjustment for discretionary policy, typically
- Takeaway: can’t really compare deficits at different points in the business cycle

Debt-to-GDP and Debt Sustainability

- Common to normalize debt by GDP: D_t/Y_t . Logic:
 - Debt ultimately gets paid off by taxing
 - Modern governments raise most revenue by taxing income flows
 - So D/Y compares debt to the tax base that would be used to pay it off
 - If wealth taxation were our main source of revenue, D/K would be a more natural measure
- Governments can roll over debt and so have a much longer horizon than you or I
- Common to assess “sustainability” of government by finances by looking at D_t/Y_t as $t \rightarrow \infty$
 - Explodes: unsustainable
 - If creditors in the distant future stop rolling debt over, you’ll need to default
 - Converges to 0: sustainable
 - If creditors in the distant future stop rolling debt over, you can pay them back with a tiny tax
 - Converges to $c > 0$: debatable
 - If creditors in the distant future stop rolling debt over, you can pay them back with a “non-tiny” tax

Debt Dynamics with $i^f > g$

- Define g to be the economy's growth rate, so $Y_t = Y_0 \cdot (1+g)^t$
- Suppose $D_{-1} = 0$ and then you run a deficit $d_0 > 0$ to fight a war or something
- If you go back to running balanced budgets after that ($d_t = 0$ for $t > 0$), you will have:

$$\frac{D_t}{Y_t} = \frac{d_0 \cdot (1 + i^f)^t}{Y_0 \cdot (1 + g)^t} = \frac{d_0}{Y_0} \cdot \left(\frac{1 + i^f}{1 + g} \right)^t$$

- So if $i^f > g$, debt will explode relative to output
 - Intuitive: you borrowed (primary deficit) so you need to pay it back at some point
- To make this plan sustainable, you need to run a primary surplus at some point to pay off debt. E.g.:
 - $d_1 = -(1+i^f) \cdot d_0$
 - or $d_2 = -(1+i^f)^2 \cdot d_0$
 - ...
 - or $d_t = -(1+i^f)^t \cdot d_0$
 - or some combination of these

Debt Dynamics with $i^f < g$

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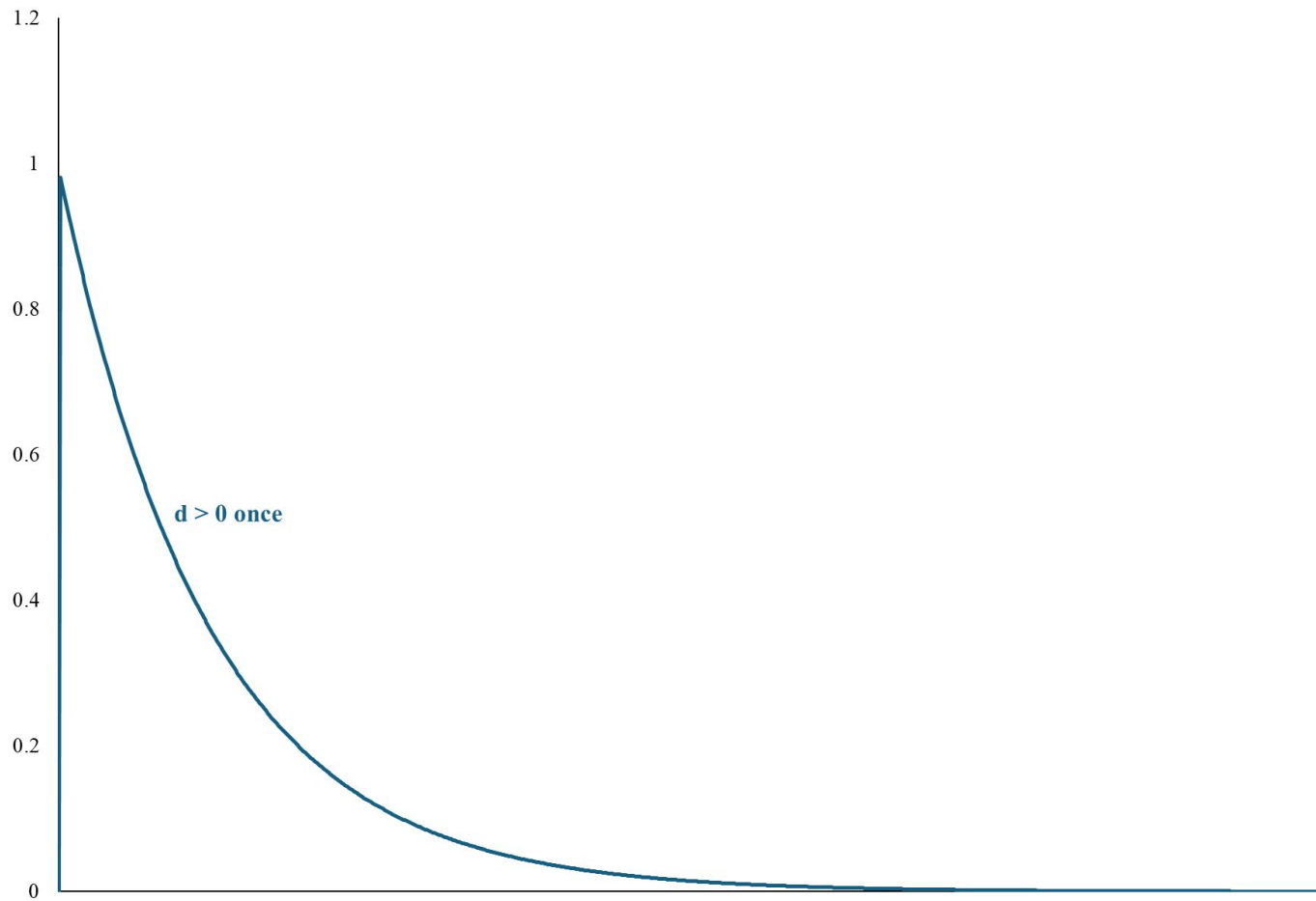
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- So if $i < g$, debt will **collapse** relative to output
 - ~~Intuitive: you borrowed (primary deficit) so you need to pay it back at some point~~
- ~~To make~~ this plan **is** sustainable, you **do not** need to run a primary surplus at ~~some any~~ point.
- **Because of the long time horizon, we can “grow out of our debt”!**
 - Maybe growth from AI will solve our debt concerns?

Debt Dynamics with $i^f < g$

Debt-to-GDP Ratio



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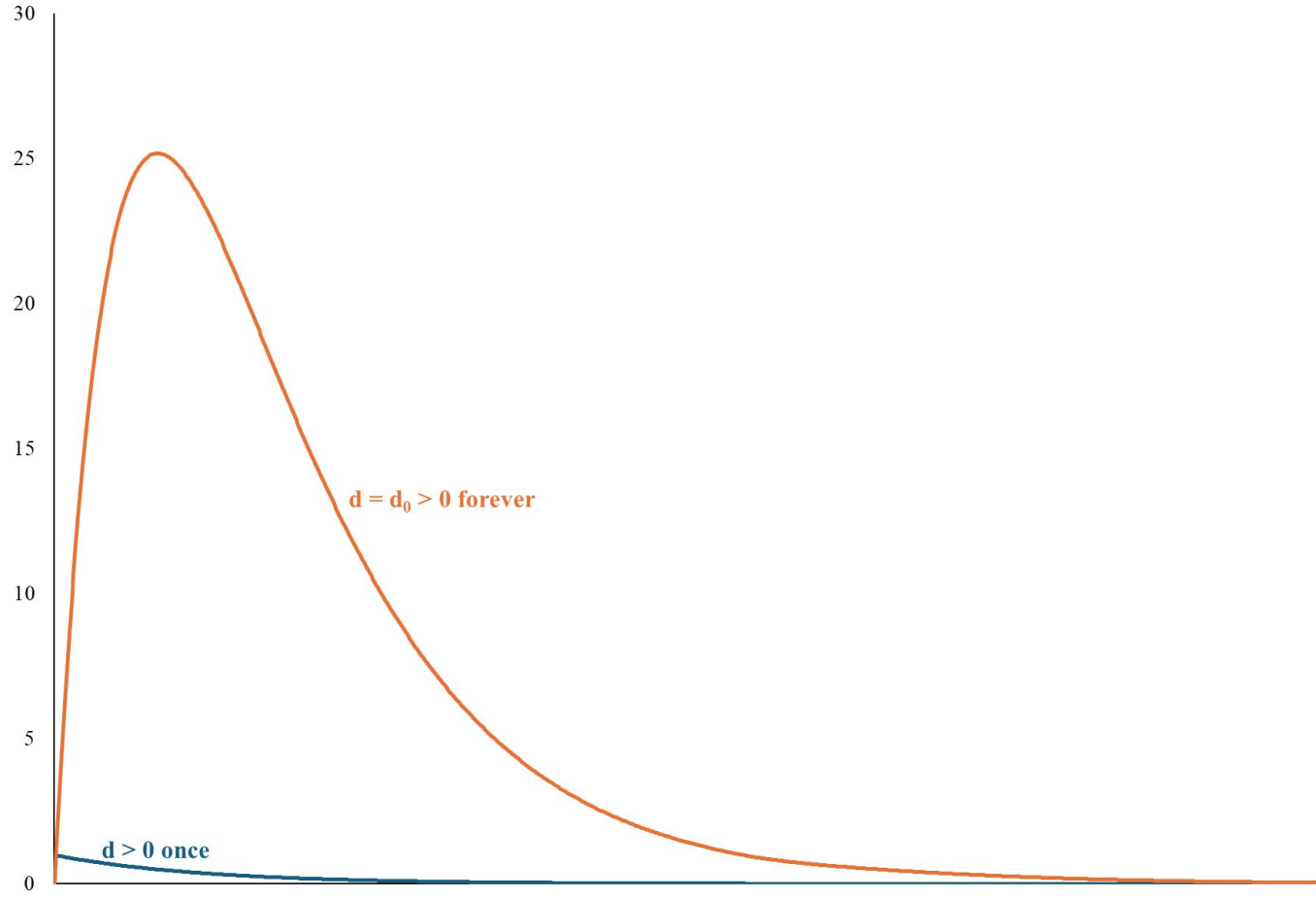
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- If you ~~go back to running balanced budgets after that ($d_t = 0$ for $t > 0$)~~ keep $d_t = d_0 > 0$ forever, you will have:

$$\frac{D_t}{Y_t} = \frac{d_0 \cdot (1 + i^f)^t}{Y_0 \cdot (1 + g)^t} + \frac{d_0 \cdot (1 + i^f)^{t-1}}{Y_0 \cdot (1 + g)^t} + \frac{d_0 \cdot (1 + i^f)^{t-2}}{Y_0 \cdot (1 + g)^t} + \dots$$

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 2. Not taking on new debts

Debt Dynamics with $i^f < g$

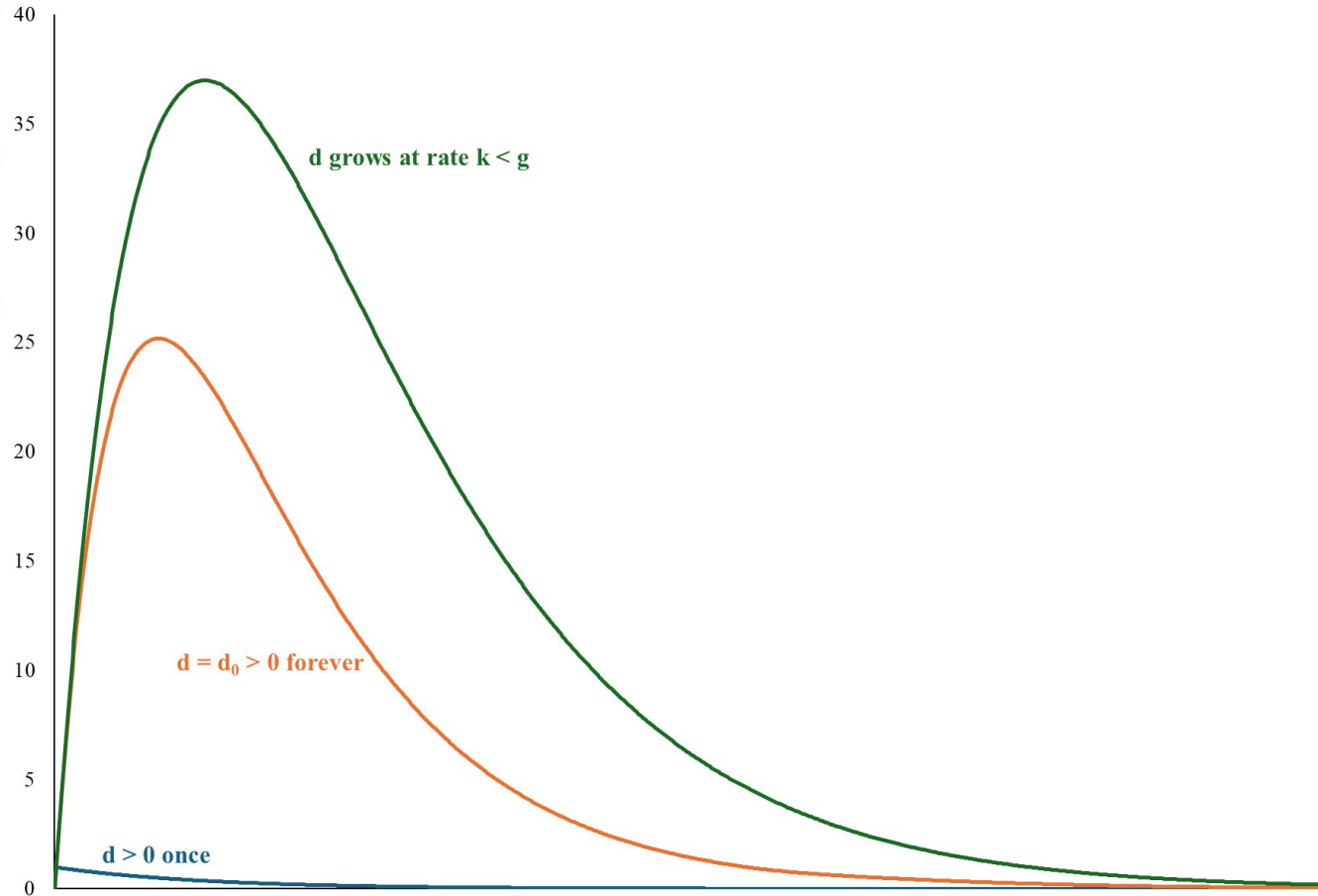
- Define g to be the economy's growth rate, so $Y_t = Y_0 \cdot (1+g)^t$
- Suppose $D_{-1} = 0$ and then you run a deficit $d_0 > 0$ to fight a war or something
- If you ~~keep $d_t = d_0 > 0$~~ let the primary deficit grow at rate $k < g$ forever, you will have:

$$\frac{D_t}{Y_t} = \frac{d_0 \cdot (1 + i^f)^t}{Y_0 \cdot (1 + g)^t} + (1 + k) \cdot \frac{d_0 \cdot (1 + i^f)^{t-1}}{Y_0 \cdot (1 + g)^t} + (1 + k)^2 \cdot \frac{d_0 \cdot (1 + i^f)^{t-2}}{Y_0 \cdot (1 + g)^t} + \dots$$

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- Because of the long time horizon, we can “grow out of our debt”!
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- With $i^f < g$, debt sustainability does not require:
 1. Paying back any debt
 2. Not taking on new debts
 3. Not taking on ever-growing new debts
 - It just requires that if you take on ever-growing new debts, their exponential growth rate be less than g

Takeaway: i^f vs g is of first-order importance for thinking about debt

- The model is clearly too simple:
 - Assumes i^f and g are constants known with certainty
 - Ignores “endogeneity” of i^f
 - D/Y is large, lenders may require a high interest rate – can lead to vicious cycles
- Nonetheless, the deeper point about growing out of debt stands
 - If i^f is small relative to g , the dynamics of debt are drastically – qualitatively! – different than if i^f is large
 - Large i^f : fiscal “discipline” requires a plan to pay off the debt
 - Small i^f : fiscal “discipline” requires deficits to just not grow too explosively

Evidence on i^f vs g

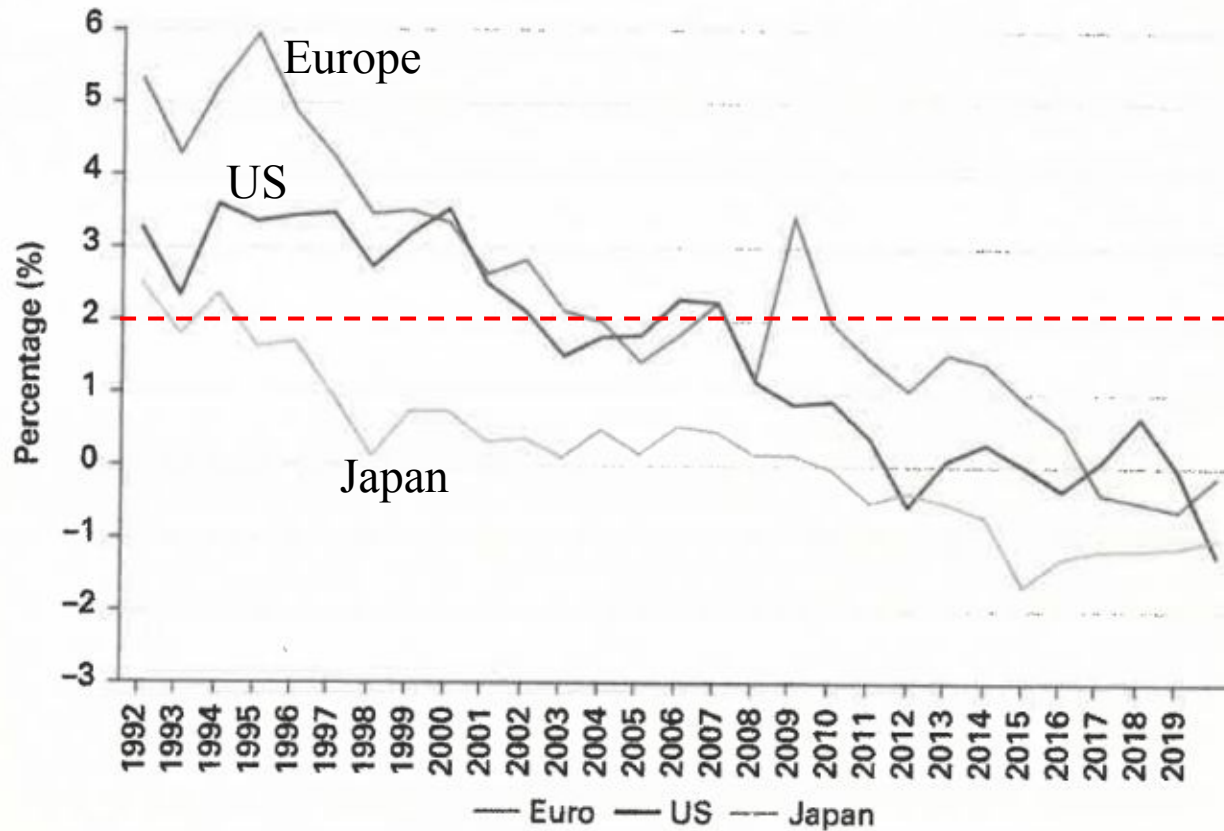
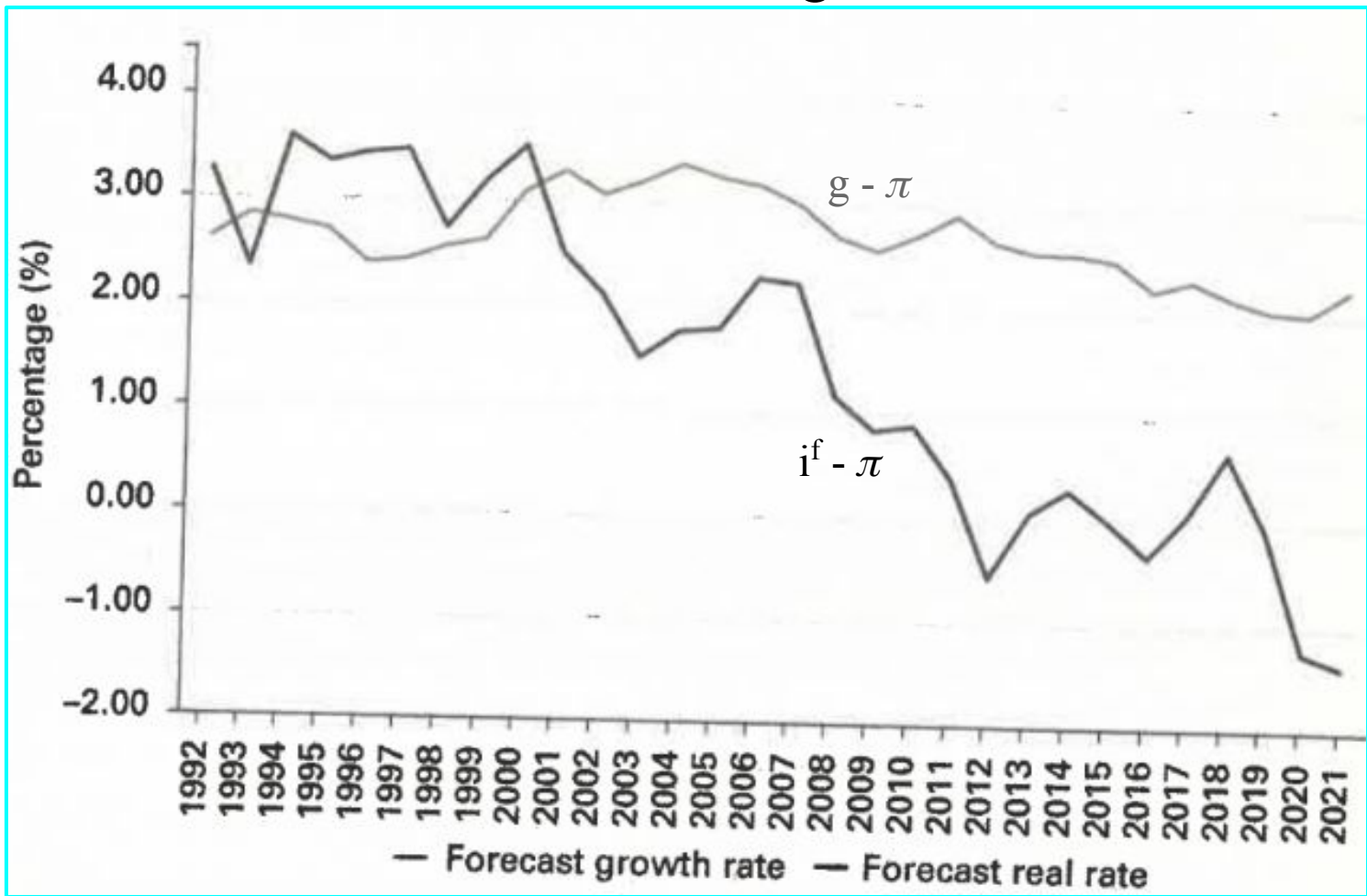


Figure 3.1

US, Euro, Japan 10-year real rates, 1992–2020.

Evidence on i^f vs g



Evidence on i^f vs g

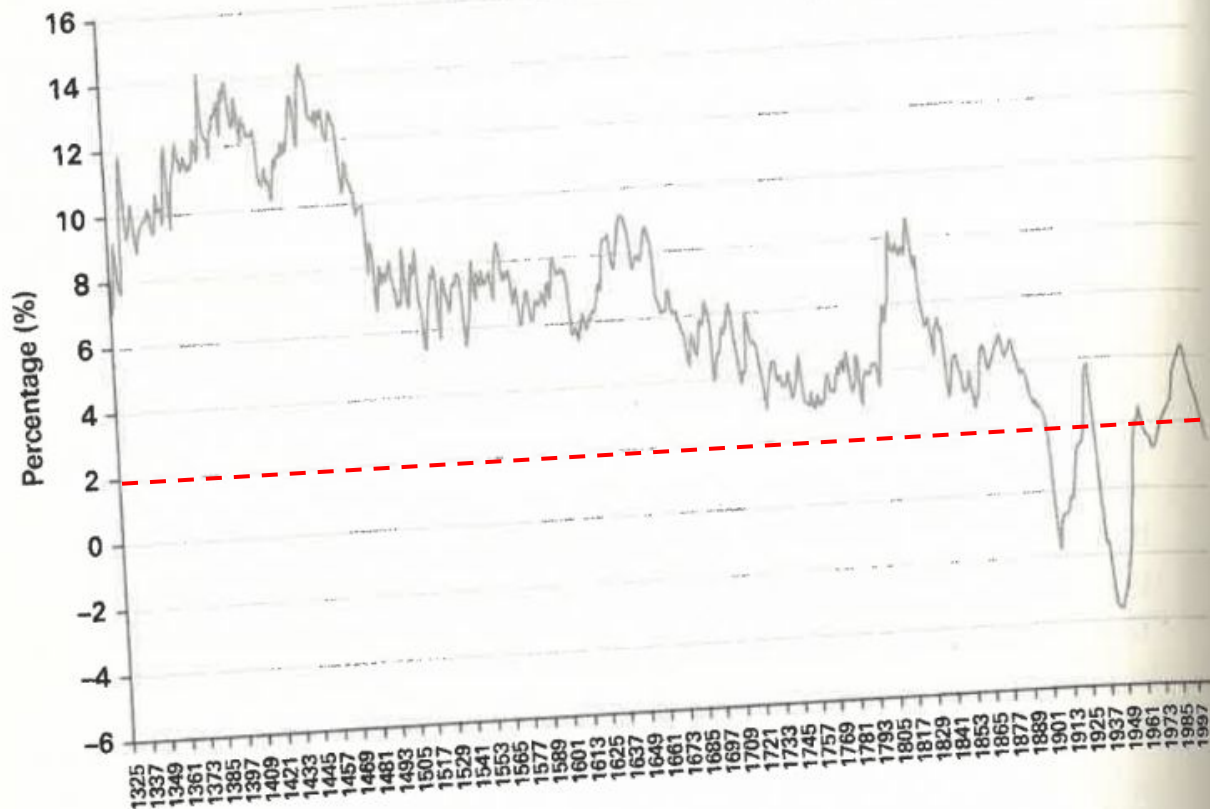


Figure 3.2

Safe real rate since 1325.

Source: Schmelzing 2020.

Inflation's Role

- We've ignored inflation, but almost all debt is denominated in dollars, so we should really normalize by nominal GDP:

$$D_t / (P_t \cdot Y_t)$$

- If we have a big burst of inflation, P spikes and debt-to-(nominal-)GDP falls.
 - So you could then get debt very low without running a primary surplus, even if $i^f > g$
 - You could then pay off the debt with a small tax increase
 - This is often referred to as “inflating away” or “monetizing” the debt
 - This is effectively defaulting, but not in a legal/technical sense
- Is it a good idea?

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 - This is effectively defaulting, but not in a legal/technical sense
- Is it a good idea?
 - It really would solve the problem...once.
 - If you do that, people will then demand higher interest rates in the future (think Fisher Equation...) to compensate them for the risk of inflation
 - So it could solve your current stock of debt, but it will make the interest payments and future borrowing more expensive
 - Plus, it requires your economy to deal with inflation, which people hate

Debt, Inflation, and Fed Independence

- This is another argument people give in favor of central bank independence
 - Earlier, we saw how Fed independence can help us avoid a “political business cycle”
- Similarly, people worry that if fiscal policymakers controlled monetary policy, they would find it too enticing to inflate the debt away so they wouldn’t have to make tough debt/spending decisions.
- If people anticipate this – which they likely would – it would lead to higher interest rates immediately and be essentially self-defeating.
- It seems very likely that the Fed will come under a lot of pressure in coming years/decades to help inflate the debt away.