Production

Fall 2023
Econ 2316, Northeastern University
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P&R: chapters 6, 7 (especially 7.1-7.4)
Emerson: chapters 6-8

Introduction

Why do firms exist?

- Modern economies produce many complex goods and services, e.g. computers
- Could imagine a production process carried out entirely by individual artisans/workers
    - Miners rent space from mine owner, dig up raw materials
    - Miners sell raw material to individual refiners
    - Refiners rent out plant space, refine raw material, sell to individual chip makers
    - Chip makers rent factory from factory owner, make chips, and sell to computer makers
    - … and so on for other components
- Each step in the process would require negotiation of prices and coordination of specs
- This process would be inefficient, costly, and frustrating
- Instead, many of these steps are “vertically integrated” into a <u>firm</u>
- A firm coordinates the actions of many individuals in a way that is set aside from market
    - Manager and worker negotiate a contract (market), and then worker does what the manager instructs
- This solves many coordination problems, but it creates other problems, like incentives
    - How does manager get worker to try hard? Constant monitoring?
- This is why modern economies have some vertical integration, but not complete integration

Economists' approach to firm production

- As with consumer theory, not intended to catch every nuance of running a business.
  - Goal is to distill and understand some of the most important elements that can then be plugged into a larger model to understand market outcomes
- Firms have some production technology, which allows them to convert inputs into output, which they then sell
  - Many analogies to consumer theory, where consumers have a utility function that converts goods into utility
    - However, there are important distinctions between consumer and producer theories
  - Key things economists consider: technology, input prices, demand for output, type of competition

Some guiding structure

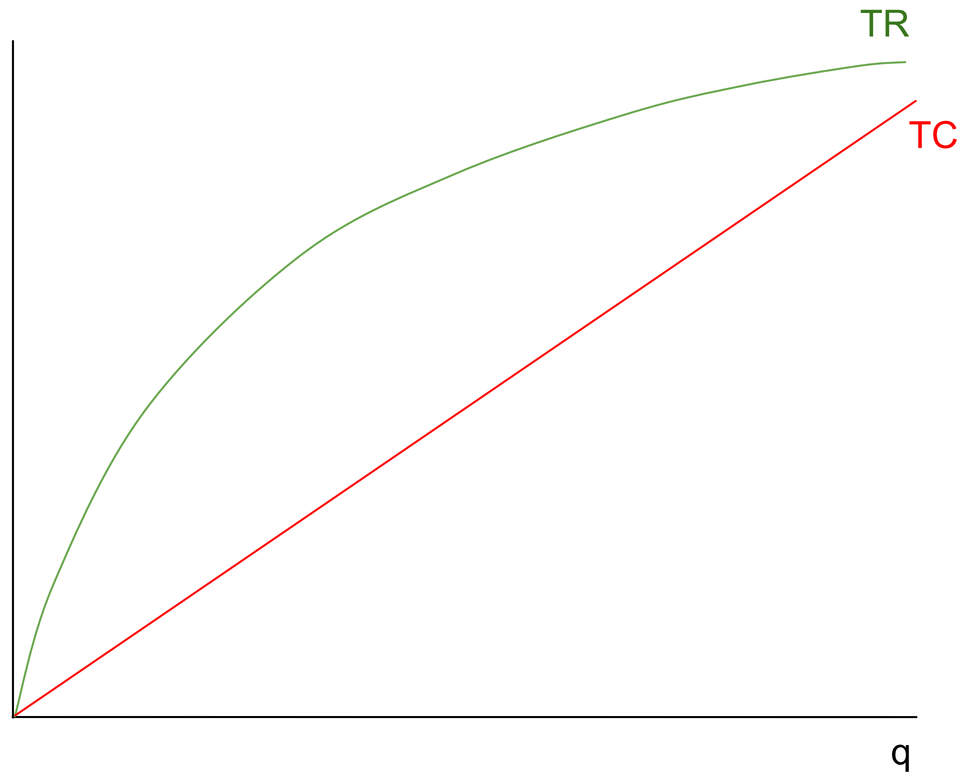
- Almost always, economists assume firms seek to maximize profits:

$$\pi(q) = TR(q) - TC(q)$$

- Choose q optimally to set derivative equal to 0:

$$MC(q) = MR(q)$$

Some guiding structure (aside)



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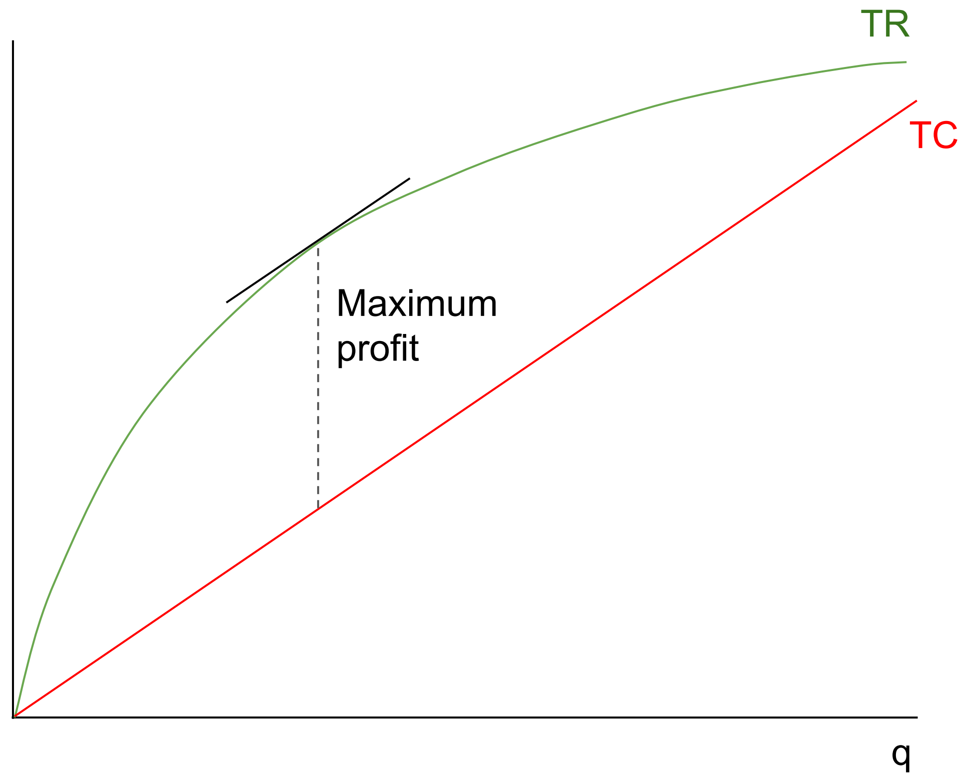

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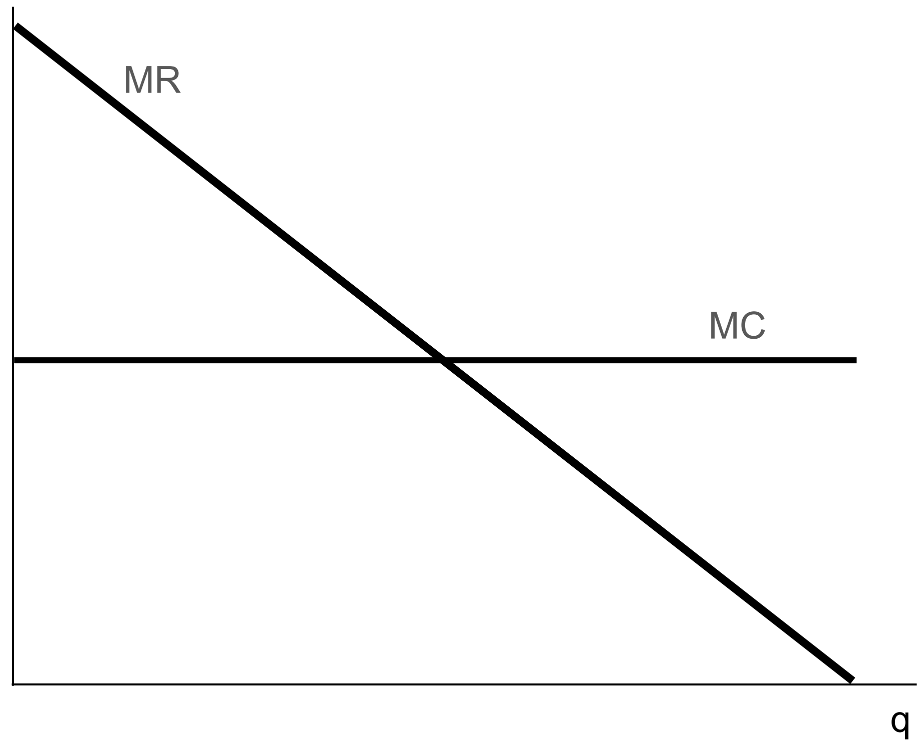

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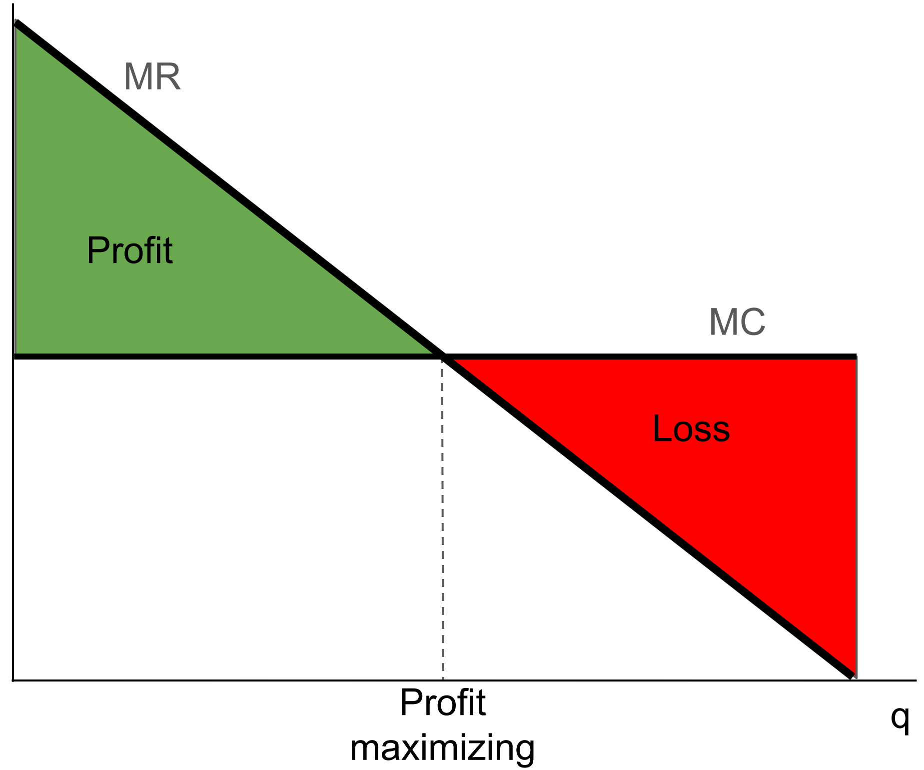

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Result of demand and market structure (we'll study these later)

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Result of technology and input costs (we study these now)

Result of demand and market structure (we'll study these later)

Technology

Technology

- A production function tells us how much output a firm can produce for different levels of inputs
- Suppose a firm uses capital (K) and labor (L) to produce output (Y). Then:

$$Y = F(K, L)$$

- Example:

$$Y = K^{0.2} L^{0.8}$$

- Distinguish between long-run and short-run
  - Long-run: both K and L can be chosen freely
  - Short-run: one input (typically K) is fixed:

$$Y = F(\bar{K}, L)$$

Marginal product

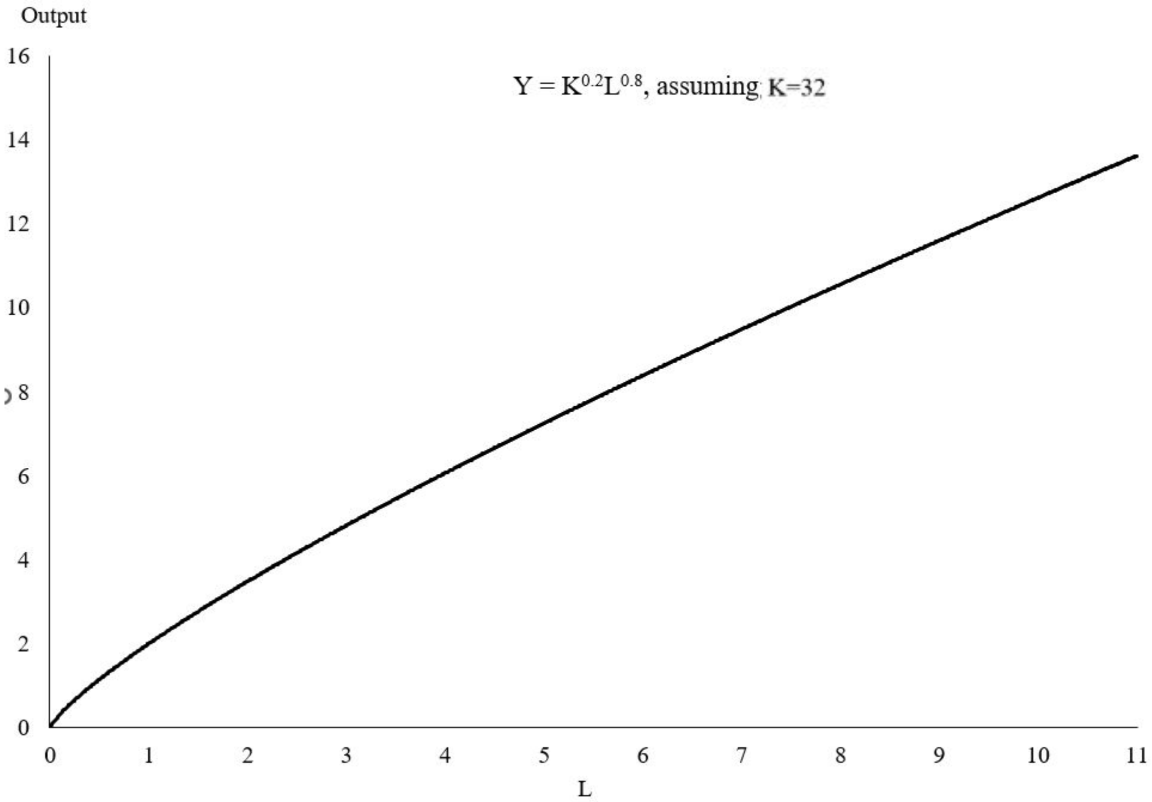
- <u>Marginal product</u> describes how the level output increases when the level of an input increases, *holding all other inputs fixed*
  - Mathematically, it is the derivative of the production function with respect to a given input:

$$MP_L = \frac{\partial F(K,L)}{\partial L} = 0.8K^{0.2}L^{-0.2}$$

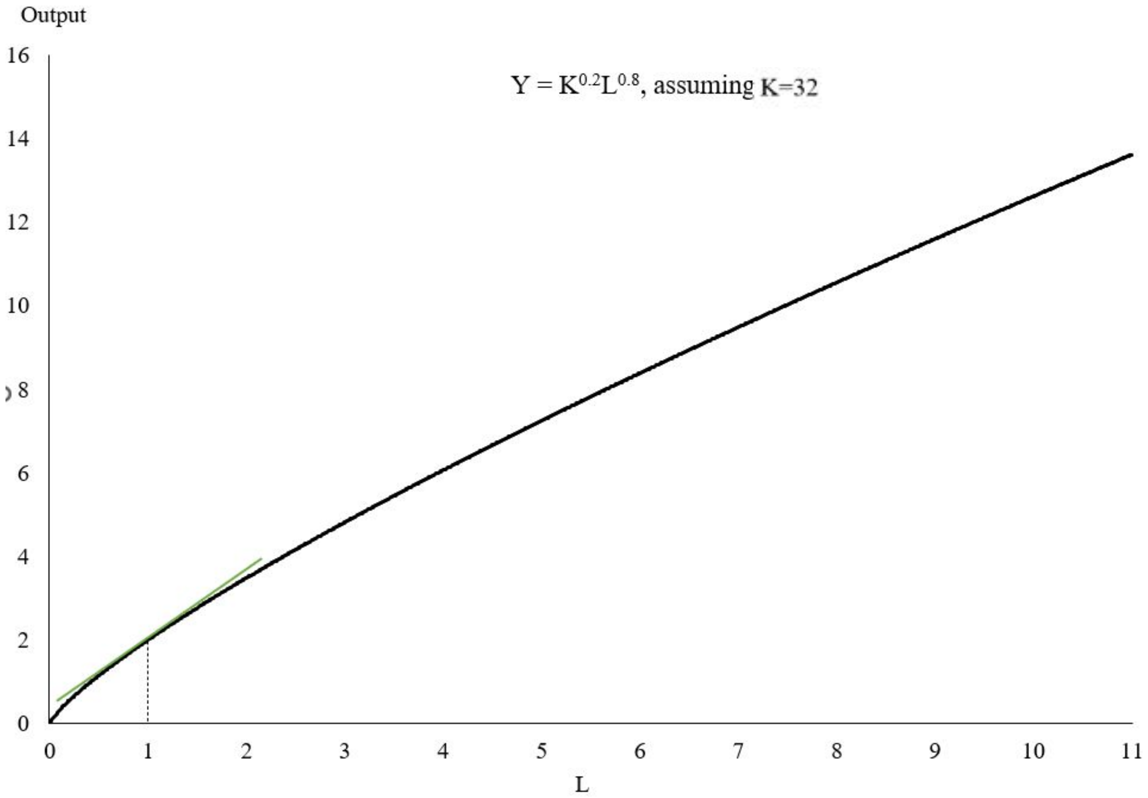
$$MP_K = \frac{\partial F(K,L)}{\partial K} = 0.2K^{-0.8}L^{0.8}$$

  - Similar to marginal utility from consumer theory
- Typically assume marginal product is decreasing, at least at high enough levels of the input
  - As your number of workers increases, it becomes harder to find productive activities for them to do
  - At low levels, more workers may *increase* marginal product, e.g. through brainstorming

Decreasing marginal product



Decreasing marginal product



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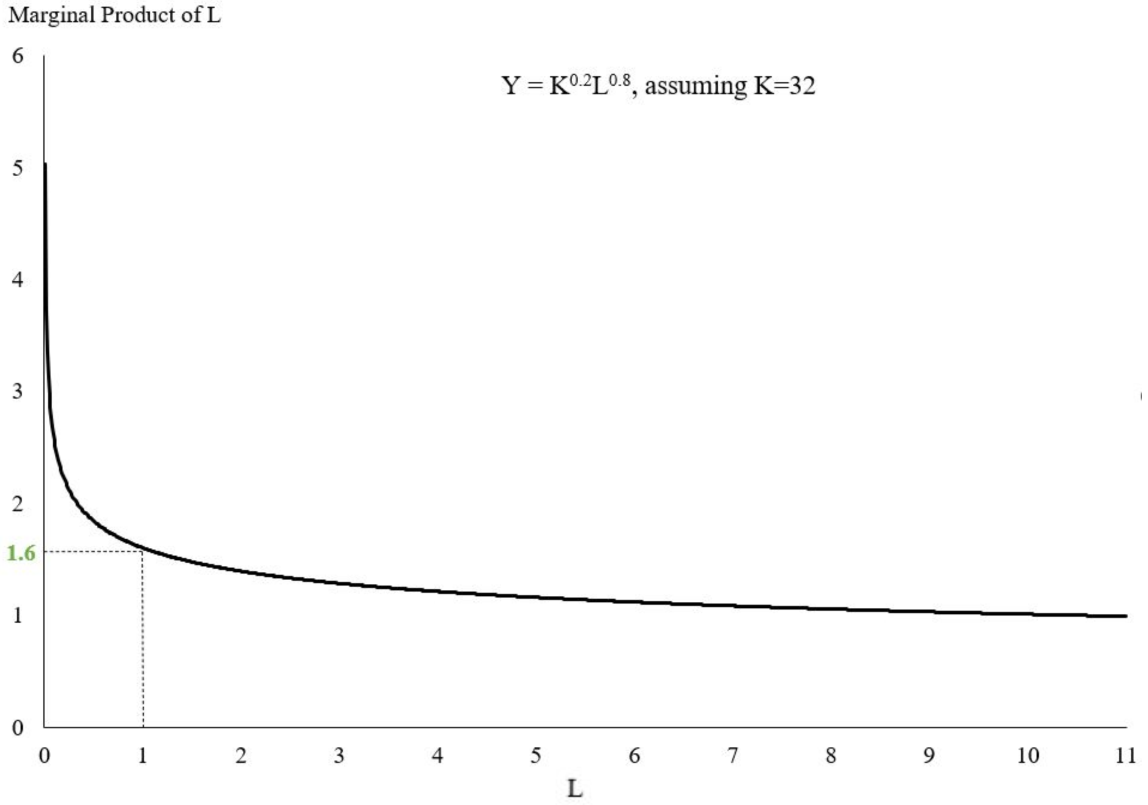

Returns to scale

- Returns to scale describes how output increases when the levels of all inputs increase, *simultaneously*
- Consider doubling both labor and capital:
  - Does output double? Then you have constant returns to scale.
  - Does output less-than-double? Then you have decreasing returns to scale.
  - Does output more-than-double? Then you have increasing returns to scale.
- Increasing returns to scale may exist at relatively low levels of output, but typically at higher levels of output, this is harder to maintain due to the difficulties of managing and coordinating so many resources

Example 1

$$Y = K^{0.2}L^{0.8}$$

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Decreasing marginal products of labor and capital

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Decreasing marginal products of labor and capital

$$F(2K,2L) = (2K)^{0.2}(2L)^{0.8} = 2^{0.2}2^{0.8}K^{0.2}L^{0.8} = 2K^{0.2}L^{0.8} = 2F(K,L)$$

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Constant returns to scale

Example: something that can be replicated separately, like retail franchising

Example 2

$$Y = KL$$

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$$MP_L = K$$

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Constant marginal products of labor and capital

$$F(2K, 2L) = (2K)(2L) = 4KL > 2F(K, L)$$

Example 2

$$Y = KL$$

$$MP_L = K$$

$$MP_K = L$$

Constant marginal products of labor and capital

$$F(2K, 2L) = (2K)(2L) = 4KL > 2F(K, L)$$

Increasing returns to scale

Example: something that feeds on itself, like R&D

Example 3

$$Y = K^{0.1}L^{0.4}$$

$$MP_L = 0.4K^{0.1}L^{-0.6}$$
$$MP_K = 0.1K^{-0.9}L^{0.4}$$

Decreasing marginal products of labor and capital

$$F(2K, 2L) = (2K)^{0.1}(2L)^{0.4} = 2^{0.5}K^{0.1}L^{0.4} < 2F(K, L)$$

Decreasing returns to scale

Example: something that exhausts resources, like mining

Costs in the Short Run

Minimum cost

- The total cost function tells us the lowest possible expenditure that the firm can incur to produce any given level of output.
- With 2 (or more) variable inputs, there are many ways to produce a given amount of output.
- For example, if $Y = KL$, then we can produce $Y = 24$ with:
  - $K = 24$, $L = 1$
  - $K = 6$, $L = 4$
  - $K = 4$, $L = 6$
  - …
- To know which is least costly, we need to know the input costs
  - w, the wage, is the per-unit cost of labor
  - r, the rental rate of capital, is the per-unit cost of capital
    - Even if you own the capital, you pay the opportunity cost of not renting it to someone else

Total cost function

$$TC(q) = \min \ rK + wL$$
$$s.t. \ F(K, L) = q$$

- In words:
  - For any quantity level q…
  - Find the combination of K and L that produces q with the minimum combined labor/capital bill
  - …and that bill is Total (minimum) Cost

Total cost in the short run

- (Minimum) cost in the short-run is fairly simple. Suppose production function is:

$$q = K^{0.5}L^{0.5}$$

- Now suppose K is fixed at K = 9. Then the short-run production function is:

$$q = 3L^{0.5}$$

- So for any level of q, there is actually only one way to produce it:

$$L = \frac{q^2}{9}$$

- So we can solve for TC(q) easily:

$$TC(q) = 9r + \frac{q^2}{9}w$$

Other cost functions

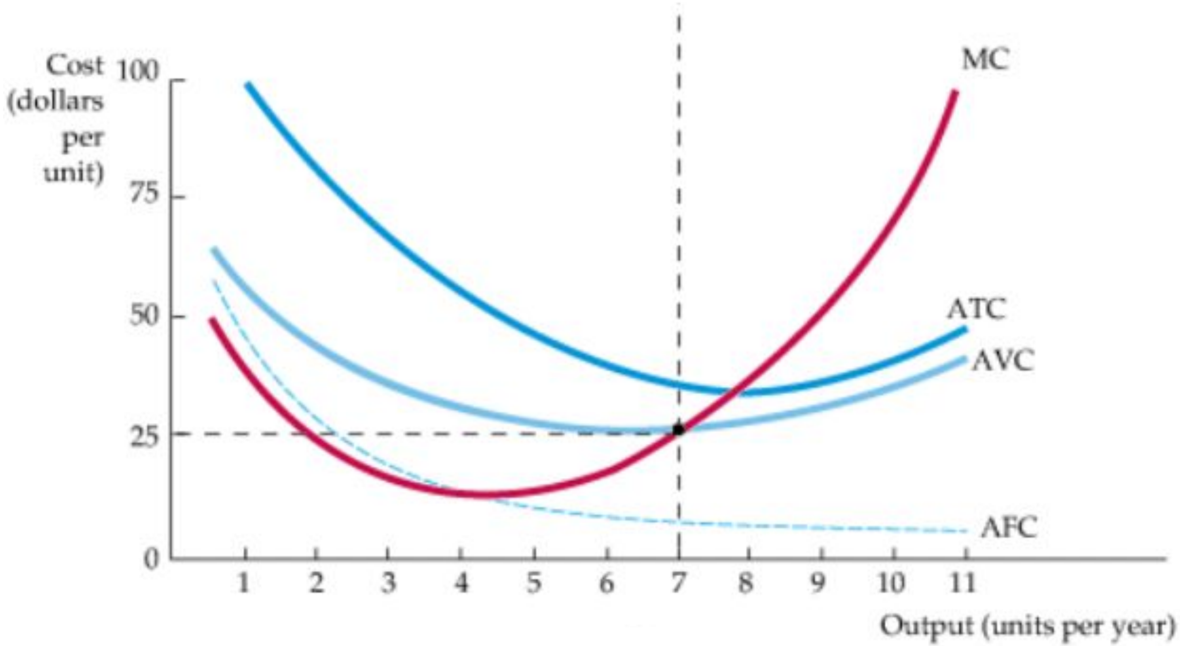
Total Cost	Total Fixed Cost	Average Fixed Cost	Total Variable Cost	Average Variable Cost	Average Total Cost	Marginal Cost
Cost of producing q	Cost of producing q=0	TFC divided by q	Difference between TC and TFC	TVC divided by q	TC divided by q	Cost of producing 1 more unit
	TFC = 9r	AFC = 9r/q	TVC = $\frac{q^2}{9}w$	AVC = $qw/9$		

$$TC(q) = 9r + \frac{q^2}{9}w$$

$$ATC(q) = \frac{9r}{q} + \frac{qw}{9}$$

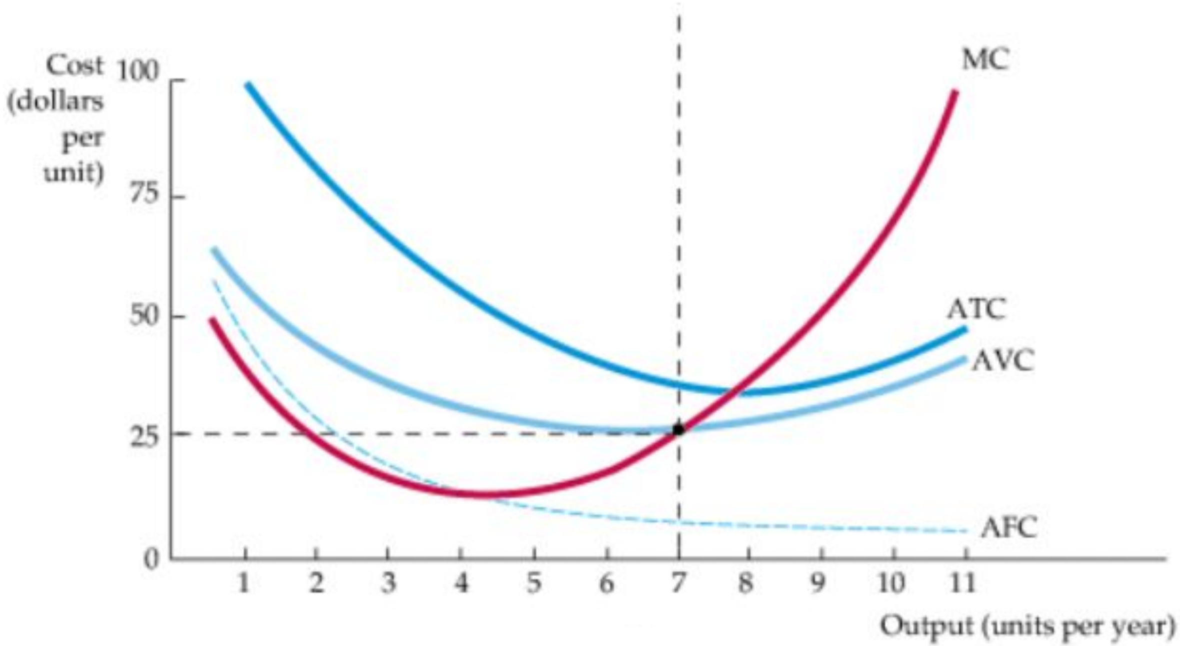
$$MC(q) = \frac{\partial TC}{\partial q} = \frac{2w}{9}q$$

Graphical Illustration



Graphical Illustration

- AFC always decreases. Why?



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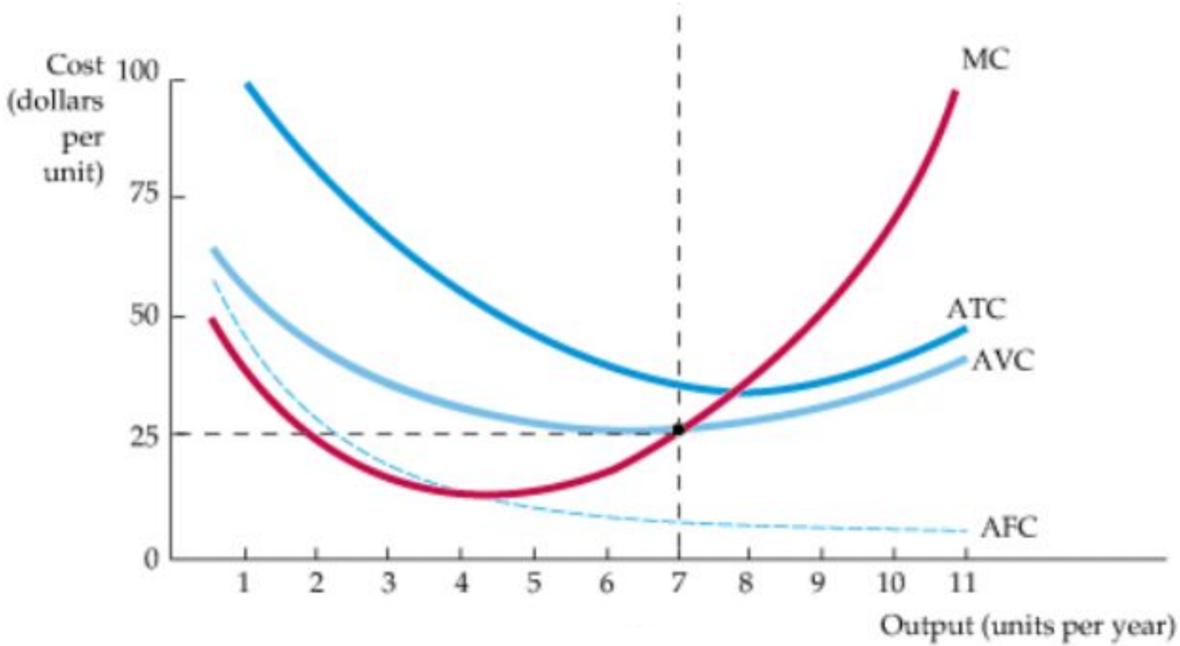
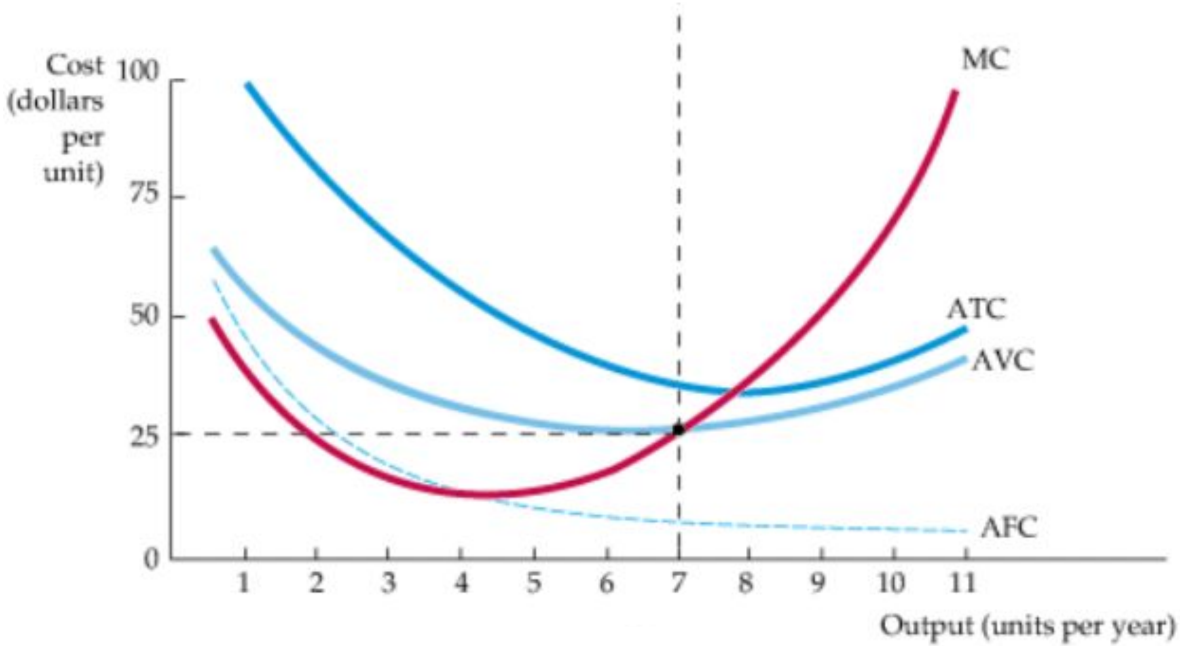
- AFC always decreases. Why?
  - Fixed costs get spread over more quantity



Graphical Illustration

- AFC always decreases. Why?
  - Fixed costs get spread over more quantity
- MC intersects minimum of AVC. Why?



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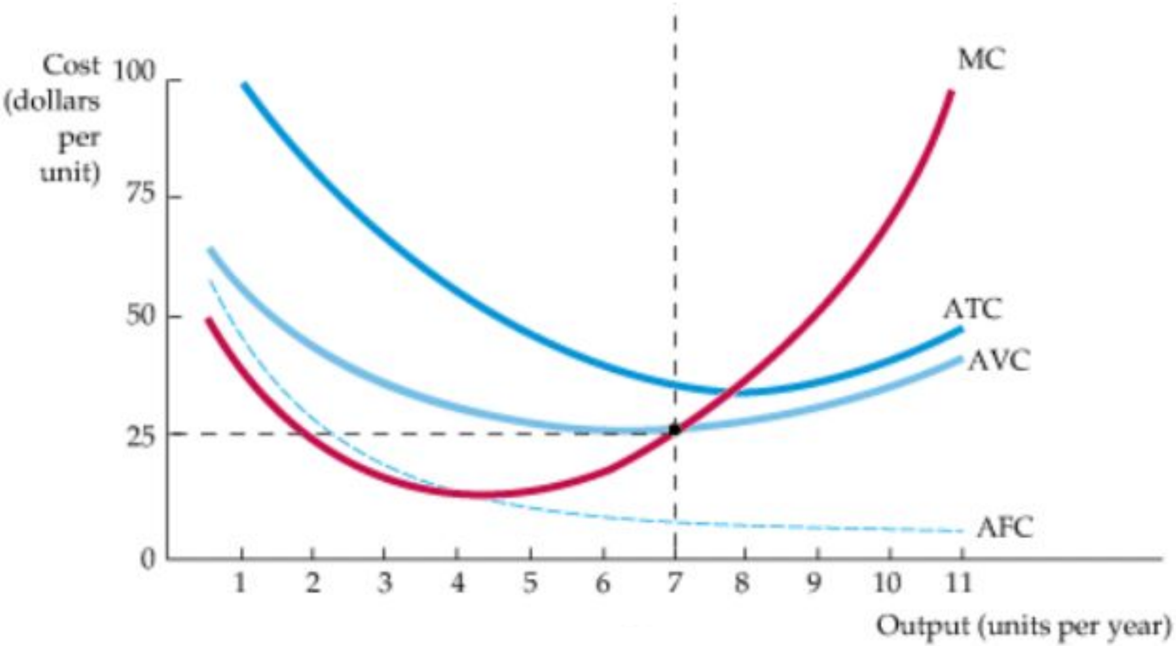
  - When next unit costs more than previous units, it pulls the average cost up.



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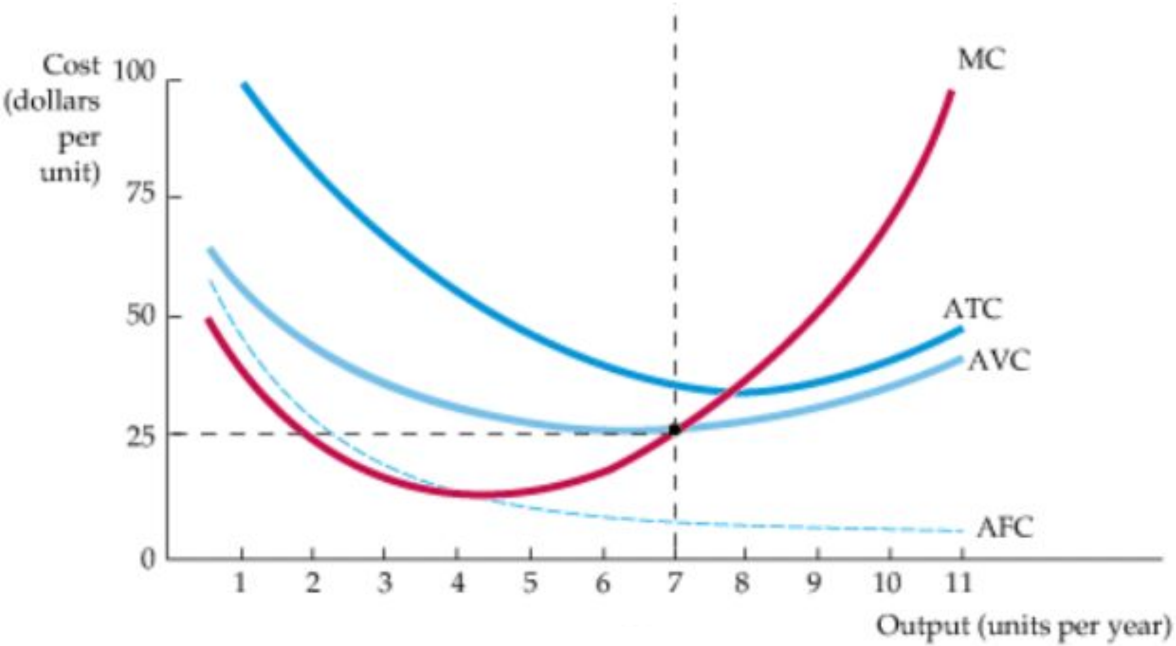
- MC intersects minimum of ATC. Why?



Graphical Illustration

- AFC always decreases. Why?
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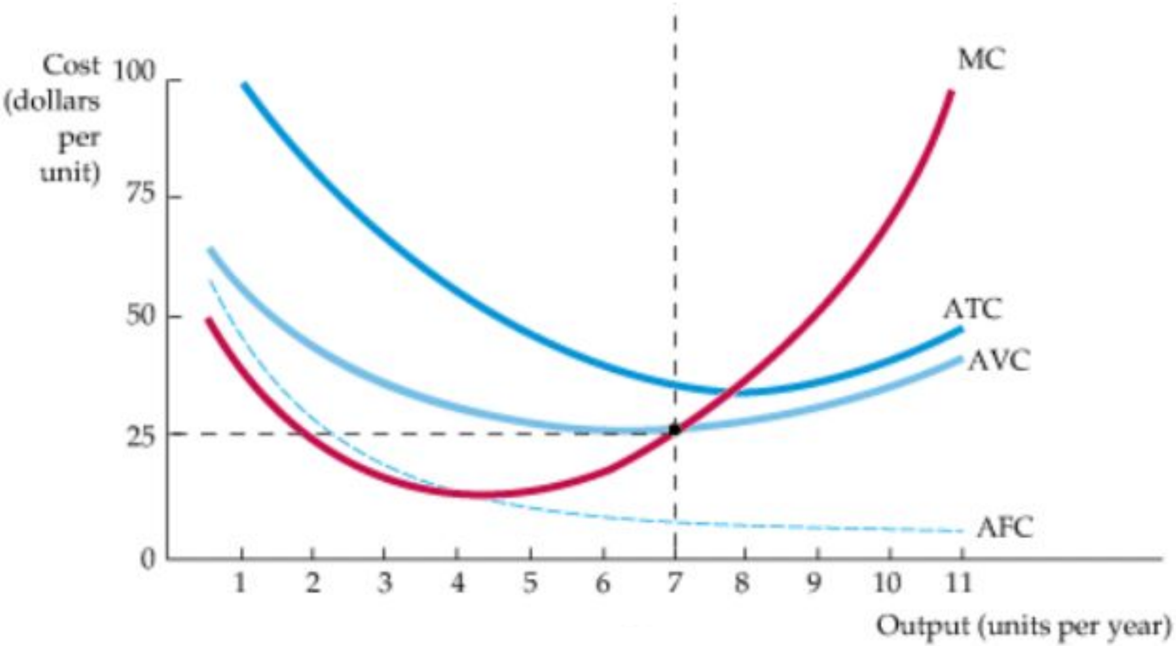
- MC intersects minimum of ATC. Why?
  - Same as above.



Costs in the Long Run

Minimum cost in the long run

- In the long run, when K and L are both variable, deriving minimum cost is more difficult
- With 2 (or more) variable inputs, there are many ways to produce a given amount of output.
- For example, if Y = KL, then we can produce Y = 24 with:
  - K = 24, L = 1, expenditure = 24r + w
  - K = 6, L = 4, expenditure = 6r + 4w
  - K = 4, L = 6, expenditure = 4r + 6w
  - …
- Need to solve the cost-minimization problem

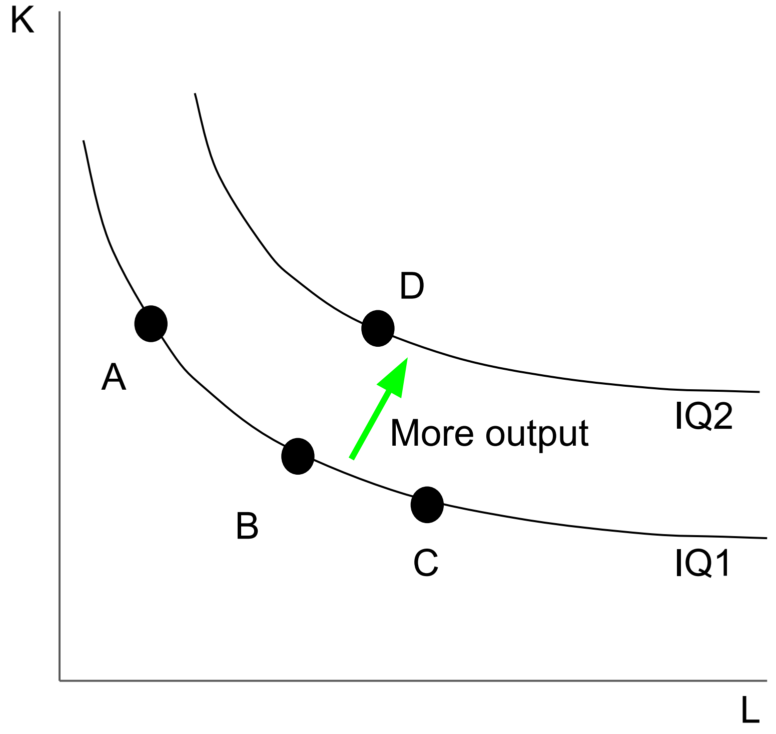
$$TC(q) = \min \; rK + wL$$
$$s.t. \; F(K, L) = q$$

Solving for minimum cost in the long run

- We will use a graphical technique with similarities to consumer theory
- Consumer theory:
  - Utility maximization
    - Reach the highest indifference curve that touches the budget set
- Producer theory:
  - Cost minimization
    - Reach the *lowest* <u>isocost line</u> that touches the <u>isoquant</u>

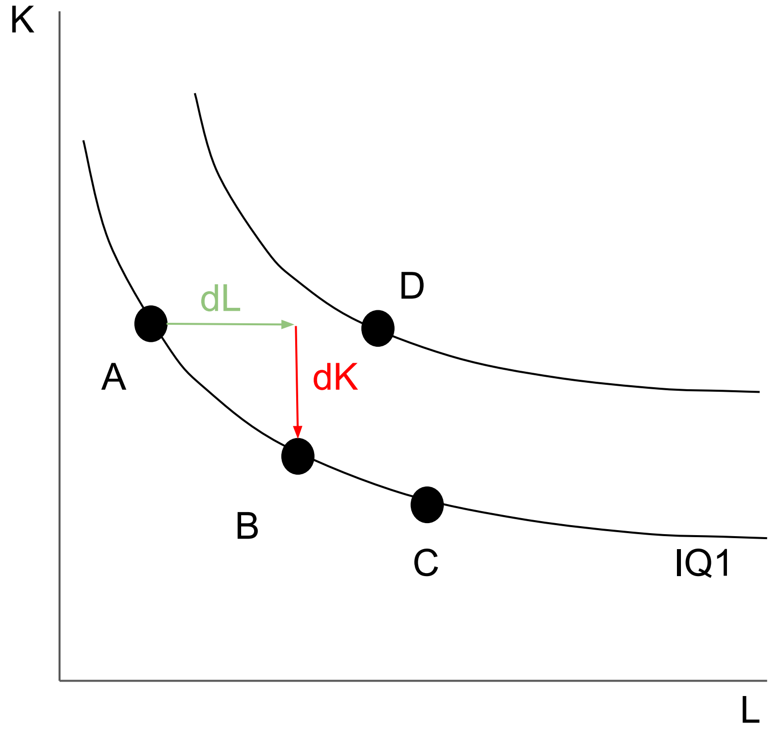
Isoquants

- Output is the same at every point – such as A, B, and C – along the IQ1 curve.
- D is on a higher isoquant and so represents more output than any point on IQ1.
- *Every* point in the plane is on some isoquant!
    - We just choose to show a select few as a way to view “slices” of this 3-D object on a 2-D page



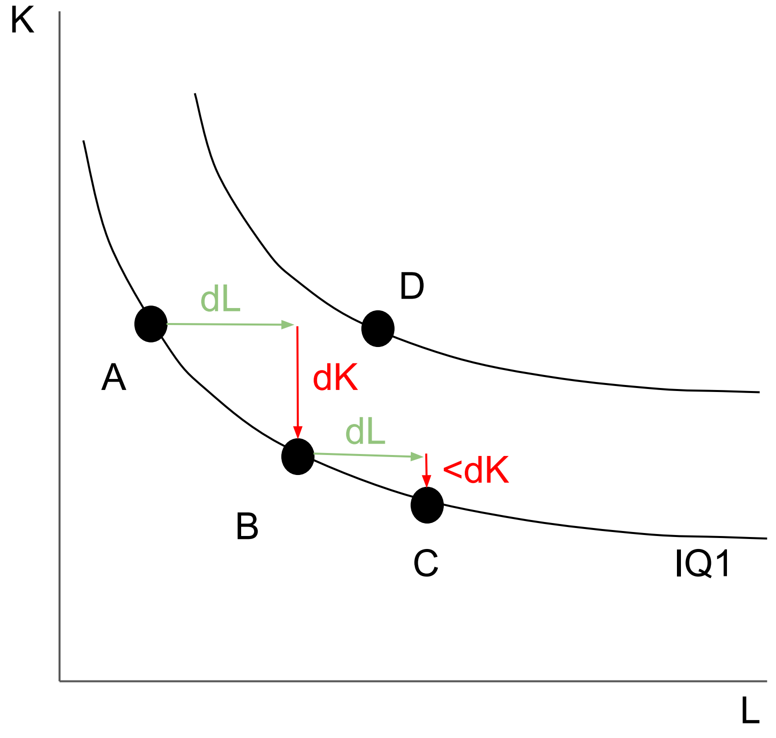
Isoquants

- Isoquants are decreasing because if you use more labor, you can use less capital to keep output constant



Isoquants

- Isoquants are often <u>convex</u> (slope flattens) because of decreasing marginal product
  - At point A, you have a lot of capital and little labor, so a little more labor can be very productive relative to a little more capital
  - At point B, you have less capital and more labor, so an increase in labor doesn't allow for as large of a cutback in capital as at point A
- Note the similarity to indifference curves in consumer theory, with decreasing marginal utility



Marginal rate of technical substitution (MRTS)

- The slope of an isoquant is called the marginal rate of technical substitution (MRTS)
- If you add one 1 unit of labor, how much capital can you cut while keeping production constant?
  - i.e. rise over run, or slope
- Note the similarity to consumer theory ("marginal rate of substitution" [MRS])
- Will now show that MRTS is the ratio of the marginal products of the two inputs

“Perturbation”/“Total Derivative”

- Suppose we allow L and K to both vary a little bit; how will output change?
- Production function is F(K,L). Change in output is:

$$dq = \frac{\partial F}{\partial L} \cdot dL + \frac{\partial F}{\partial K} \cdot dK = MP_L \cdot dL + MP_K \cdot dK$$

- L and K both change, so to see how each one affects output, you just multiply by the marginal product of each one, then add them together.

Finding the isoquant's slope

- The premise of the isoquant is that L and K change but output does not
- So let's set the total derivative (dq) equal to 0

$$dq = MP_L \cdot dL + MP_K \cdot dK = 0$$

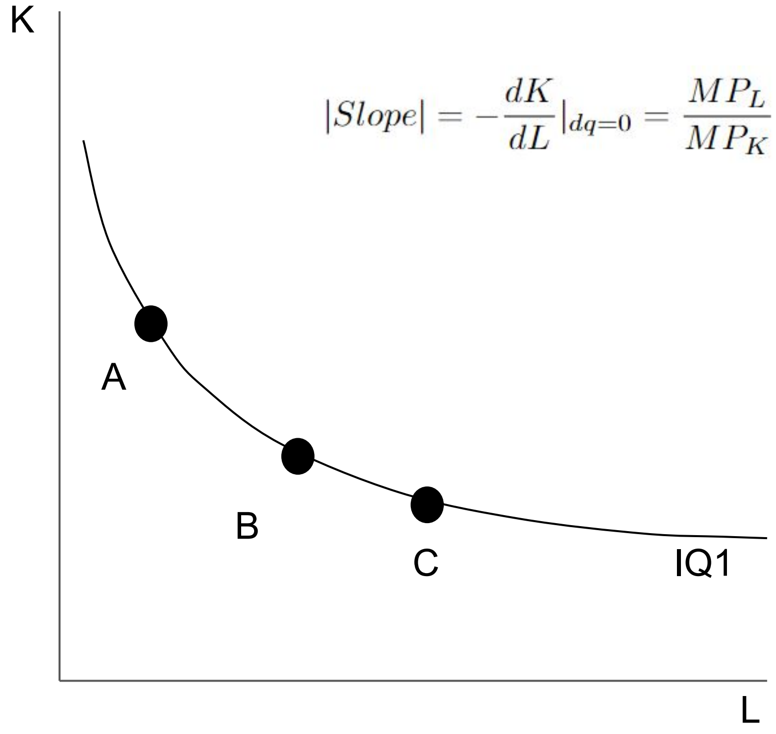
- And now we can find the slope: how much K must change per unit of change of L to keep output constant:

$$-\frac{dK}{dL}\Big|_{dq=0} = \frac{MP_L}{MP_K}$$

Visualizing the MRTS

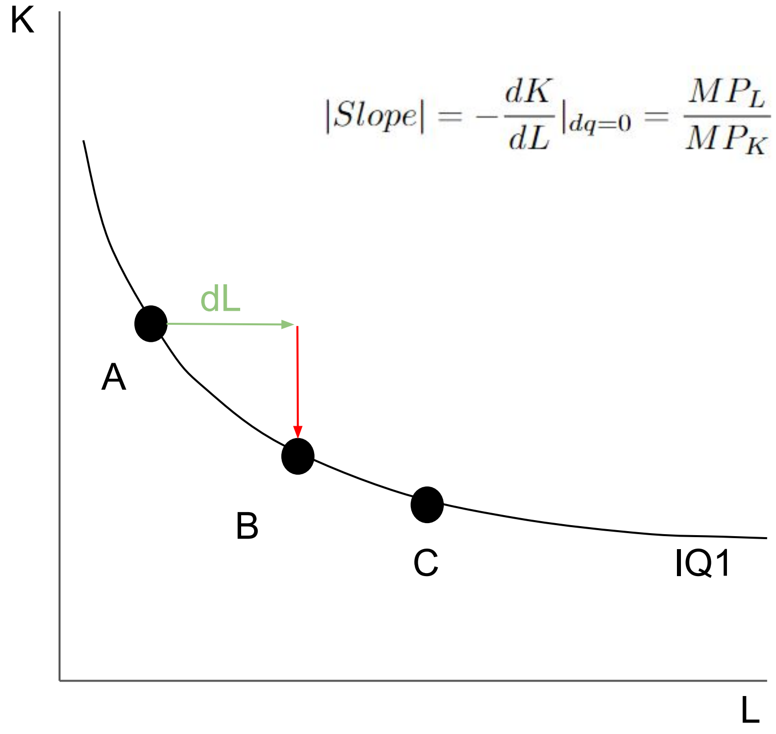
- (The magnitude of) the slope of an isoquant is the ratio of the marginal products

$$|Slope| = -\frac{dK}{dL}|_{dq=0} = \frac{MP_L}{MP_K}$$



Visualizing the MRTS

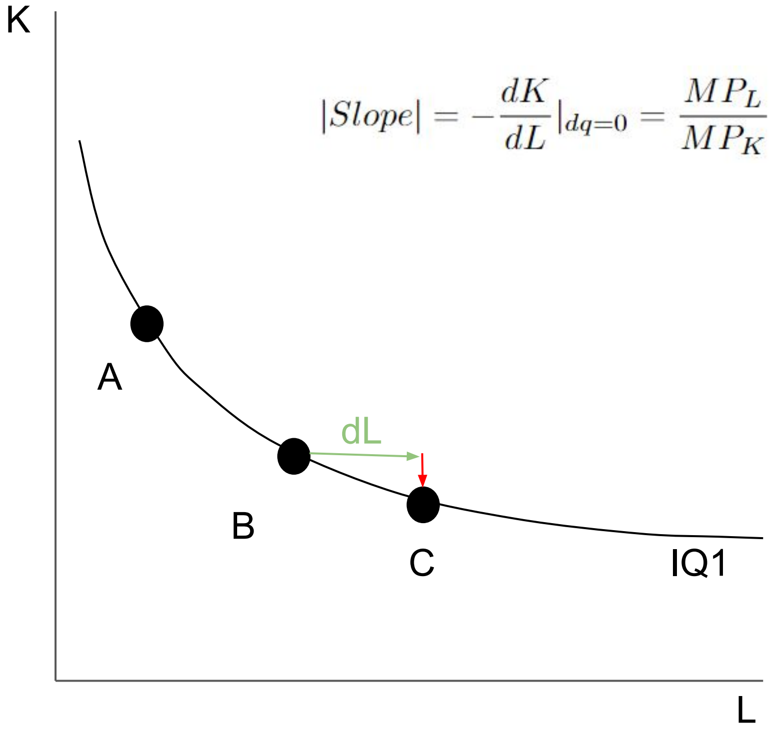
- If the marginal product of L is high relative to K (because L is low and K is high), you can give up a lot of K if you add another unit of L.
  - Therefore, the isoquant will be steep.



$$|Slope| = -\frac{dK}{dL}|_{dq=0} = \frac{MP_L}{MP_K}$$

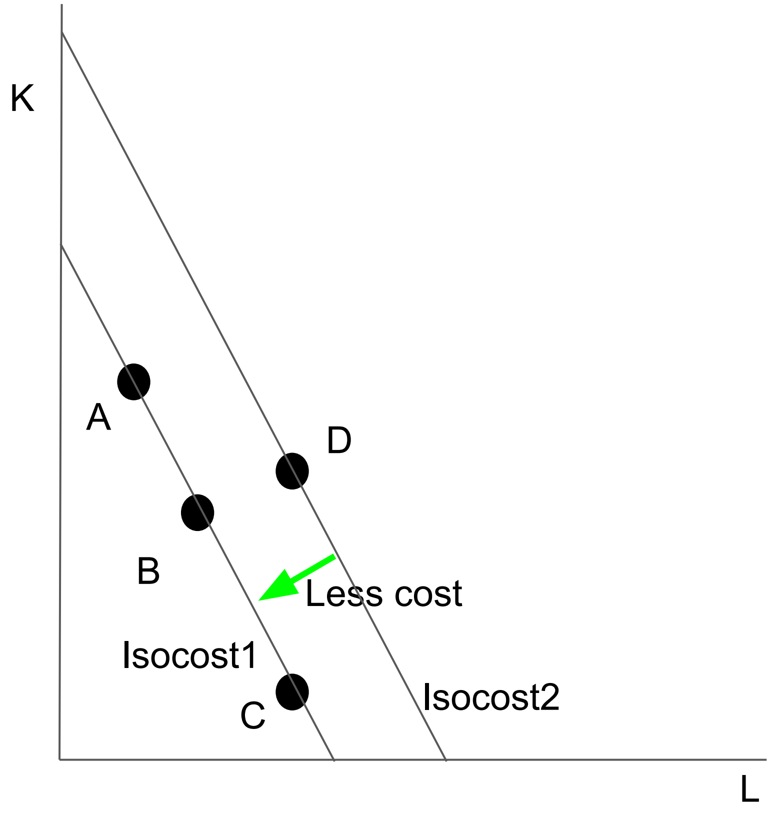
Visualizing the MRTS

- If the marginal product of L is low relative to K, adding a unit of L does not allow you to give up much K.
  - Therefore, the isoquant will be flat.



Isocost lines

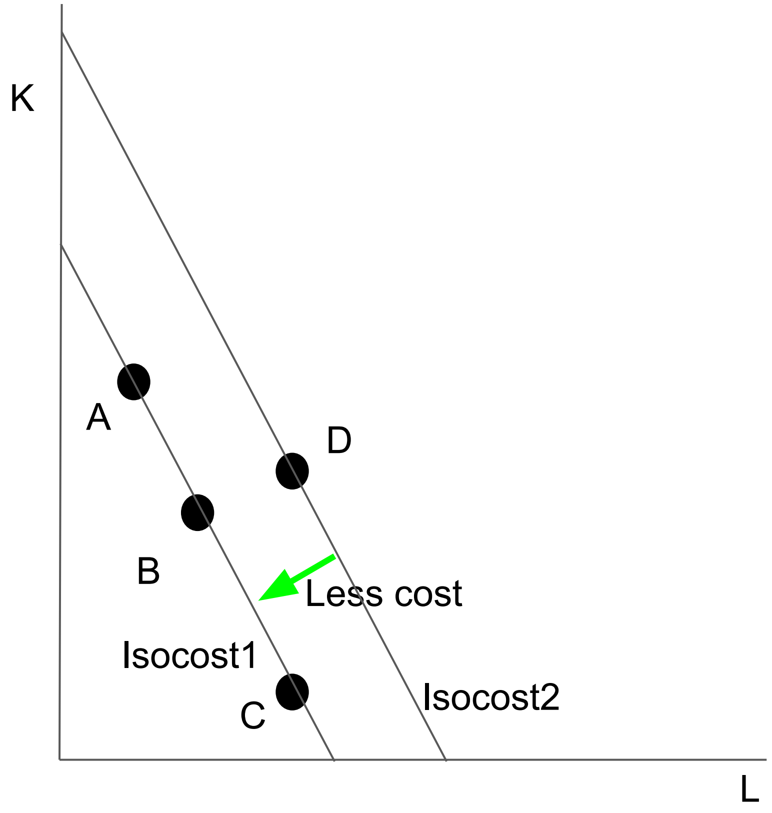
- Isoquants give us a way to visualize how much output each (L, K) produces
- Isocost lines tell us how much they cost
- Points A, B, and C all cost the same amount to produce
- D is on a higher isocost line, so it costs more



Isocost lines

- The slope of an isocost line:
  - If you add a unit of L, how much must you change K to keep cost constant?
- The slope is -w/r:
  - Adding 1 L increases cost by $w
  - Each unit of K costs $r, so to save $w, I must decrease K by w/r

$$w \cdot 1 + r \cdot -\frac{w}{r} = 0$$



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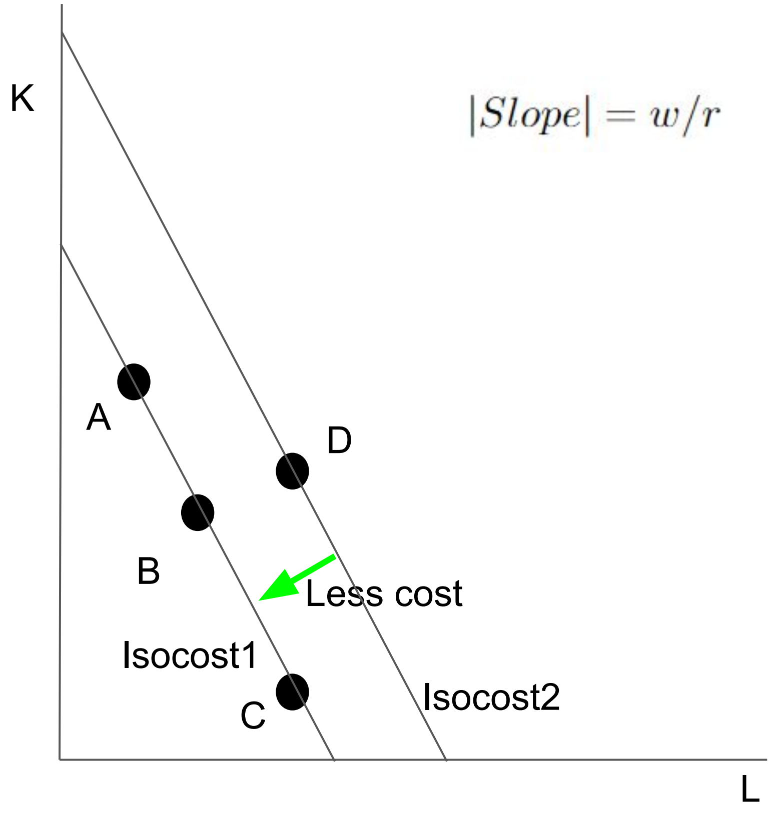
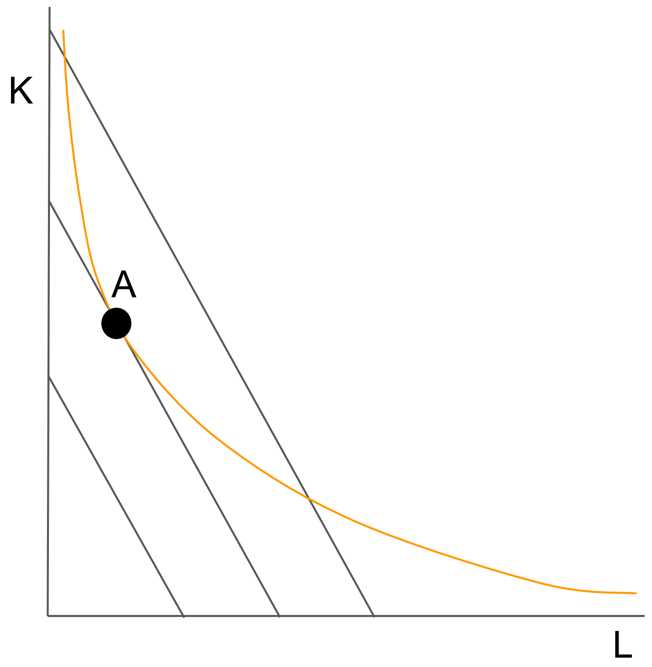


Solution to the cost minimization problem

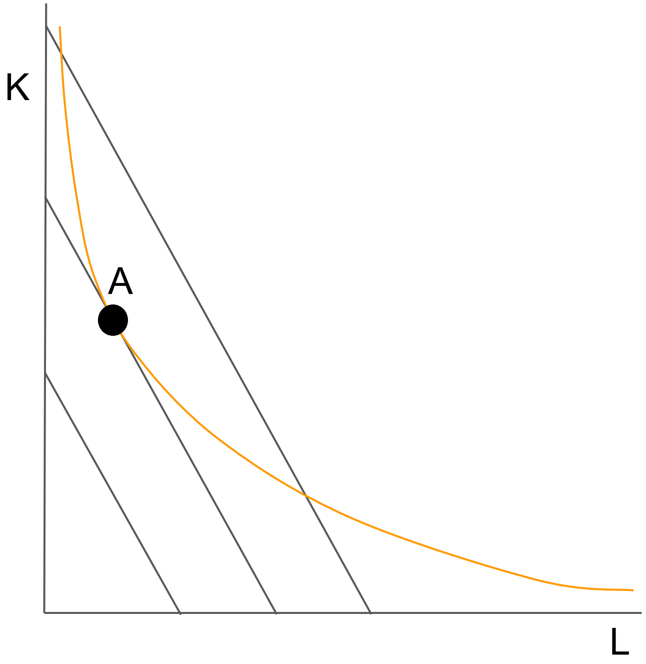
- The solution is at the tangency point between the isoquant and an isocost curve (A)
- Any other point on the isoquant is associated with a higher isocost curve
- Alternatively, any point with lower cost is below the isoquant (i.e. does not generate enough output)



Cost minimization condition

- Tangency implies: $$\frac{MP_L}{MP_K} = w/r$$

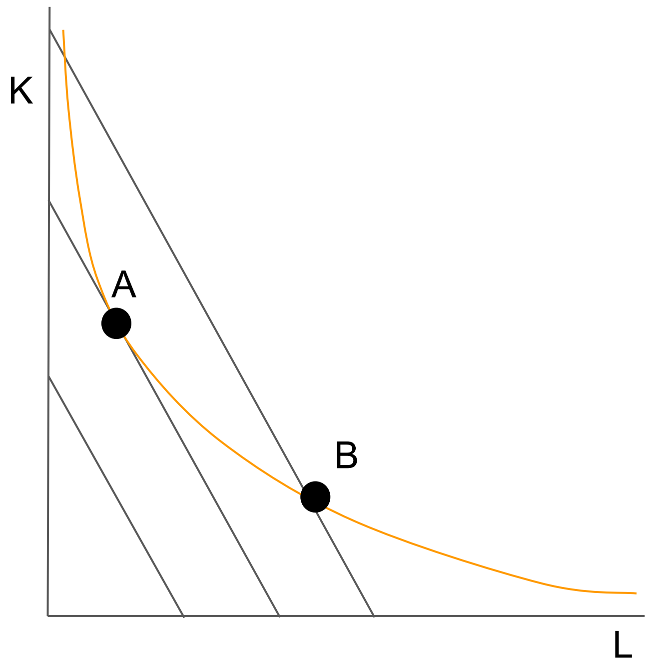
  or $$\frac{MP_L}{w} = \frac{MP_K}{r}$$
- An additional dollar spent on labor yields the same output as an additional dollar spent on capital – that's optimality



Cost minimization condition

- Tangency implies: $\frac{MP_L}{MP_K} = w/r$

  or $\frac{MP_L}{w} = \frac{MP_K}{r}$
- At point B, $\frac{MP_L}{w} < \frac{MP_K}{r}$, so you can lower cost by shifting production away from labor, toward capital (back towards A)



Calculating a long-run total cost curve (with example)

Suppose we have a production function of $Y = K^{0.2}L^{0.8}$ and $r = 1$ and $w = 2$

1. Calculate MP_L and MP_K

$$MP_L = 0.8K^{0.2}L^{-0.2} \qquad MP_K = 0.2K^{-0.8}L^{0.8}$$

2. Set $\frac{MP_L}{MP_K} = w/r$

$$\frac{MP_L}{MP_K} = \frac{0.8K^{0.2}L^{-0.2}}{0.2K^{-0.8}L^{0.8}} = 4\frac{K}{L} = 2 = w/r$$

3. Solve for L in terms of K (or vice versa)

$$L = 2K$$

Calculating a long-run total cost curve (with example)

Suppose we have a production function of $Y = K^{0.2}L^{0.8}$ and $r = 1$ and $w = 2$

4. Plug back into production function to solve for K (or L) in terms of q

$$q = K^{0.2}(2K)^{0.8} \rightarrow K = \frac{q}{2^{0.8}}$$

5. Use step #3 to solve for L in terms of q

$$L = 2K = 2 \cdot \frac{q}{2^{0.8}} = 2^{0.2}q$$

6. Use w and r to compute total cost

$$TC = rK + wL = 1 \cdot \frac{q}{2^{0.8}} + 2 \cdot \left(2 \cdot \frac{q}{2^{0.8}}\right) = \frac{5q}{2^{0.8}}$$

Other cost measures

$$TC(q) = \frac{5q}{2^{0.8}}$$

$$TFC = AFC = 0$$

$$TVC(q) = \frac{5q}{2^{0.8}},\ AVC(q) = \frac{5}{2^{0.8}}$$

$$ATC(q) = \frac{5}{2^{0.8}}$$

$$MC(q) = \frac{5}{2^{0.8}}$$

$$AVC(q) = ATC(q) = MC(q)$$

Other cost measures

$$TC(q) = \frac{5q}{2^{0.8}}$$

$$TFC = AFC = 0$$

$$TVC(q) = \frac{5q}{2^{0.8}}, \ AVC(q) = \frac{5}{2^{0.8}}$$

$$ATC(q) = \frac{5}{2^{0.8}}$$

$$MC(q) = \frac{5}{2^{0.8}}$$

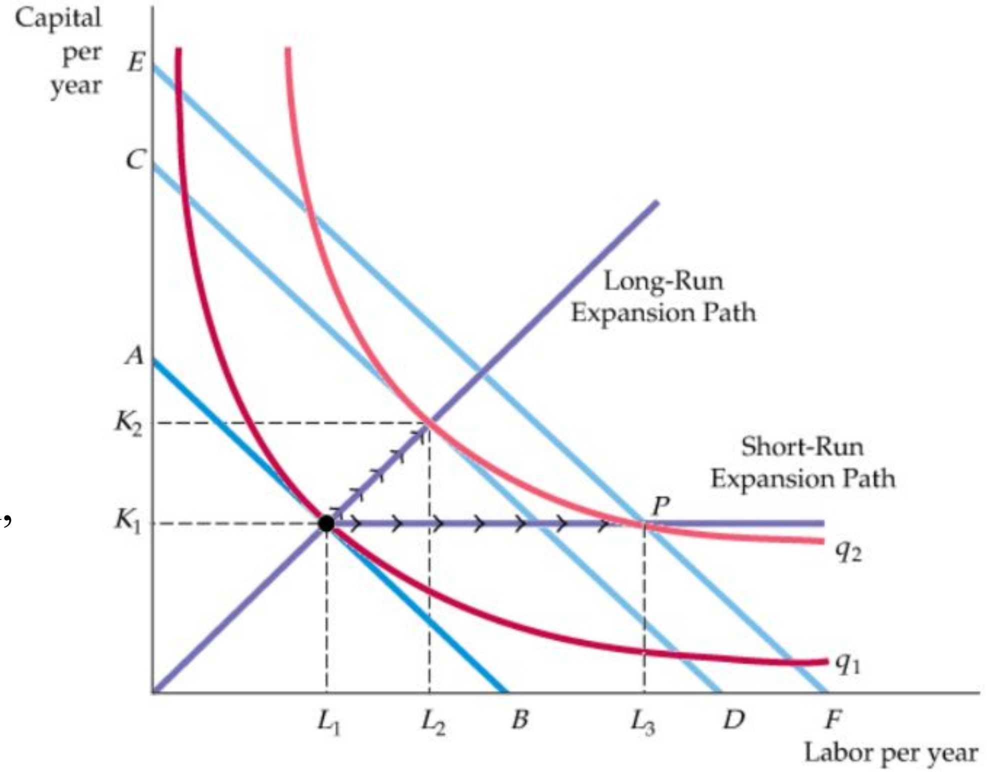
$$\boxed{AVC(q) = ATC(q) = MC(q)}$$ Why?

Comparing costs in the short run vs long run

- We know that for any output level, q, it is less costly to produce it when all inputs are flexible (long run) than when some are fixed (short run)
- Why?

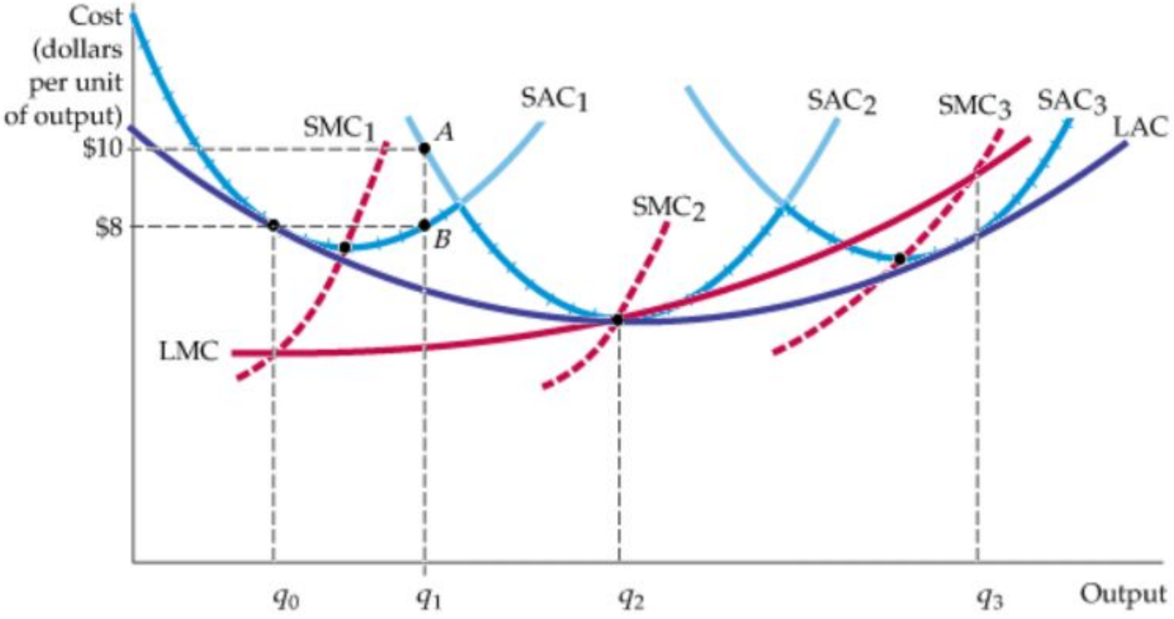
Comparing costs in the short run vs long run

- We know that for any output level, q, it is less costly to produce it when all inputs are flexible (long run) than when some are fixed (short run)
- Why?
    - If you want to produce q2, select (L2, K2) in the long-run to minimize cost
    - But if you're stuck at K1 in the short-run, your cost will be higher
    - *Maybe* you got lucky in the short run and already had K2; even then, short run cost is not lower than long run cost (they are equal)



Comparing costs in the short run vs long run (2)

- Long-run ATC is the “lower envelope” short-run ATCs
  - In the short run, you are stuck on <u>a single one</u> of the blue cost curves, each associated with a different level of K
  - In the long run, you get to choose which blue cost curve to be on
  - For a given quantity, you would always choose the blue curve with the lowest cost

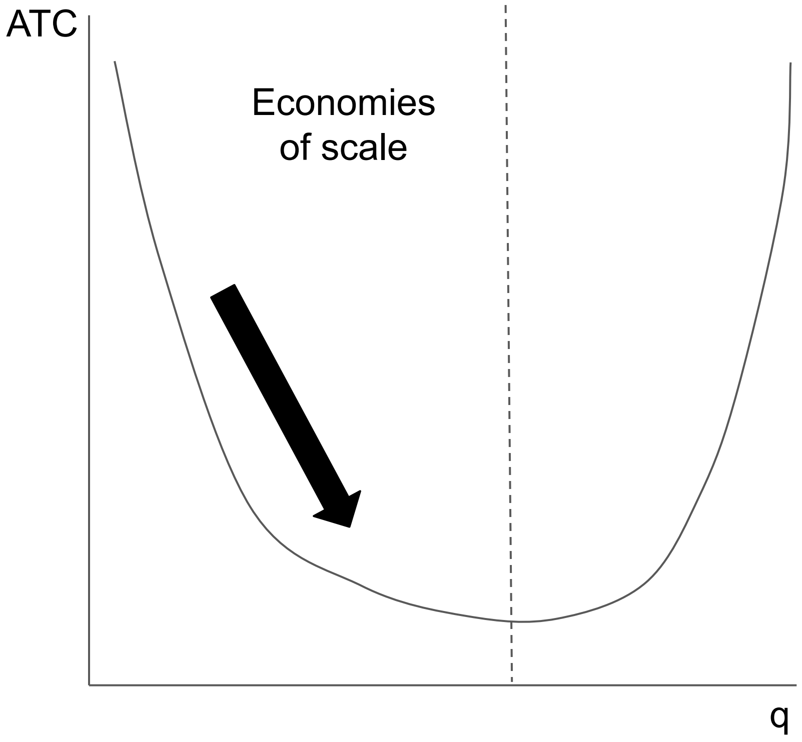


Economies of scale

- As firm increases output, does ATC go up or down?
  - Down: economies of scale
  - Up: diseconomies of scale
- This is different from returns to scale!
  - Returns to scale is purely about technology
    - Double all inputs, what happens to output?
  - Economies of scale incorporates cost-minimization
    - To double your output, you might increase some inputs more than others, which is incorporated into the cost function
- Increasing returns to scale implies economies of scale
  - If you doubling inputs more-than-doubles the output, your average cost will fall
    - And perhaps you can do even better by increasing some inputs more than others
  - But economies of scale does not imply increasing returns to scale
- Does decreasing returns to scale imply diseconomies of scale?

Economies of scale (2)

- We typically think that economies of scale exist when q is low
  - Improved specialization and flexibility
  - Learning-by-doing
  - Bulk purchases of inputs



Economies of scale (2)

- We typically think that economies of scale exist when q is low
  - Improved specialization and flexibility
  - Learning-by-doing
  - Bulk purchases of inputs
- We typically think that diseconomies of scale exist when q is high
  - Some inputs may be limiting (short run)
  - Managing more people doing more things requires expensive managerial/coordination costs