Game Theory

Fall 2023
Econ 2316, Northeastern University
Prof. Josh Abel

P&R: chapter 13 (especially 13.1-13.7)
Emerson: chapter 17

Introduction

- Game Theory is the formal study of <u>strategic interactions</u>
    - My behavior affects you, yours affects me
    - So we need to anticipate each other’s actions and plan accordingly
- Game Theory applies to many settings outside of standard economic markets
    - Social interactions (e.g. how to choose where to meet up with friends)
    - Geopolitics (e.g. nuclear arms race)
    - Sports (e.g. penalty kicks)
- While it is applied in many vastly different settings, Game Theory is unified by a set of tools and solution concepts

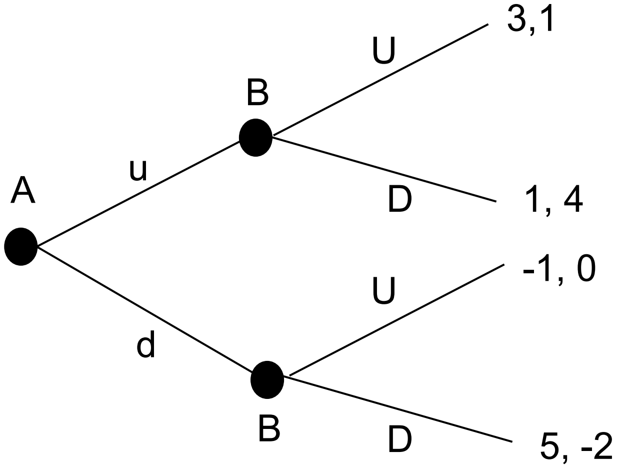
Introduction (2)

- Because players have to anticipate each other’s actions, strategic situations can be quite complex
- Goal of Game Theory is to find a reasonable, predictable outcome
  - But there might not be one!
- Broadly speaking, we will focus on two sets of games
  - Sequential (players move one after another)
  - Simultaneous (players move at the same time)

Sequential Games

Sequential games

- Sequential are often analyzed with a “game tree”



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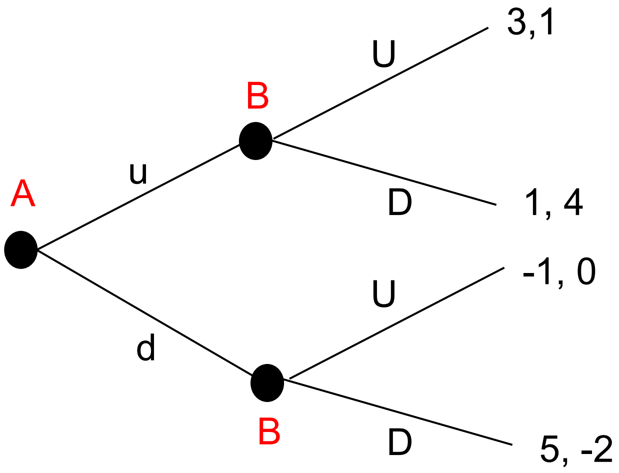

There are 2 players,
A and B

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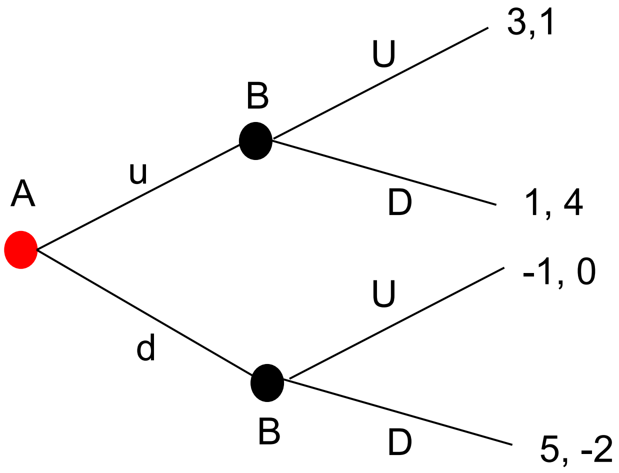

The game starts at player A's decision node

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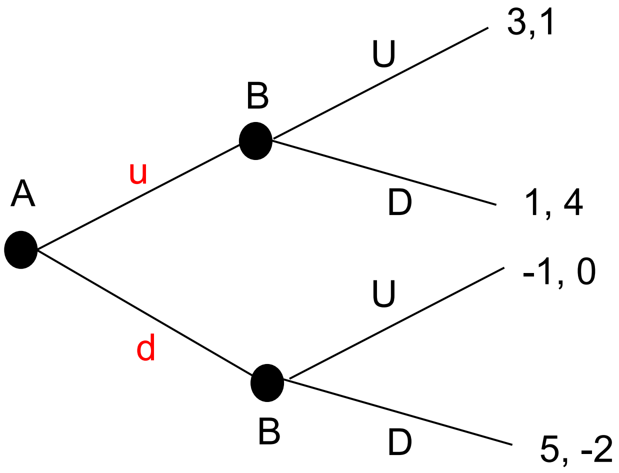

Player A has to choose between 2 actions, u and d, at that node

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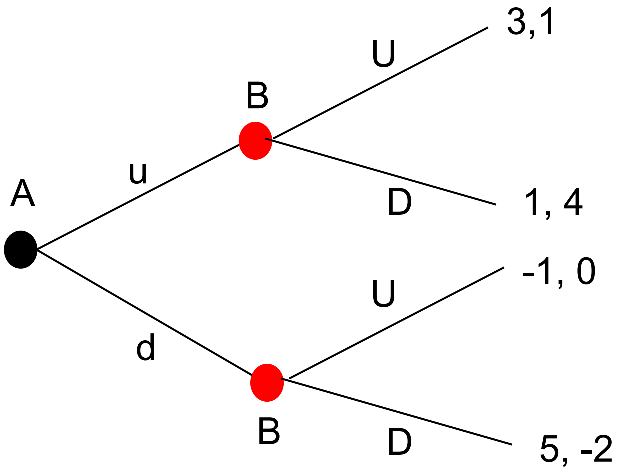

Depending on which action player A chose, the game moves to one of player B’s decision nodes

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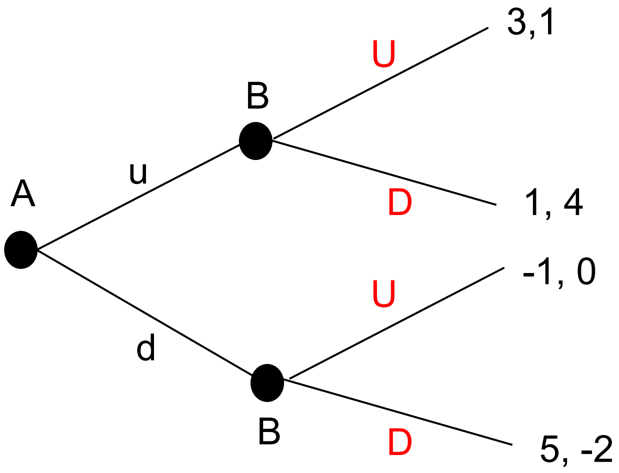

Player B has 2 actions to choose between at each of her decision nodes: U and D

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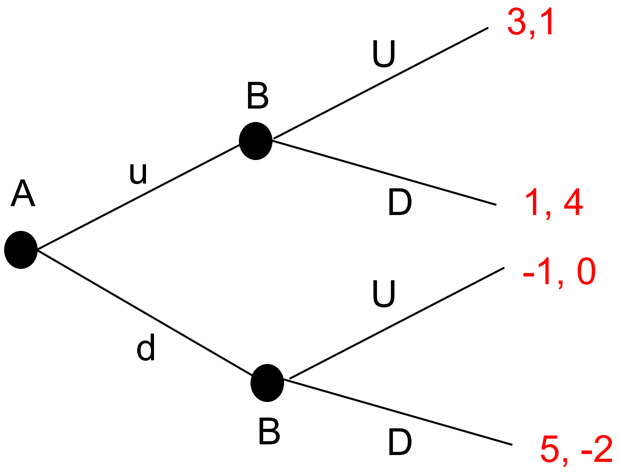

The game ends, and the players receive the payoffs corresponding to the actions chosen during the game

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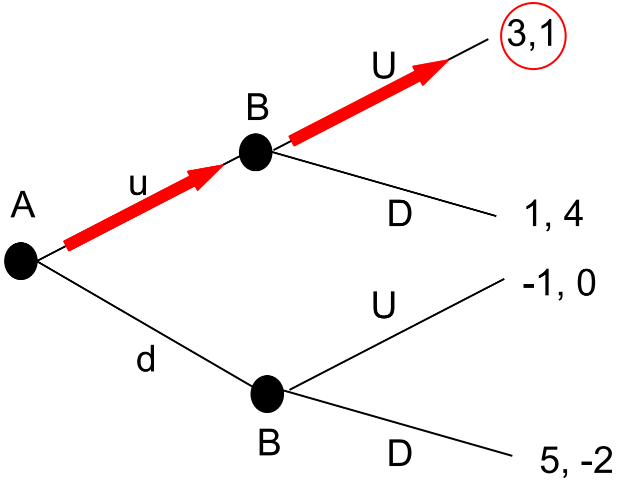

If player A chooses “u” and player B chooses “U” then player A receives a payoff of 3 and player B gets 1

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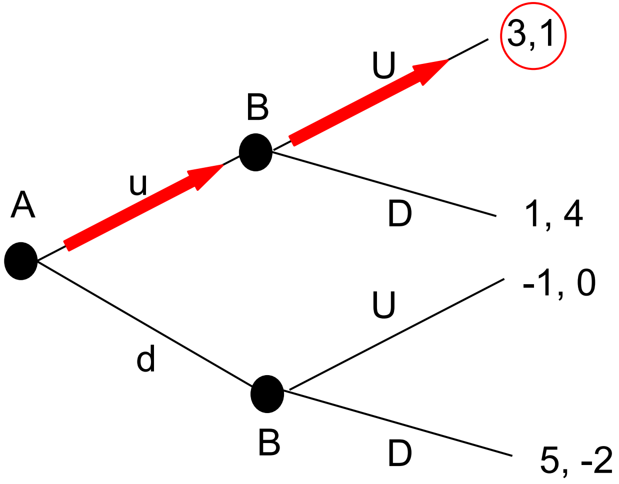
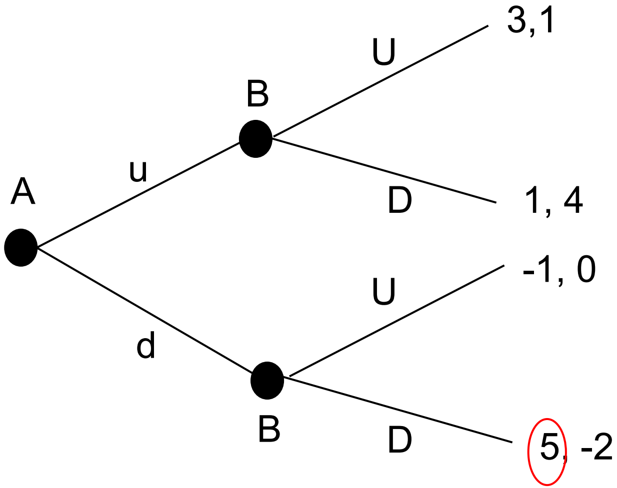


If player A chooses "u" and player B chooses "U" then player A receives a payoff of 3 and player B gets 1

- If the players are rational, is this the outcome we would expect?

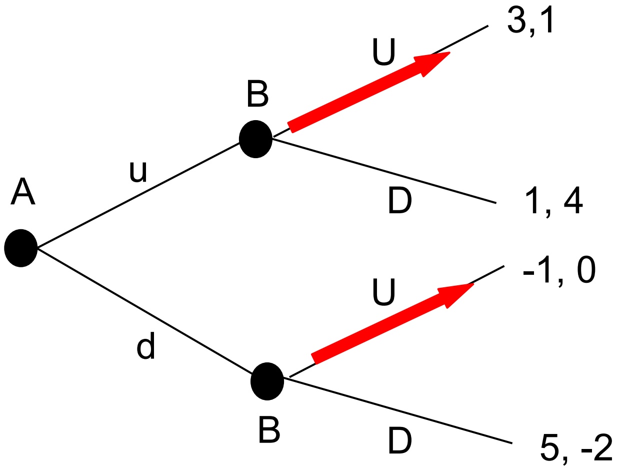
Thinking through A's choice

- How can A know what to do if he doesn't know what B will do?



Thinking through A's choice

- How can A know what to do if he doesn't know what B will do?
- If he knew B would choose U…



Thinking through A's choice

- How can A know what to do if he doesn't know what B will do?
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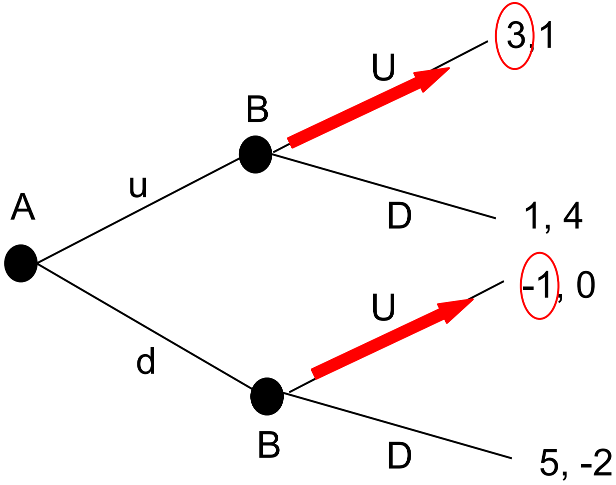
- …he'd be choosing between payoffs of 3 and -1…



Thinking through A’s choice

- How can A know what to do if he doesn’t know what B will do?
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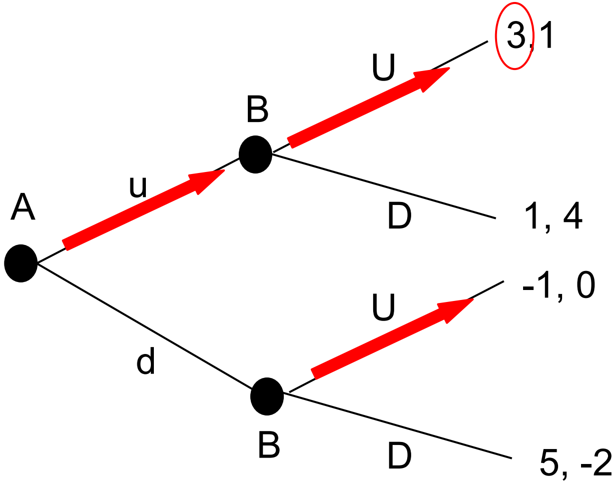

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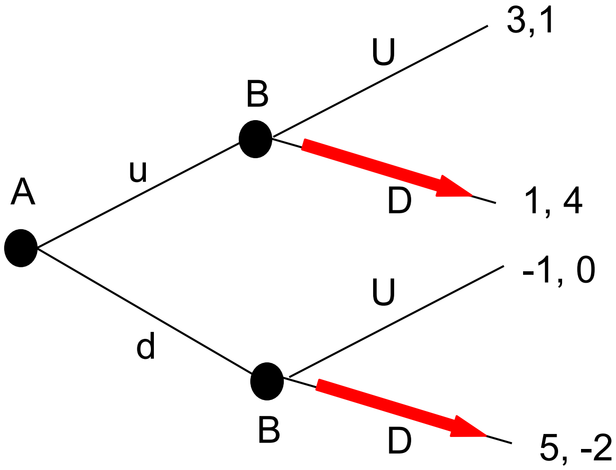
- But if he knew B would choose D…



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- If he knew B would choose U…
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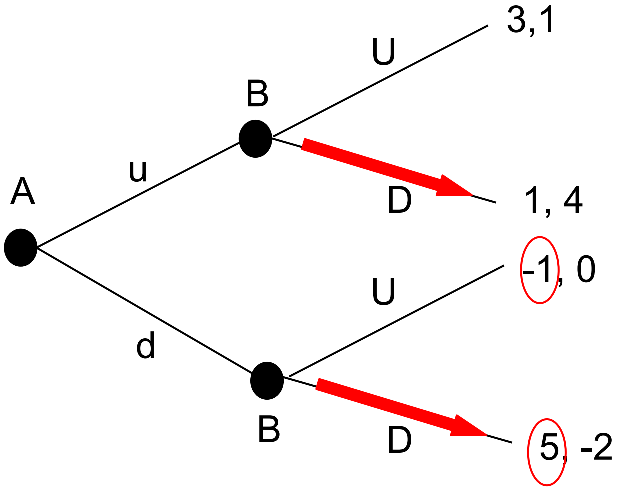
- …so he'd choose u!
- But if he knew B would choose D…
- …he'd be choosing between payoffs of 1 and 5…



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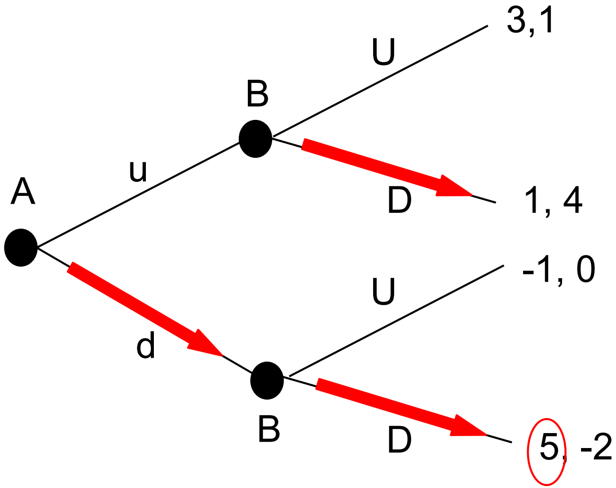

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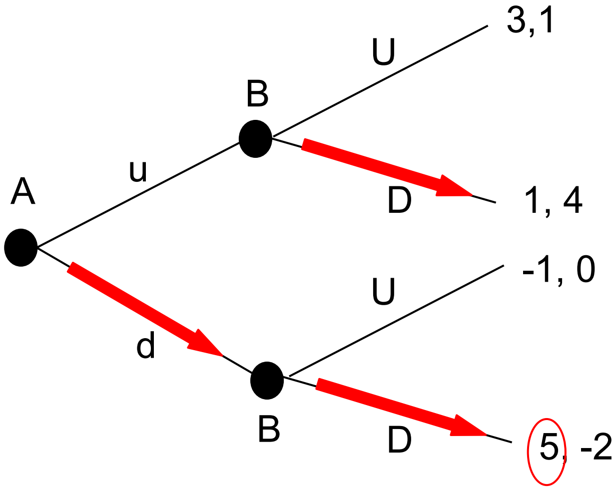
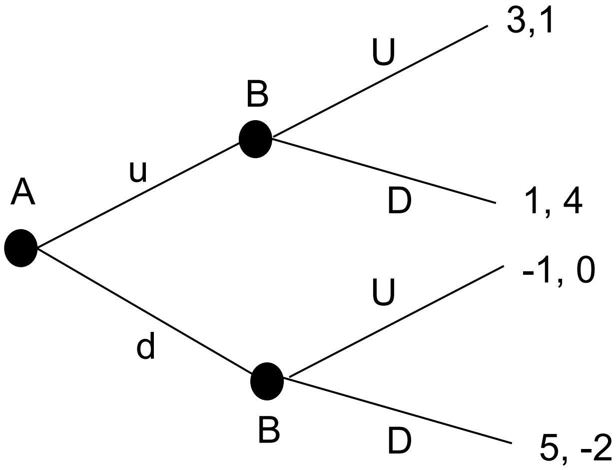


So without understanding B’s choice, we cannot say much about A’s choice.

Thinking through B's choices

- B's decisions are simpler than A's
  - Because B ends the game, she doesn't have to anticipate any of A's actions



Thinking through B's choices

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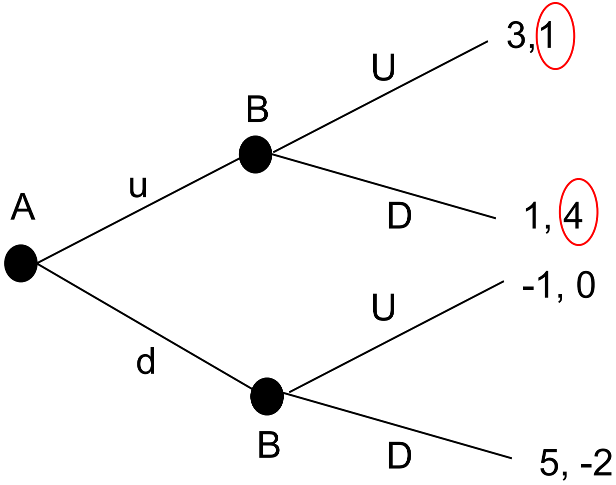
- If B is at her upper node, she gets 1 or 4



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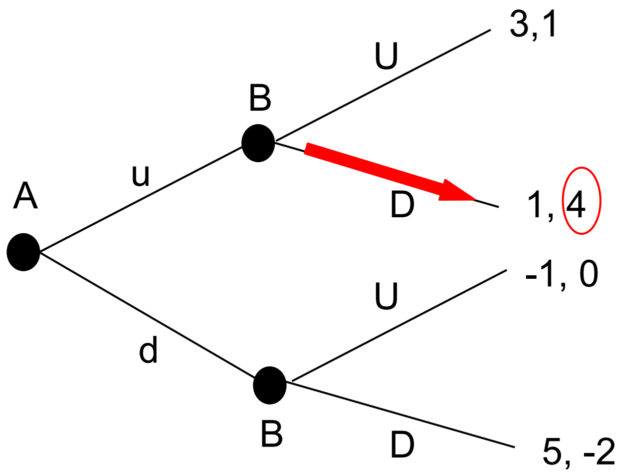

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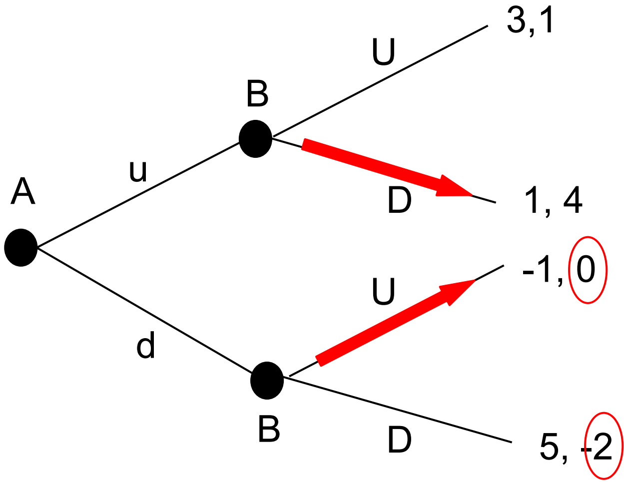

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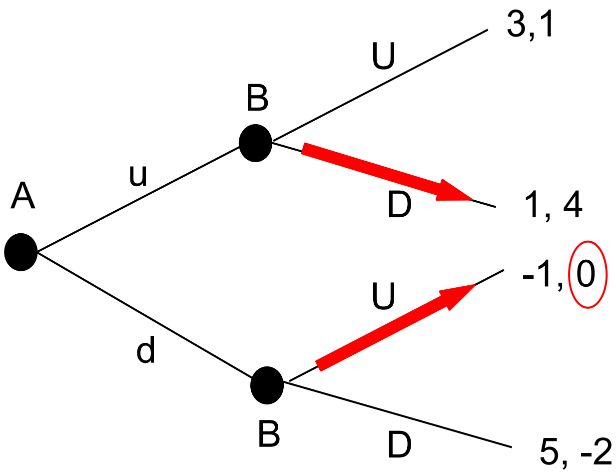
    - So she would choose U



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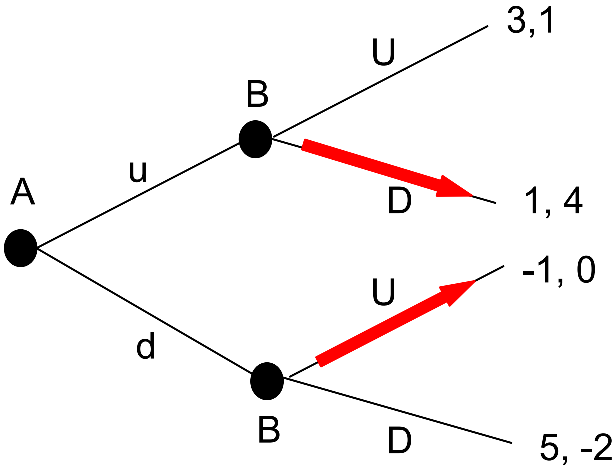
- If B is at her upper node, she gets 1 or 4
  - So she would choose D
- If B is at her lower node, she gets 0 or -2
  - So she would choose U
- So we have solved B's problem:
  - Choose D at the upper node and U at the lower node



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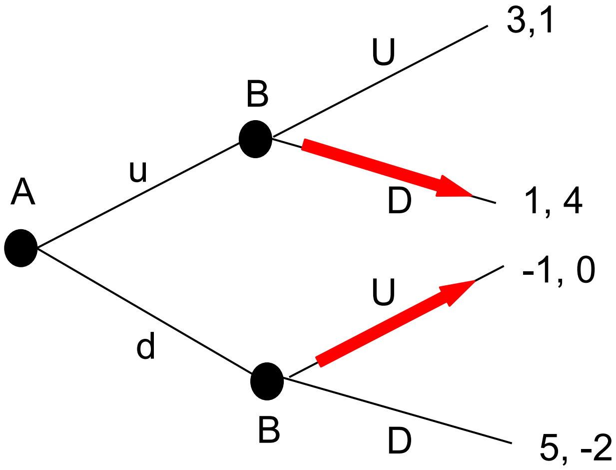
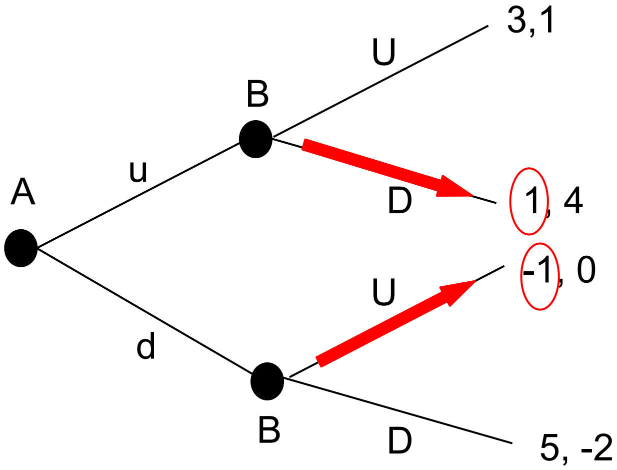
- If B is at her lower node, she gets 0 or -2
    - So she would choose U
- So we have solved B's problem:
    - Choose D at the upper node and U at the lower node
- Given this, what action should A choose: u or d?



Solving a sequential game

- Since A can now anticipate B’s actions, he sees that he is choosing between 1 and -1



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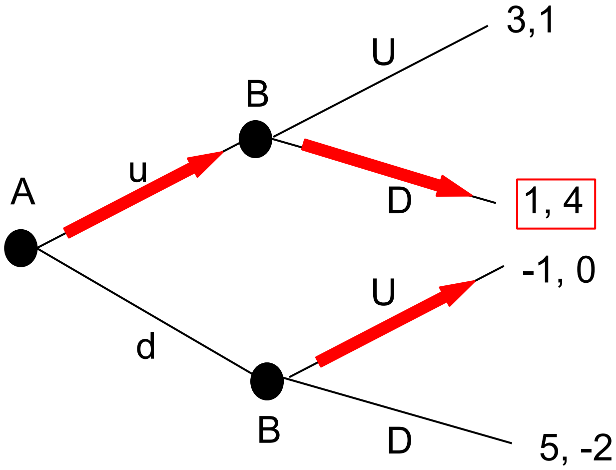
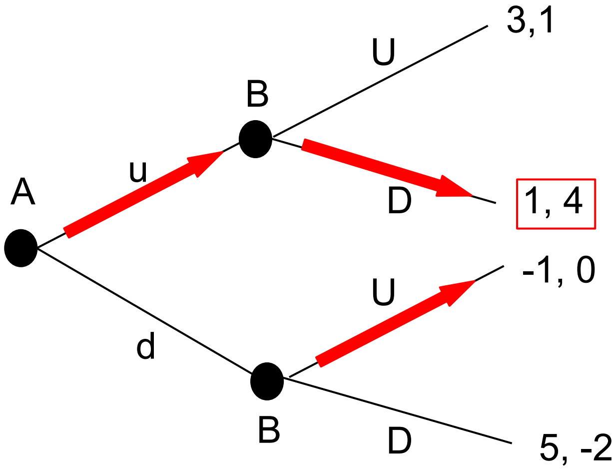


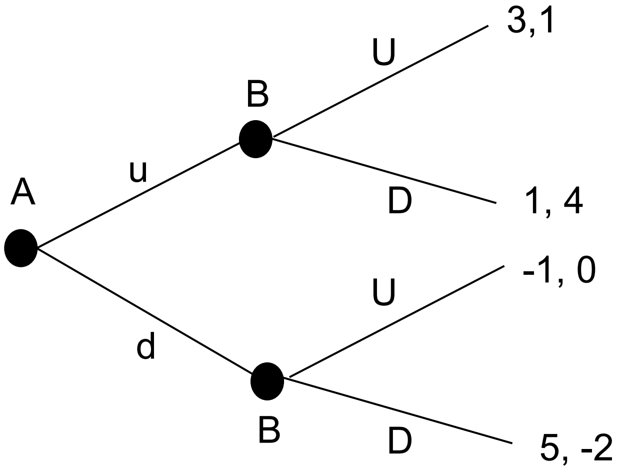
Solving a sequential game

- Since A can now anticipate B’s actions, he sees that he is choosing between 1 and -1
- He will choose u
- This demonstrates the most reliable way to solve a sequential game: <u>backwards induction</u>
  - ○ Start at the end, and work backwards



Strategies

- A player's <u>strategy</u> indicates what choice s/he will make at ***every*** decision node



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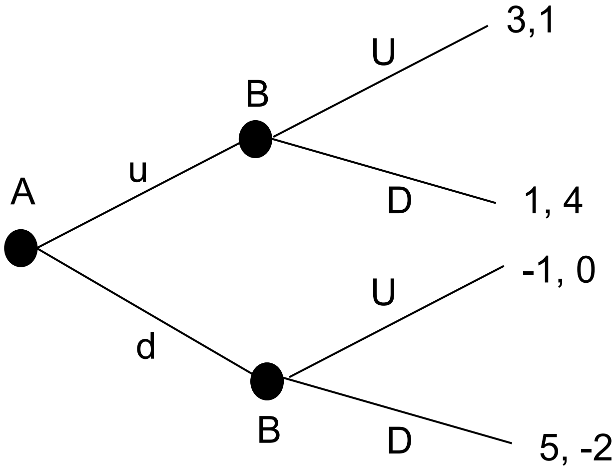
- Player A has 2 potential strategies:



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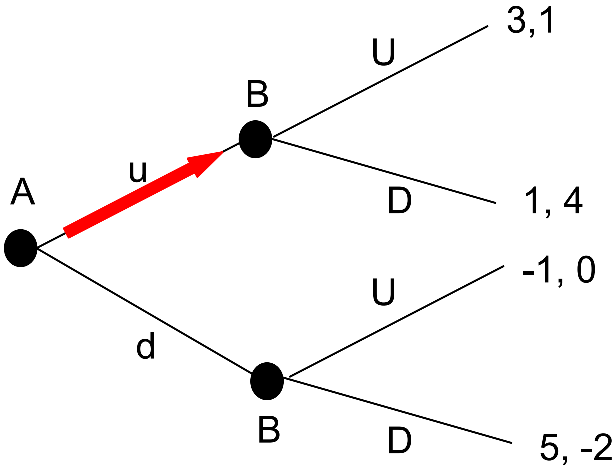

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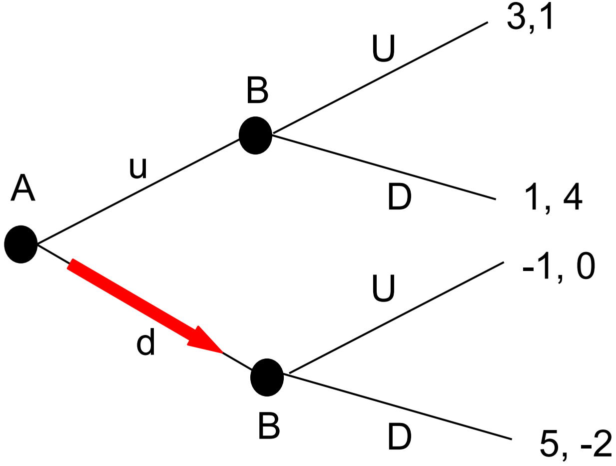
- Player A has 2 potential strategies:
  - u
  - d



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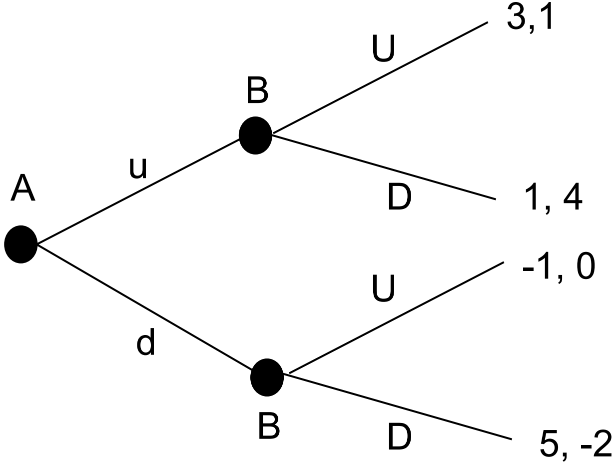
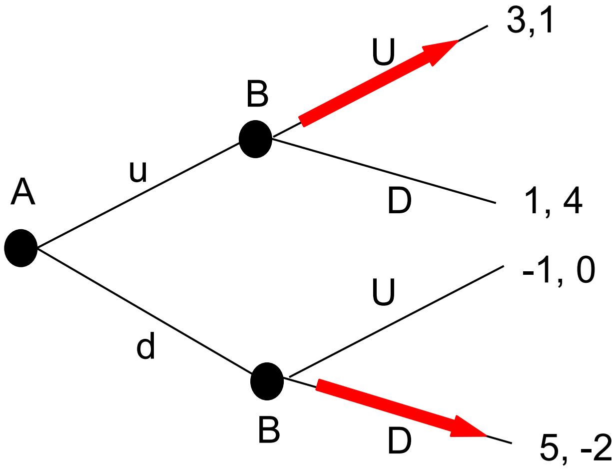


Strategies

- A player's strategy indicates what choice s/he will make at ***every*** decision node
- Player A has 2 potential strategies:
    - u
    - d
- Player B has 4 potential strategies:
    - U (at the top node) D (at the bottom node)



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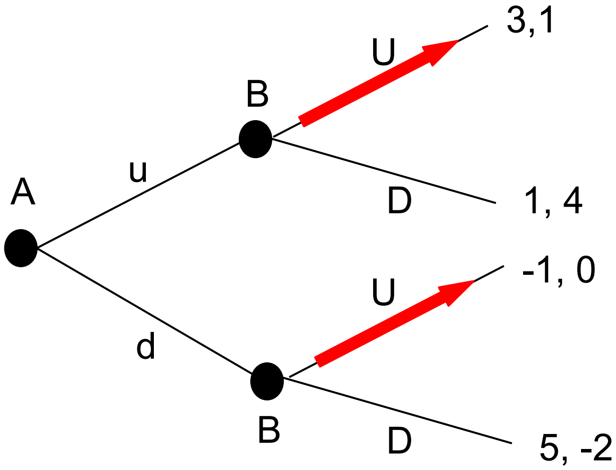

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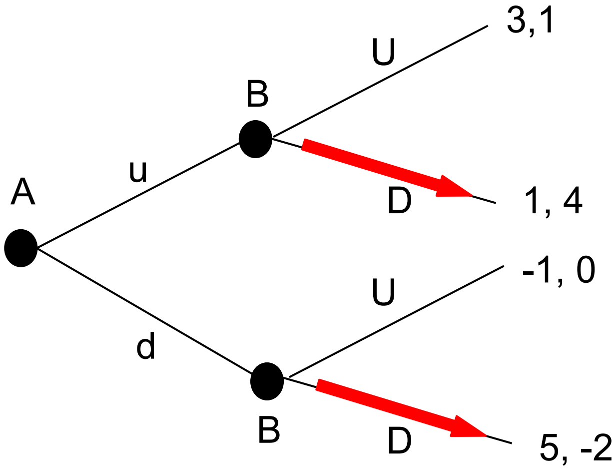
  - DU
  - DD



Strategies

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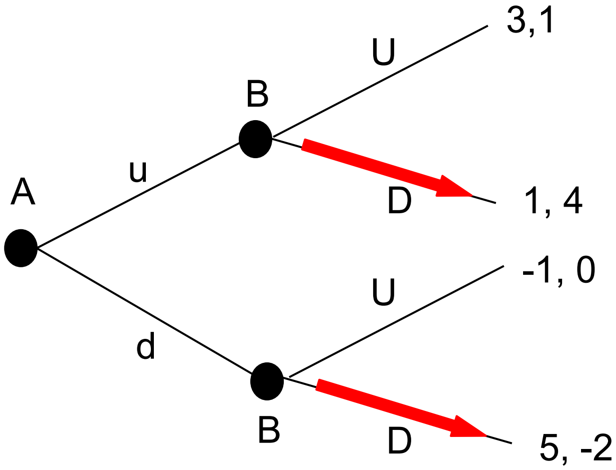
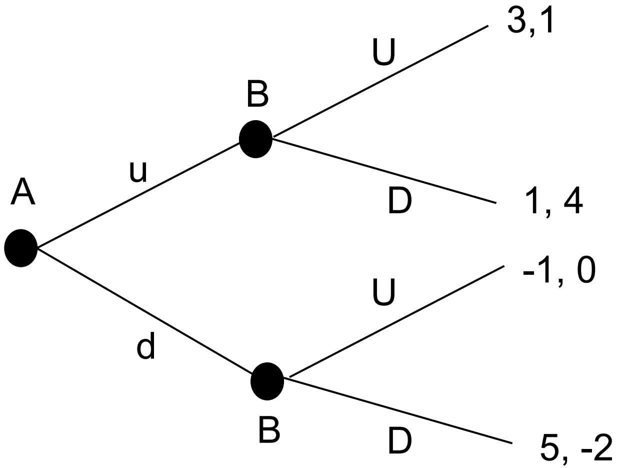
- Player A has 2 potential strategies:
  - u
  - d
- Player B has 4 potential strategies:
  - U (at the top node) D (at the bottom node)
  - UU
  - DU
  - DD
- "D" is not a strategy for player B: we need to know what she will do at ***every*** decision node



Nash Equilibrium

- A Nash Equilibrium is a set of strategies, one for each player, such that no player can increase its payoff by unilaterally deviating to a different strategy
    - "Given your choice, my choice is optimal. (And same for you.)"
- This is a foundational concept in Game Theory



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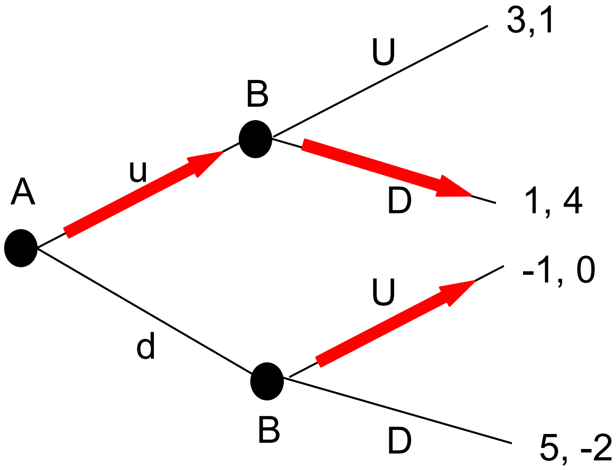
- Backwards induction will always find a NE
  - NE: {u, DU}



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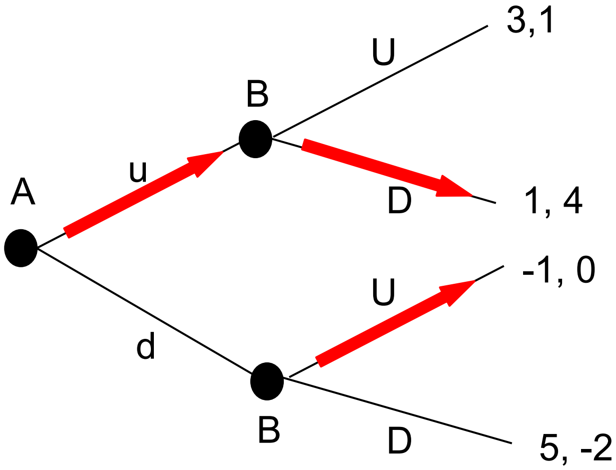
- Note that (1,4) is the payoffs resulting from the NE, but (1,4) is not the NE itself; *the strategies are*



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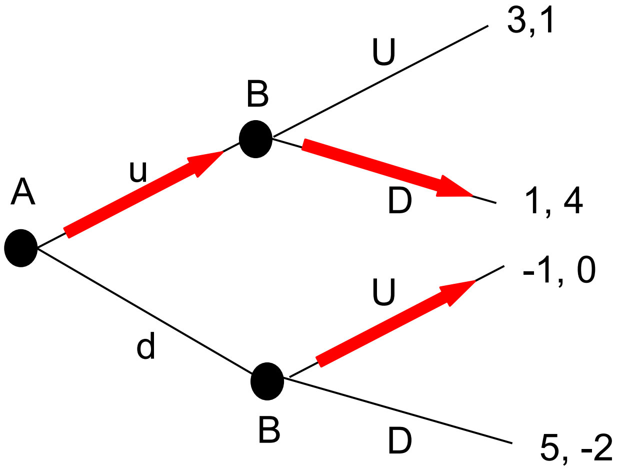
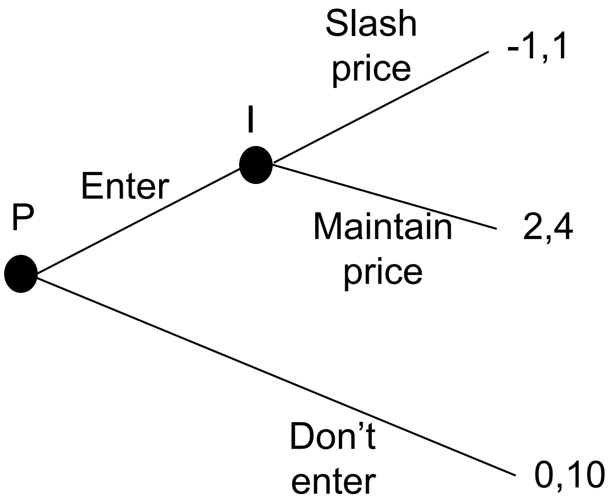
- We will see games with multiple NEs, 0 NEs, or just 1



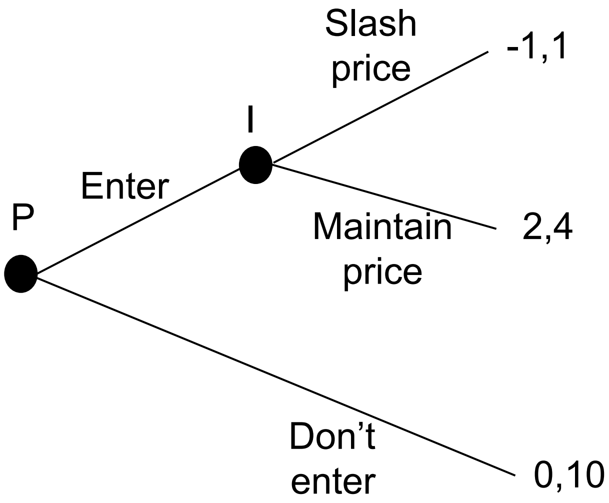
Market entry game

- Consider a market with an incumbent (I) and a potential entrant (P).
- P must decide whether to enter the market.
- If it chooses not to, the game ends with I maintaining his payoff of 10 (and P receiving 0)
- If P Enters, I must decide whether to cut price to compete fiercely, or maintain a higher price
    - If the former, the payoffs are -1 for P and 1 for I
    - If the latter, the payoffs are 2 for P and 4 for I
        - (P's payoffs are lower because of the start-up cost)



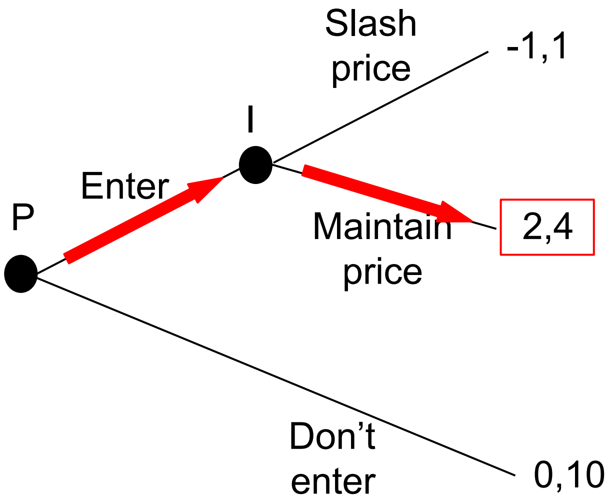
Market entry game - backward induction solution

- Find a Nash Equilibrium via backward induction



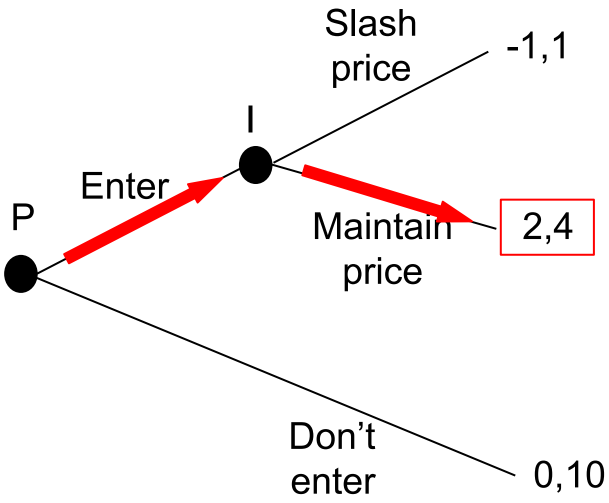
Market entry game - backward induction solution

- Find a Nash Equilibrium via backward induction
  - NE = {Enter, Maintain price}



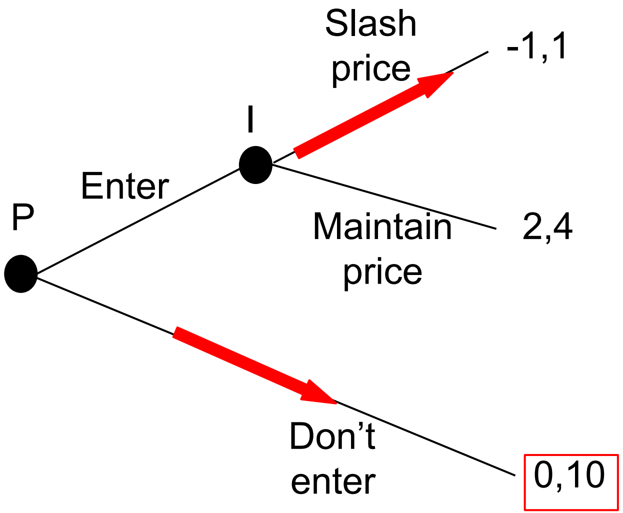
Market entry game - another solution

- Find a Nash Equilibrium via backward induction
    - NE = {Enter, Maintain price}
- Can you find another NE?



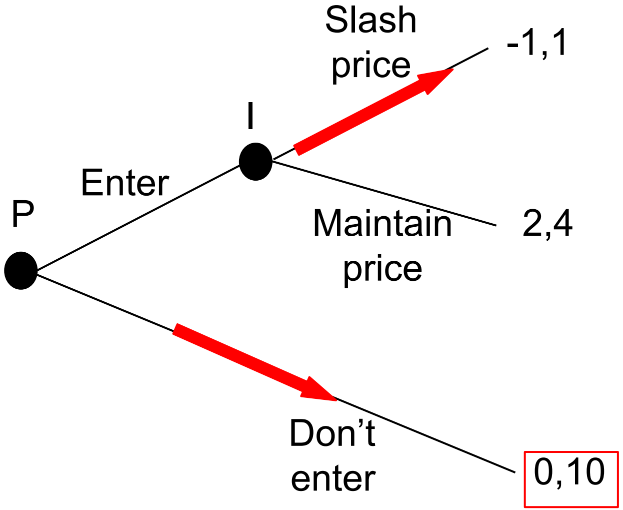
Market entry game - another solution

- Find a Nash Equilibrium via backward induction
  - NE = {Enter, Maintain price if P enters}
- Can you find another NE?
  - Alternative NE = {Don't enter, Slash price if P enters}
  - Story: I threatens P with a price war if P enters. P believes the threat and so does not enter the market.



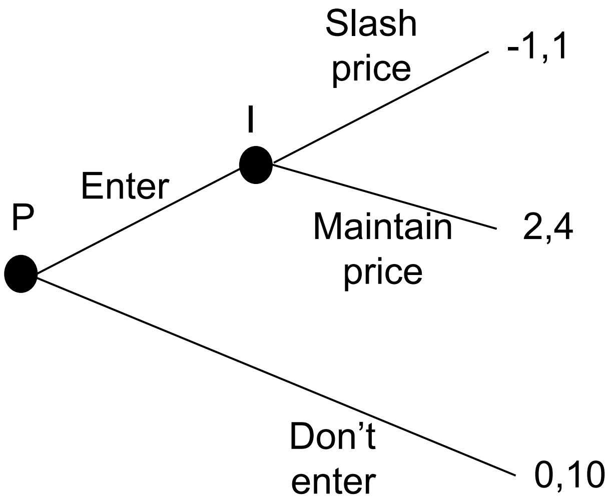
Market entry game - another solution

- Find a Nash Equilibrium via backward induction
    - NE = {Enter, Maintain price if P enters}
- Can you find another NE?
    - Alternative NE = {Don't enter, Slash price if P enters}
    - Story: I threatens P with a price war if P enters. P believes the threat and so does not enter the market.
- Which equilibrium do you think is more likely to occur?



Refining the set of Nash Equilibria

- Found 2 NE: {Enter, Maintain}, {Don’t enter, Slash}



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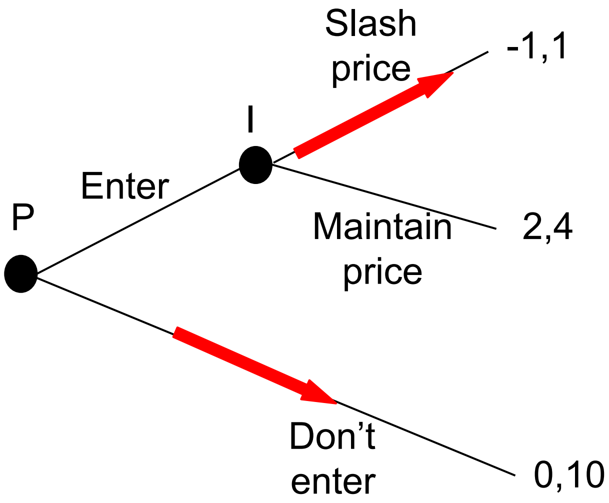
  - Still a NE because the game never reaches that node!



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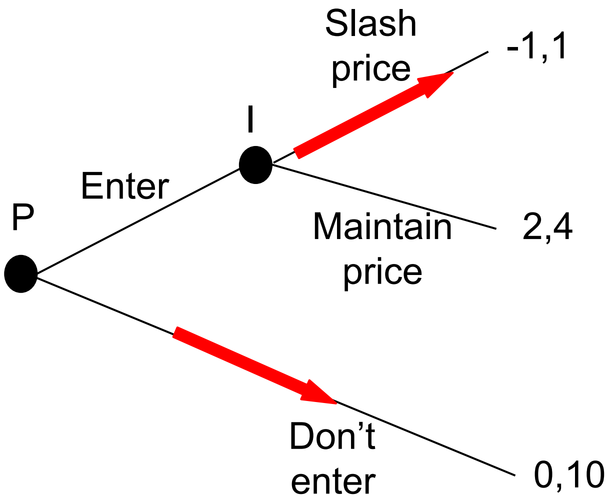

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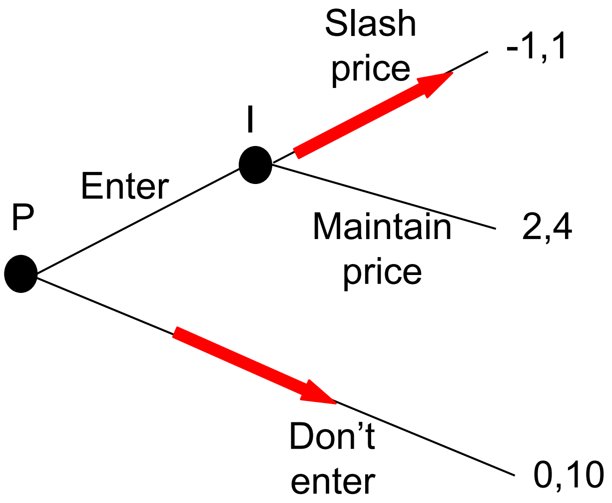

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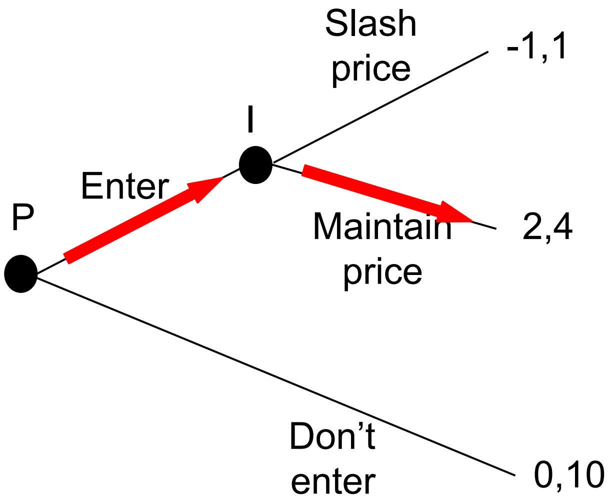
    - It is Subgame Perfect: rational choice at every decision node



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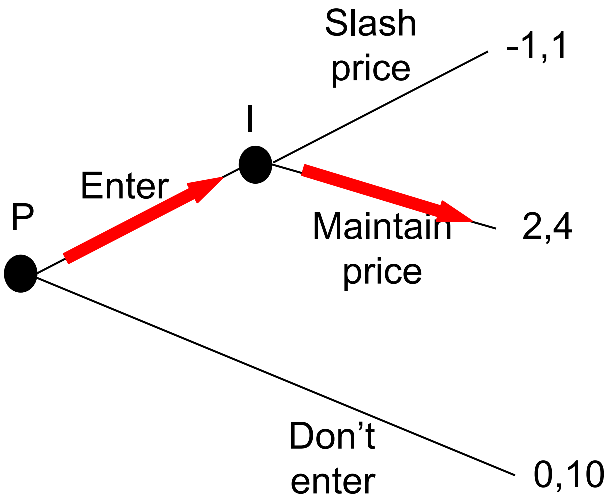
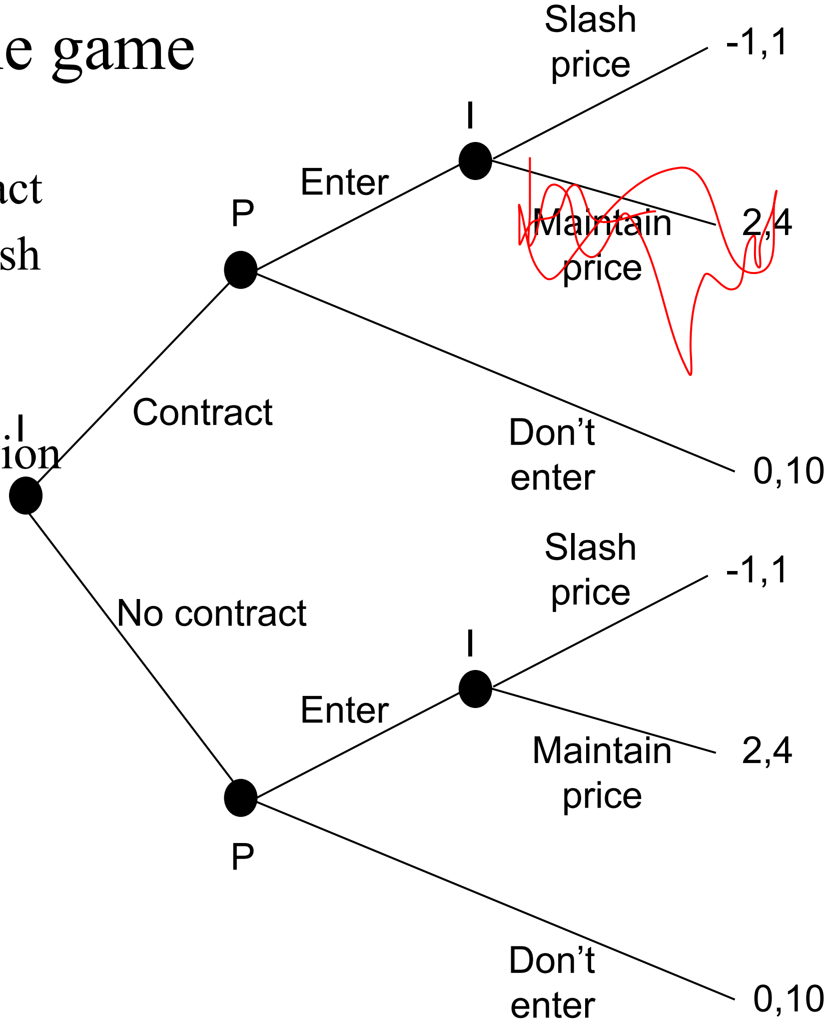
- So {Enter, Maintain} seems more reasonable!
  - It is <u>Subgame Perfect</u>: rational choice at every decision node
- Food for thought: what if you had seen the incumbent use a price war before to drive out competition. How would that impact your prediction?



Manipulating the game

- Suppose that, before P chooses, I can sign a contract with wholesalers saying that if P enters, it will Slash prices
- The game tree now shows I's contracting decision first, then P's entry decision, then I's pricing decision
- What path will this game follow?



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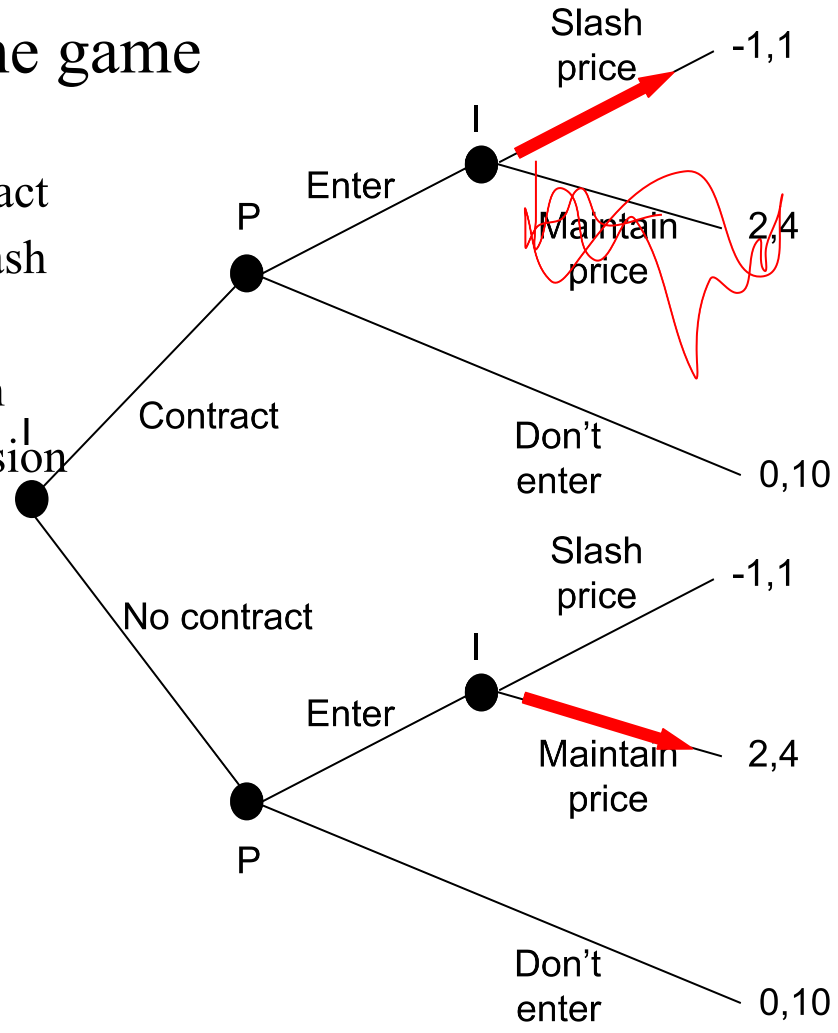

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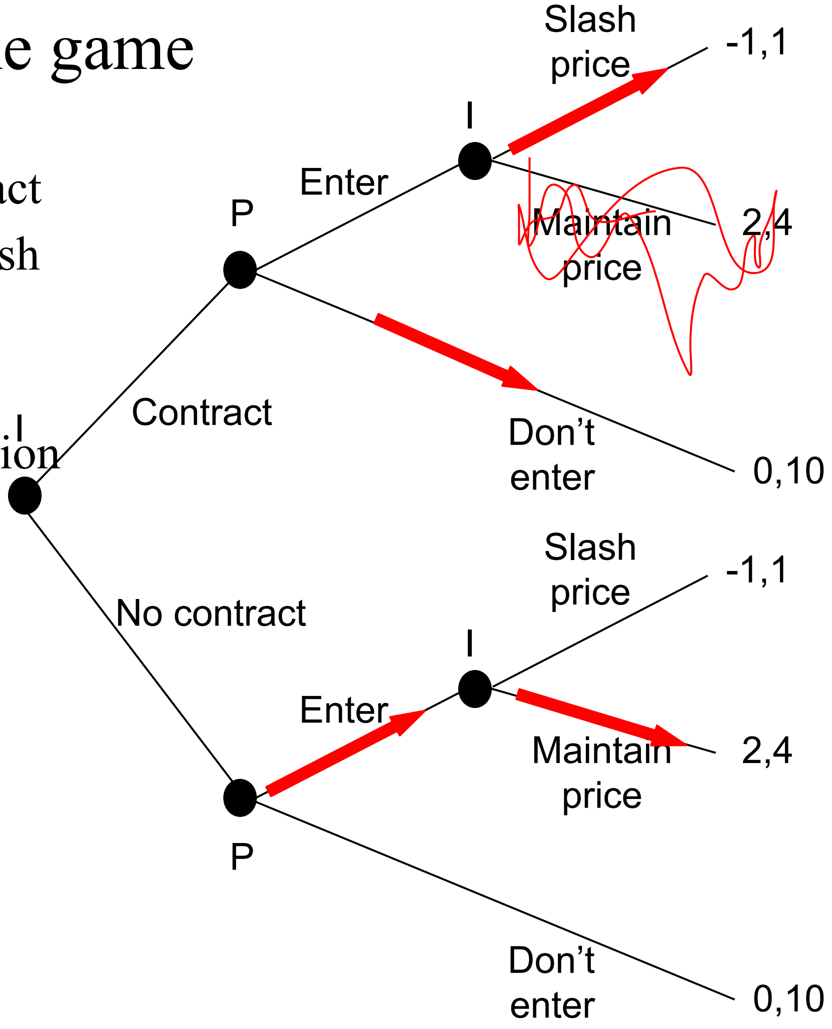

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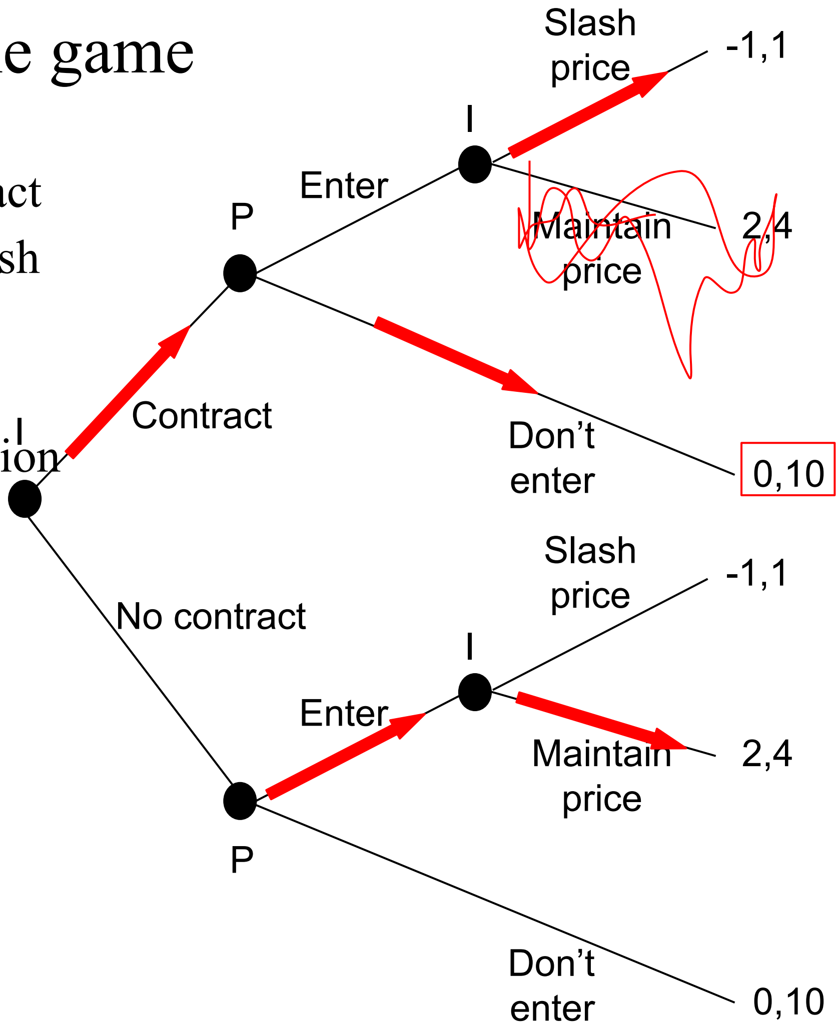

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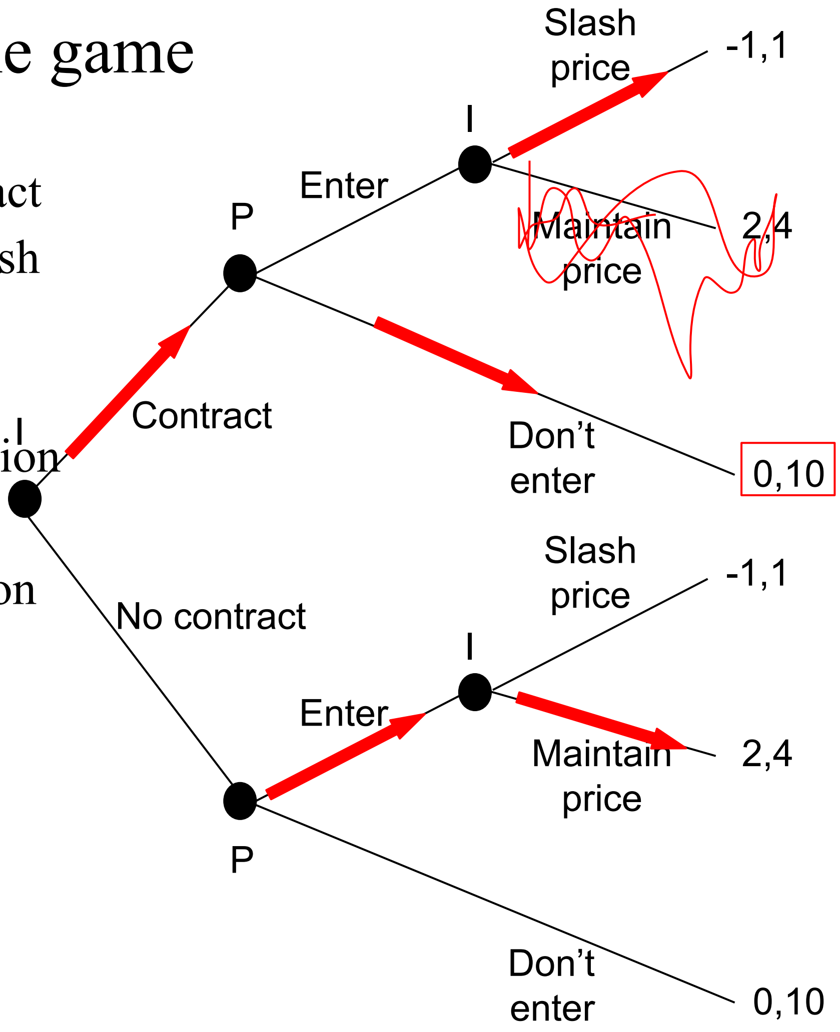
- I will want to sign the contract, removing the option to Maintain a high price



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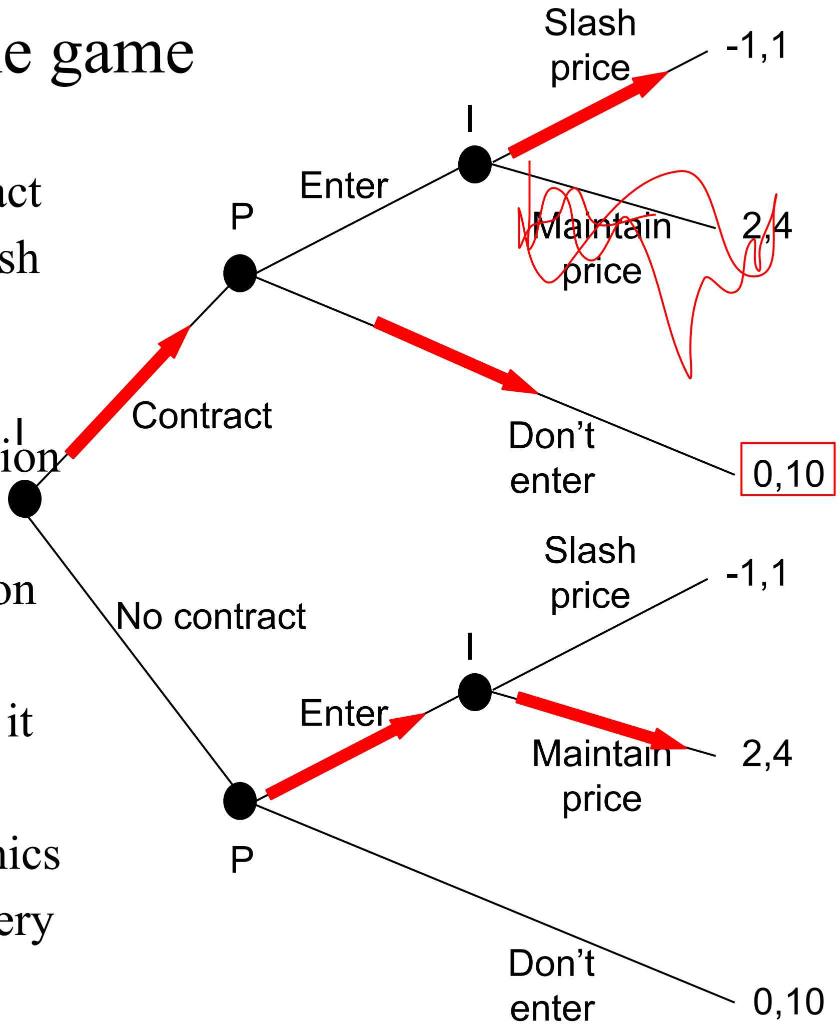
- The game tree now shows I's contracting decision first, then P's entry decision, then I's pricing decision
- What path will this game follow?
- I will want to sign the contract, removing the option to Maintain a high price
    - This is counter-intuitive – I does better when it has fewer choices?!
        - That doesn't happen in standard economics
    - In a strategic setting, a <u>commitment</u> can be very powerful: it made the threat credible



https://www.youtube.com/watch?v=2yfXgu37iyI

Simultaneous Games

Simultaneous games

- In simultaneous games, players must choose their strategies before observing any actions of other players
- These games are often analyzed with a game matrix

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

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- There are 2 players, A and B

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- Player A has two potential strategies: U and D

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- There are 2 players, A and B
- Player A has two potential strategies: U and D
- Player B has two potential strategies: L and R

		Player B	
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- The players play their selected strategies, and then the game ends with the corresponding payoffs

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- Player A has two potential strategies: U and D
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- The players play their selected strategies, and then the game ends with the corresponding payoffs
- So if A chooses "U" and B chooses "R," A will receive 3 and B will receive 5

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		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

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- Player A has two potential strategies: U and D
- Player B has two potential strategies: L and R
- The players play their selected strategies, and then the game ends with the corresponding payoffs
- So if A chooses "U" and B chooses "R," A will receive 3 and B will receive 5
- Should we expect that outcome, or something else?

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”
    - He would then essentially be choosing between 2 and 3

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”
    - He would then essentially be choosing between 2 and 3
    - So he would choose “D”

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”
    - He would then essentially be choosing between 2 and 3
    - So he would choose “D”
- If instead A knew B would choose “R”
    - He would then essentially be choosing between 3 and 1

Player B

		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”
    - He would then essentially be choosing between 2 and 3
    - So he would choose “D”
- If instead A knew B would choose “R”
    - He would then essentially be choosing between 3 and 1
    - So he would choose “U”

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through A’s choice

- Suppose A knew B would choose “L”
    - He would then essentially be choosing between 2 and 3
    - So he would choose “D”
- If instead A knew B would choose “R”
    - He would then essentially be choosing between 3 and 1
    - So he would choose “U”

So A can’t know what to choose unless he knows what B will choose

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through B’s choice

- Suppose B knew A would choose “U”

		Player B	
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Thinking through B's choice

- Suppose B knew A would choose “U”
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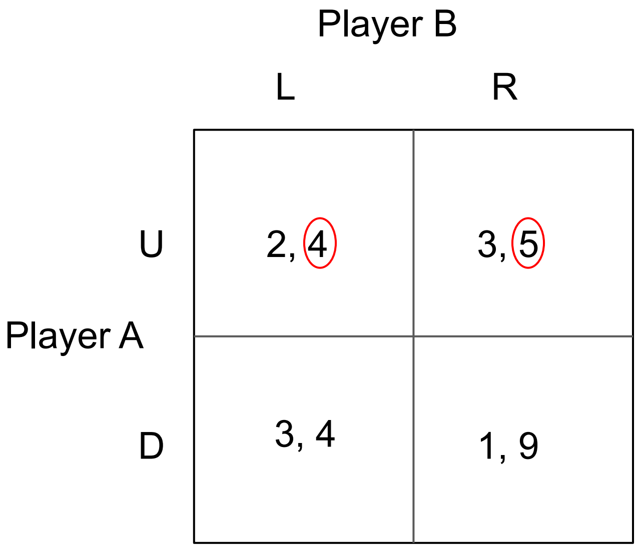

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- Suppose B knew A would choose “U”
    - She would then essentially be choosing 4 and 5
    - So she would choose “R”

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Thinking through B's choice

- Suppose B knew A would choose "U"
  - She would then essentially be choosing 4 and 5
  - So she would choose "R"
- If instead B knew A would choose "D"
  - She would then essentially be choosing between 4 and 9

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    - She would then essentially be choosing 4 and 5
    - So she would choose “R”
- If instead B knew A would choose “D”
    - She would then essentially be choosing between 4 and 9
    - So she would choose “R”

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		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Thinking through B’s choice

- Suppose B knew A would choose “U”
  - She would then essentially be choosing 4 and 5
  - So she would choose “R”
- If instead B knew A would choose “D”
  - She would then essentially be choosing between 4 and 9
  - So she would choose “R”
- So in this game, B does not actually have to worry about A’s choice – R is better than L regardless!
- R is player B’s dominant strategy

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

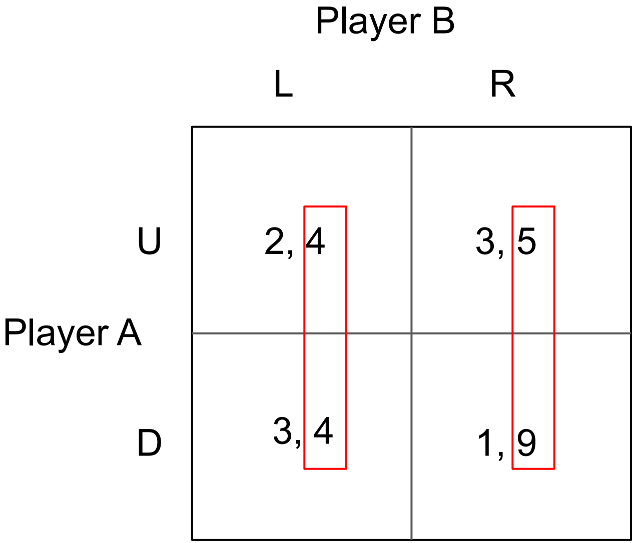
Dominant strategies

- A player has a dominant strategy if one strategy yields a better outcome than all other strategies, regardless of the strategies played by other players

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Dominant strategies

- A player has a dominant strategy if one strategy yields a better outcome than all other strategies, regardless of the strategies played by other players
- In this game, “R” is B’s dominant strategy because her number in the right column is higher than the left, *in every row*



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		L	R
Player A	U	2, 4	3, 5
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Dominant strategies

- A player has a dominant strategy if one strategy yields a better outcome than all other strategies, regardless of the strategies played by other players
- In this game, “R” is B’s dominant strategy because her number in the right column is higher than the left, *in every row*
- Dominant strategies simplify analysis because B does not have to predict A’s action
- So if both players have dominant strategies, we know they will play those strategies
- And even if just one player has a dominant strategy, it can help to solve the game!

		Player B	
		L	R
Player A	U	2, 4	3, 5
	D	3, 4	1, 9

Solving this simultaneous game

- Now that we know B’s dominant strategy is “R,” A can predict with confidence that B will play “R”

Player B

Player A

	L	R
U	2, 4	3, 5
D	3, 4	1, 9

Solving this simultaneous game

- Now that we know B’s dominant strategy is “R,” A can predict with confidence that B will play “R”
- This leaves A to choose between 3 and 1

		Player B	
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Player A	U	2, 4	3, 5
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- So he will choose “U”

		Player B	
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Player A	U	2, 4	3, 5
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Solving this simultaneous game

- Now that we know B’s dominant strategy is “R,” A can predict with confidence that B will play “R”
- This leaves A to choose between 3 and 1
- So he will choose “U”
- And our Nash Equilibrium is {U, R}
    - A would do worse by choosing D
    - B would do worse by choosing L

		Player B	
		L	R
Player A	U	2, 4	$\boxed{3, 5}$
	D	3, 4	1, 9

Solving this simultaneous game

- In general, if there are no dominant strategies, you must go through each quadrant and check whether it is a Nash Equilibrium
    - You might find none
    - You might find multiple
    - You might find just one

		Player B	
		L	R
Player A	U	a, b	c, d
	D	e, f	g, h

Night out

- Suppose two friends planned to get dinner together, but they didn’t specify the restaurant: Pizza or Seafood
- Neither one cares which food they get, they just want to eat together.
- Can you find any NE here?

		Player B	
		Pizza	Seafood
Player A	Pizza	1, 1	0, 0
	Seafood	0, 0	1, 1

Night out

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- Can you find any NE here?
- There are 2: {Pizza, Pizza} and {Seafood, Seafood}

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Night out

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- Neither one cares which food they get, they just want to eat together.
- Can you find any NE here?
- There are 2: {Pizza, Pizza} and {Seafood, Seafood}
- It’s hard to make a prediction here
    - Either one could happen; or neither…

		Player B	
		Pizza	Seafood
Player A	Pizza	$\boxed{1, 1}$	0, 0
	Seafood	0, 0	$\boxed{1, 1}$

Focal points

- This game is a coordination game: the two players want to coordinate actions
- The game can be “solved” if one of the NE becomes a “focal point”

		Player B	
		Pizza	Seafood
Player A	Pizza	$\boxed{1, 1}$	0, 0
	Seafood	0, 0	$\boxed{1, 1}$

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- Or maybe the pizza was so good last time that it makes sense to go back
    - If they both realize that, Pizza could be predictable

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Player A	Pizza	1, 1	0, 0
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    - If they both realize that, Seafood could become a predictable outcome
- Or maybe the pizza was so good last time that it makes sense to go back
    - If they both realize that, Pizza could be predictable
- Better yet, if they could send a signal (e.g. text) to each other, they could solve it for sure!

Player B

Player A	Pizza	Seafood
Pizza	1, 1	0, 0
Seafood	0, 0	1, 1

A tough coordination game

You and your friend agreed to meet in New York City tomorrow, but you didn’t set a time or place.

You only have one shot and all you care about is seeing them (i.e. choosing the same time and place).

Where do you go, and when?

Chicken

- Two players drive their cars at each other at full speed. Each player has the option to Continue or Swerve.
- If they both Continue, they get into a horrible car crash
- If one Continues and the other Swerves, the former looks brave and the other looks like a "chicken"
- If both Swerve, they both look like chickens.
- Can you find any NE here?

		Player B	
		Continue	Swerve
Player A	Continue	-10, -10	1, -1
	Swerve	-1, 1	-1, -1

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- If they both Continue, they get into a horrible car crash
- If one Continues and the other Swerves, the former looks brave and the other looks like a "chicken"
- If both Swerve, they both look like chickens.
- Can you find any NE here?
- There are 2: {Continue, Swerve}, {Swerve, Continue}
    - Which is likely to happen – if either?

		Player B	
		Continue	Swerve
Player A	Continue	-10, -10	1, -1
	Swerve	-1, 1	-1, -1

Possible solutions to the Chicken game

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    - If one driver removes his steering wheel so he can’t turn, what do you predict as an outcome?

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  - What if one driver has a reputation as a tough risk-taker, and the other is known to be cautious?

vs

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- Commitment
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    - These reputations can cause a focal point, where the wild driver Continues and the cautious one Swerves

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- Reputation
  - What if one driver has a reputation as a tough risk-taker, and the other is known to be cautious?
    - These reputations can cause a focal point, where the wild driver Continues and the cautious one Swerves

- Or maybe they'll use mixed strategies. Or maybe Game Theory just can't help us make a good prediction here. It depends on the facts on the ground…

Hide-and-seek

- Consider a game of hide-and-seek, where the Hider chooses either the basement or attic, and the Seeker does the same
- If they choose the same place, the Hider is caught and the Seeker wins
- If they choose different places, the Hider wins
- Can you find any NE here?

		Seeker	
		Attic	Basement
Hider	Attic	0, 1	1, 0
	Basement	1, 0	0, 1

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- Can you find any NE here?
- There are none!

Seeker

		Attic	Basement
Hider	Attic	0, 1	1, 0
	Basement	1, 0	0, 1

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- If they choose the same place, the Hider is caught and the Seeker wins
- If they choose different places, the Hider wins
- Can you find any NE here?
- There are none!
- If you were playing as Hider, what would you do?
    - Always choose Attic?
    - Always choose Basement?
    - Something else?

		Seeker	
		Attic	Basement
Hider	Attic	0, 1	1, 0
	Basement	1, 0	0, 1

Mixed strategies

- Players can – and often do – play <u>mixed strategies</u>, choosing randomly between their actions
    - Good idea if the other player would hurt you if they knew your action

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Prisoners’ Dilemma

- Two oligopolists decide whether to Collude or Compete.
- If they both Collude, total profit is maximized
- If they both Compete, total profit is minimized
- If one Competes and one Colludes, the former steals most of the market
- Can you find any NE here?

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		Collude	Compete
Player A	Collude	5, 5	1, 7
	Compete	7, 1	2, 2

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- Frustrating for the oligopolists because they could both do better if they both collude
    - Might we ever see them reach this "better" outcome?

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Repeated games

- Suppose this same game were repeated twice.
- What if both players use the following strategy: “I will Collude in Round 1; in Round 2, I will Collude if the other firm Colluded in Round 1, but I will Compete if it Competed in Round 1”?
- Is this a NE with collusion in both rounds?

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Repeated games

- Naive analysis might think this will work.
- If they follow the strategy, each gets 5+5 = 10.
- If one deviates to Compete in the first round, the best it can get is 7+2 = 9.
- So shouldn’t they collude in both rounds?
- Let’s use backward induction.

		Player B	
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→

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Repeated games

- Start with Round 2

	Player B	Player B
	Collude	Compete
Player A: Collude	5, 5	1, 7
Player A: Compete	7, 1	2, 2

	Player B	Player B
	Collude	Compete
Player A: Collude	5, 5	1, 7
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Repeated games

- Start with Round 2
    - Round 2 is the end of the game, so it’s a normal Prisoners’ Dilemma – they’ll Compete

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Repeated games

- Start with Round 2
    - Round 2 is the end of the game, so it's a normal Prisoners' Dilemma – they'll Compete
- What about Round 1?

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		Collude	Compete
Player A	Collude	5, 5	1, 7
	Compete	7, 1	2, 2

		Player B	
		Collude	Compete
Player A	Collude	5, 5	1, 7
	Compete	7, 1	$\boxed{2, 2}$

Repeated games

- In Round 1, both players know that the other will Compete in Round 2
- Therefore, they have no incentive to play nice in Round 1

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Repeated games

- In Round 1, both players know that the other will Compete in Round 2
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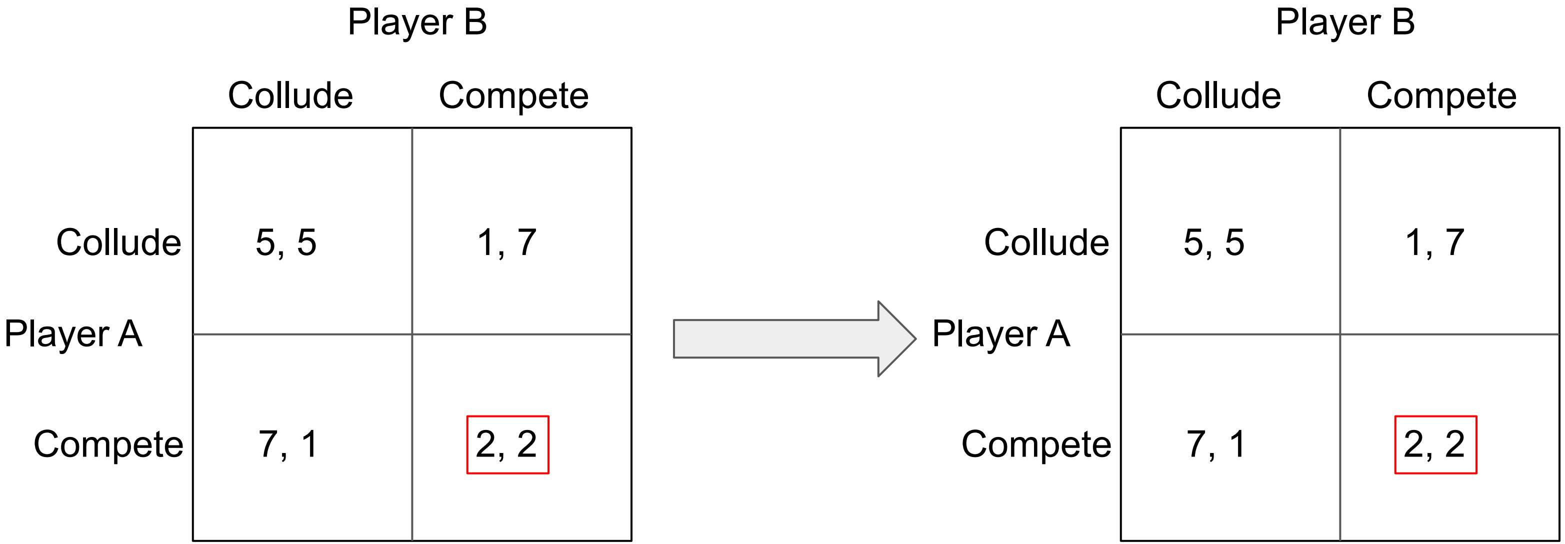
- Both will compete



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Repeated games

- In Round 1, both players know that the other will Compete in Round 2
- Therefore, they have no incentive to play nice in Round 1
- Both will compete
- If PD is repeated a ***finite*** number of times, there will be no collusion in any of the rounds!

		Player B	
		Collude	Compete
Player A	Collude	5, 5	1, 7
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Infinitely-repeated games

- The analysis is different if the game is – or may be – repeated infinitely
- If you never *know* that you're in the final round, it's never obvious that you should Compete
- Therefore, the possibility of earning 5, 5, 5,... instead of 7, 2, 2, 2,… may induce you to Collude.
  - The logic of collusion does not unravel in the same way it did with finite rounds

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		Collude	Compete
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Collusion in practice

- So what are the factors that cause us to see collusion in some industries and not others?
- High likelihood of future games
    - If our industry isn't likely to last, why should I not try to steal the market from you now?
- Stable market conditions
    - I need to know when you chose Compete vs Collude. If demand jumps around a lot, I may get low demand because you undercut me, or because demand fell.
- Observable actions/monitoring
    - Same as above: I'll keep up my end of the bargain if you know when I cheat you
- Relatively few firms
    - Easier to form an agreement and monitor adherence to the agreement

Summary of solutions

Game	**# NE**	**Additional concepts**
Hide and seek	0	Mixed strategies
Night out	2	Focal point, communication
Chicken	2	Reputation, commitment
Prisoners' Dilemma	1	Repeated games

Game Theory summary

- Game Theory provides rigorous way to unwind complex strategic situations
- In some cases, you can find a reasonable solution (reliable prediction)
- In others, you might not solve it, but the analysis will lay bare the issues at play and potentially still be fruitful
    - E.g. need for randomization, or importance of focal points

https://www.youtube.com/watch?v=LJS7Igvk6ZM

Caution about “solutions”

- Just because *you* can solve a game, real players may not play that way
- Consider chess

THAT LOOK WHEN...

Instagram @crusoe_dachshund

YOU KNOW YOU’RE GOING TO WIN

Caution about “solutions”

- Just because *you* can solve a game, real players may not play that way
- Consider chess
- Chess has a “solution” (i.e. optimal strategies) but we’ll probably never find it
- Even if you knew the solution, would that help you predict how a game of chess will unfold between two people playing in a park?

As games get more complex, the less likely it is that your analytical solution will line up with how real players actually play the game.