Time, Risk, and Finance

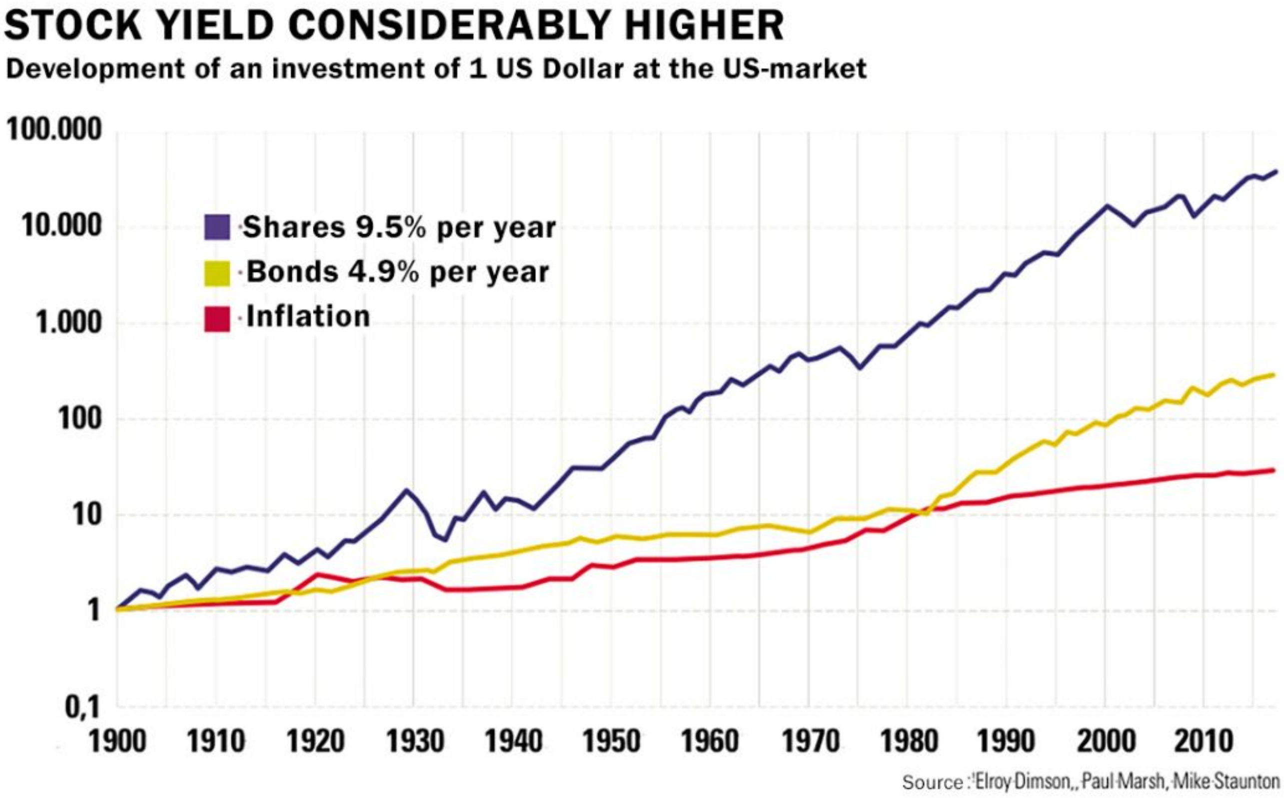
Fall 2023
Econ 2316, Northeastern University
Prof. Josh Abel

P&R: chapters 5 and 15 (especially 15.2-15.3)
Emerson: chapter 23

Introduction

- Benefits and costs of some decisions unfold over time
- Sometimes, a decision must be made before its costs and/or benefits are known
- <u>Finance</u> is the study of such decisions
- It has extremely broad applications
    - Stock and bond markets
    - Insurance
    - Buying a home; mortgage?
    - Investing in education; student loans?
    - Building a factory
    - Funding research
    - …
- Despite varied applications, some tools and insights are universal

Returns on stocks and bonds



Time

Time Value of Money

- Suppose you have a savings account with a 1% annual interest rate
- Someone offers you two choices:
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  - You can have $1 in a year
  - Which do you choose?

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  - Which do you choose?
- If you accept the $1 today, you can save it and it becomes $1.01 in a year
  - Choose $1 today!

Money today is worth more than money in the future

(assuming interest rates are positive)

Time Value of Money

- Financial assets offer a stream of future payments:
  - $100, ten years from now (bond with balloon payment)
  - $100, every year for ten years (bond)
  - $100, every year for eternity (annuity)
  - 1% of Apple's earnings, forever (stock)
  - The value of your house, if your house burns down (insurance)
- Valuing such assets *today* requires discounting those future payments
  - Because money in the future is less valuable than money today

Discounting example

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$$(x \cdot 1.01) \cdot 1.01 = 1$$

$$x = \frac{1}{1.01 \cdot 1.01} \approx 0.98$$

Present discounted value

- Consider an asset offering payments of $D_1,\ D_2,\ D_3, ...$
- The <u>present discounted value</u> (PDV) is the sum of the discounted series of payments, given by:

$$\text{PDV}_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + ...$$

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- If D is constant and the payments last n years:

$$PDV_0 = \frac{D}{r} - \frac{D}{r \cdot (1+r)^n}$$

Asset pricing

- If markets are competitive and the payments are risk-free, an asset's price should equal its PDV:

$$P_0 = PDV_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + \ldots$$

- Otherwise, arbitrage (profit without risk) would be possible
  - Much of financial theory is based on the simple assumption that arbitrage is impossible

Returns

- A little algebra yields:

$$P_0 = \frac{D_1}{(1+r)} + \frac{D_2}{(1+r)^2} + \frac{D_3}{(1+r)^3} + ... = \frac{D_1 + P_1}{1+r}$$

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    - Dividend yield: “Dividend” per dollar invested
    - Capital gain: Increase in price per dollar invested

Dividend yield vs capital gains

- Consider 2 companies:

- Long-time titan of the consumer market

- Just broke into the consumer market
- Pioneer of coming wave of electric vehicles

<u>Which company would have higher dividend yield, and which would have more capital gains?</u>

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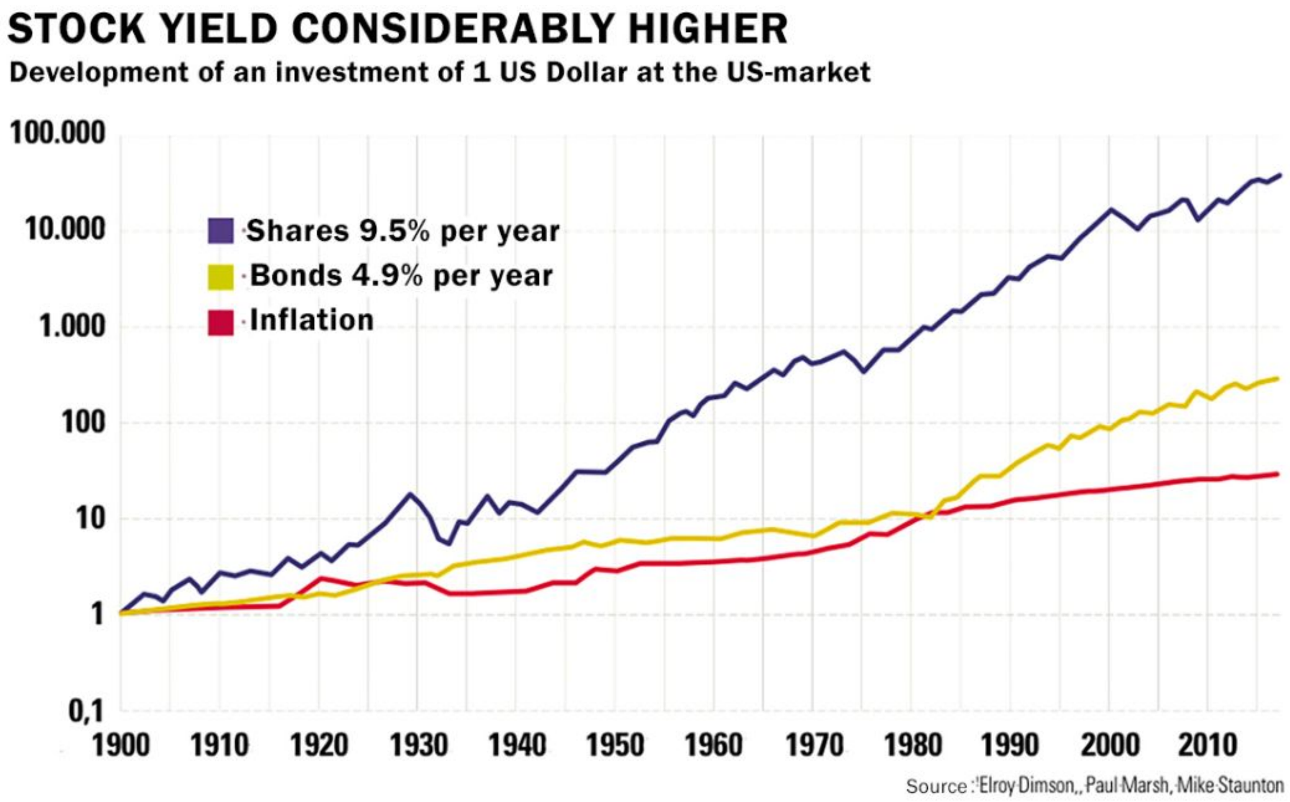
In the absence of risk, all assets must offer the same return.

Not all assets give the same return



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- What if the stakes were $10,000 rather than $1. Would you agree to play?

Risk aversion

- We typically assume that economic actors are <u>risk averse</u>
  - I.e. they prefer a sure thing to a risky outcome
- The $10,000 coin flip leaves you with the same amount of money on average, but it introduces a lot of risk/uncertainty, so most people would decline
- Risk aversion is responsible for many important phenomena that we observe
  - Insurance
  - Strict liability laws
  - Financial market returns
- To understand this, we will return to utility theory
- But first, let's talk about uncertainty

Expected value

- If we flip a fair coin 2 times, # of Heads is an uncertain outcome
  - 0, 1, or 2
  - Probabilities: 25%, 50%, and 25%, respectively
- The <u>expected value</u> is the probability-weighted average
  - E[X] = 0.25*0 + 0.50*1 + 0.25*2 = 1

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- While "average" is the same, the second gamble seems riskier – extreme outcomes more likely

Variance

- The "spread" of an uncertain outcome is typically summarized by the <u>variance</u>
  - Probability-weighted average of squared deviations from expected value
- Independent flips
  - $var(X) = 0.25 \cdot (0-1)^2 + 0.5 \cdot (1-1)^2 + 0.25 \cdot (2-1)^2 = 0.25 + 0 + 0.25 = 0.5$
- Correlated flips
  - $var(X) = 0.5 \cdot (0-1)^2 + 0 \cdot (1-1)^2 + 0.5 \cdot (2-1)^2 = 0.5 + 0 + 0.5 = 1$
- As we suspected, correlated flips lead to more risk

Covariance

- Final statistical concept is covariance, a measure of the comovement of 2 outcomes
- It is the probability-weighted average of the product of deviation from means
- Consider each individual flip: #Heads is 0 or 1, with an average of 0.5

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Independent

Flip 2 (Z)

		0	1
Flip 1 (Y)	0	25% Y-E[Y] = -0.5 Z-E[Z] = -0.5	
	1		

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cov(Y,Z) = 0.25*(-0.5)*(-0.5) + 0.25*(-0.5)*(0.5)
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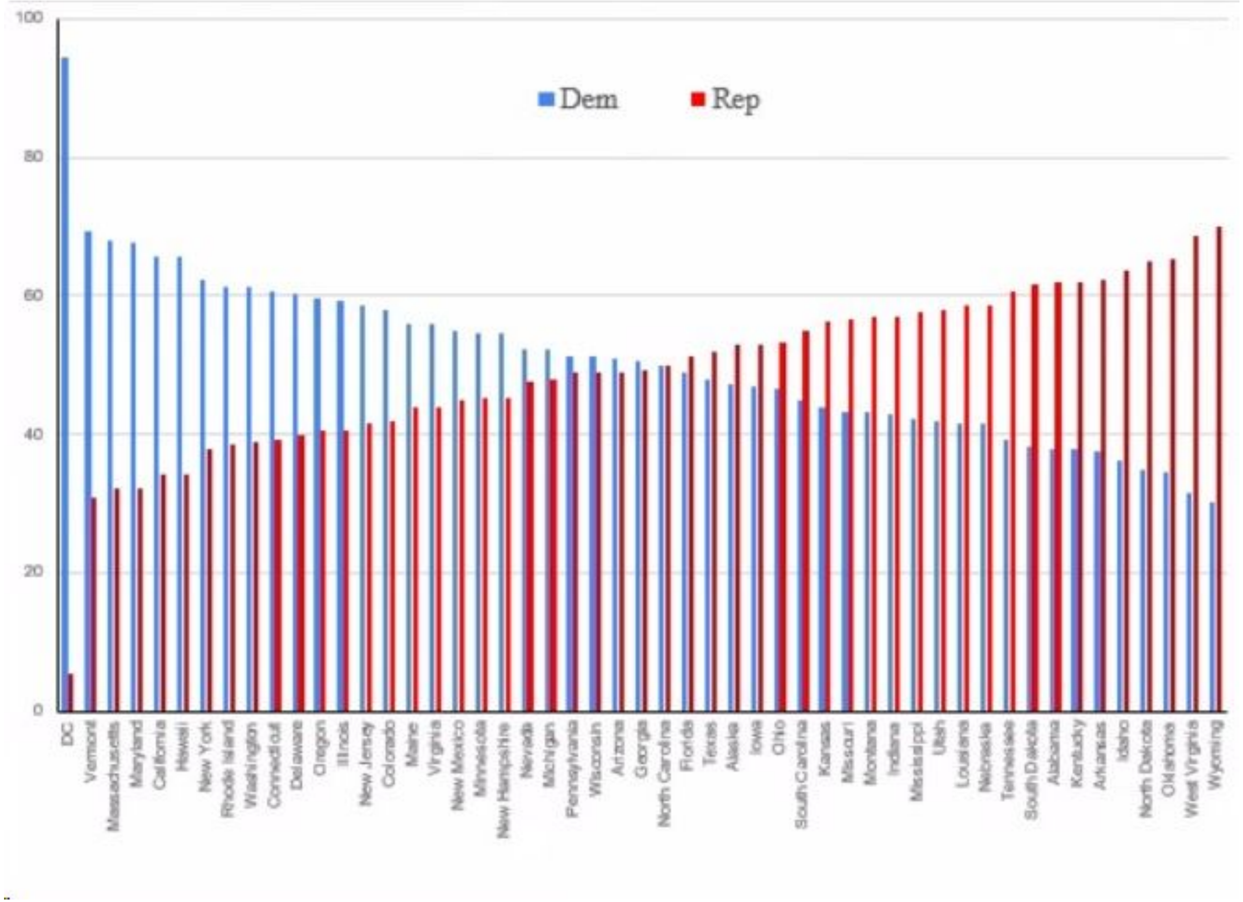
+ 0.25*(0.5)*(-0.5) + 0.25*(0.5)*(0.5) = 0

Correlated

Flip 1 (Y) \ Flip 2 (Z)	0	1
0	50% Y-E[Y] = -0.5 Z-E[Z] = -0.5	0% Y-E[Y] = -0.5 Z-E[Z] = 0.5
1	0% Y-E[Y] = 0.5 Z-E[Z] = -0.5	50% Y-E[Y] = 0.5 Z-E[Z] = 0.5

cov(Y,Z) = 0.5*(-0.5)*(-0.5) + 0*(-0.5)*(0.5)
+ 0*(0.5)*(-0.5) + 0.5*(0.5)*(0.5) = 0.25

Vote By Party and State



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- So does Rep share

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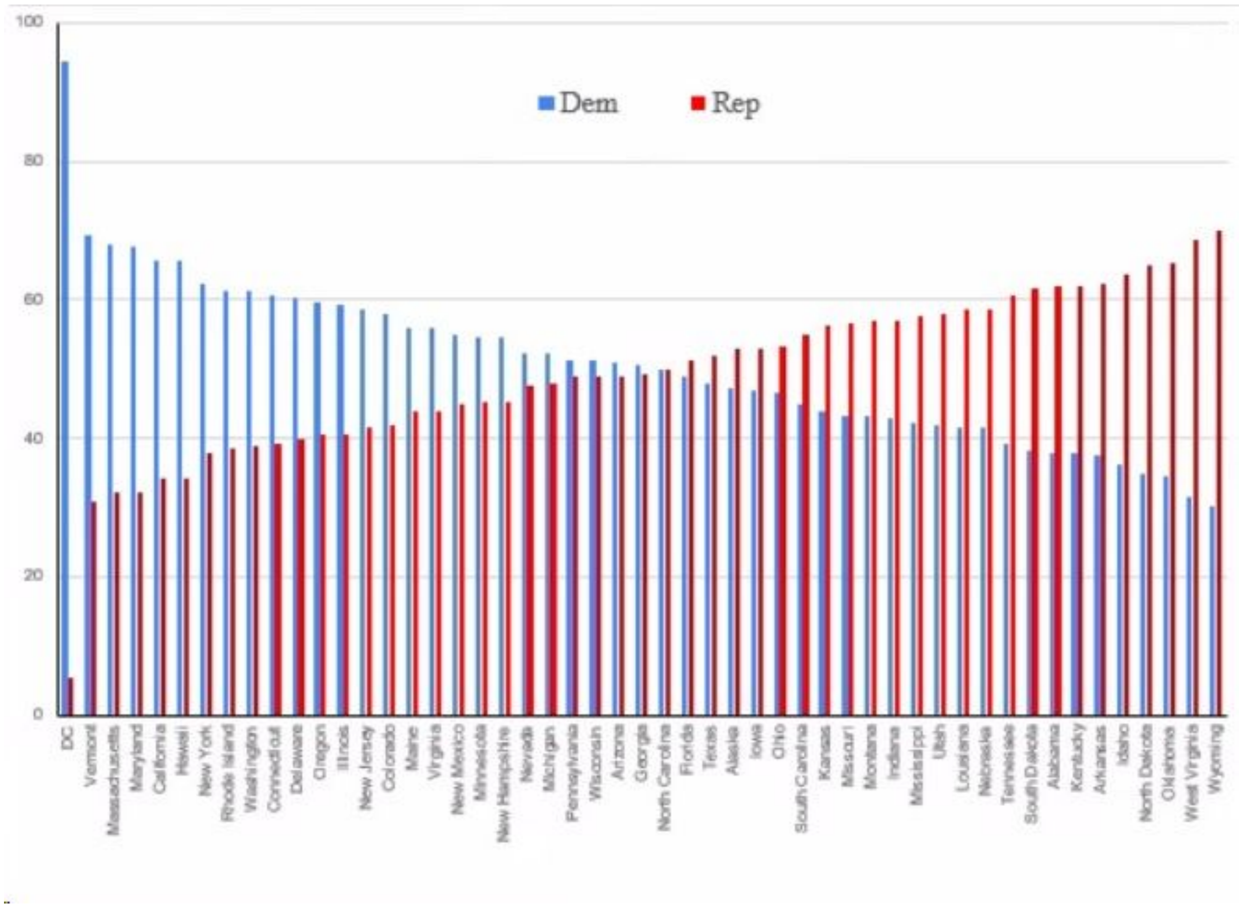

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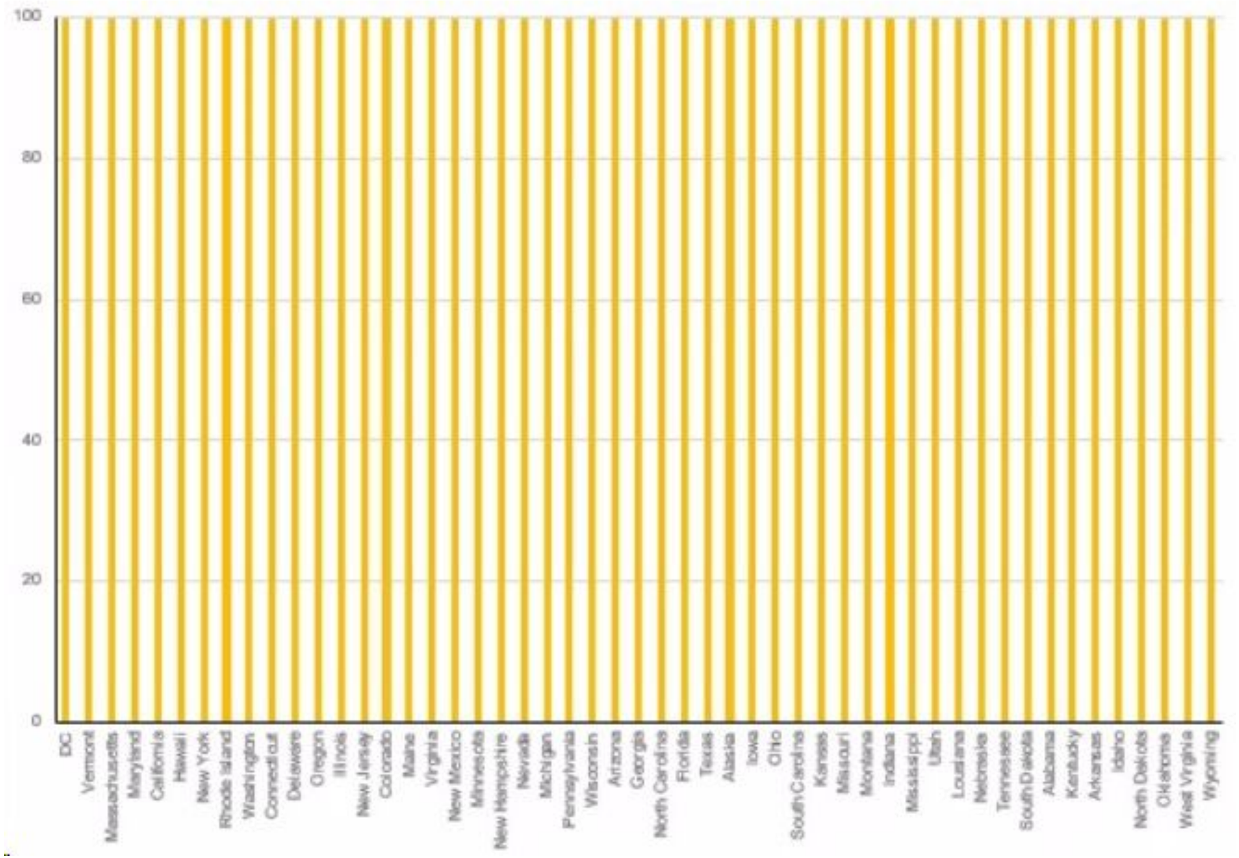

- Dem share varies a lot from state to state
- So does Rep share
- But very negative correlation
- So their sum hardly varies at all

Utility of income

- Let’s consider a new type of utility function: U(Y)
- It has only one argument, income (Y)
- In the background, the consumer is taking that income and solving the consumer choice problem we covered earlier (indifference curves, MRS, etc.)
- But we will “abstract away” from all of that right now – U(Y) summarizes where she ends up after maximizing her utility
    - Often referred to as an <u>indirect utility function</u>

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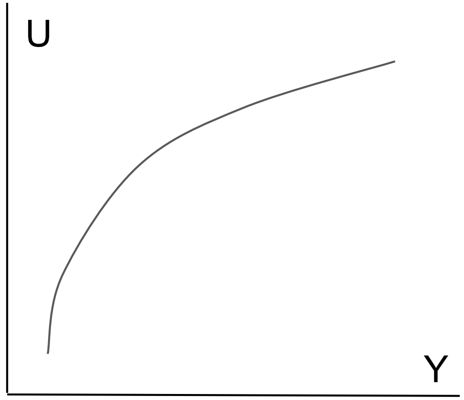
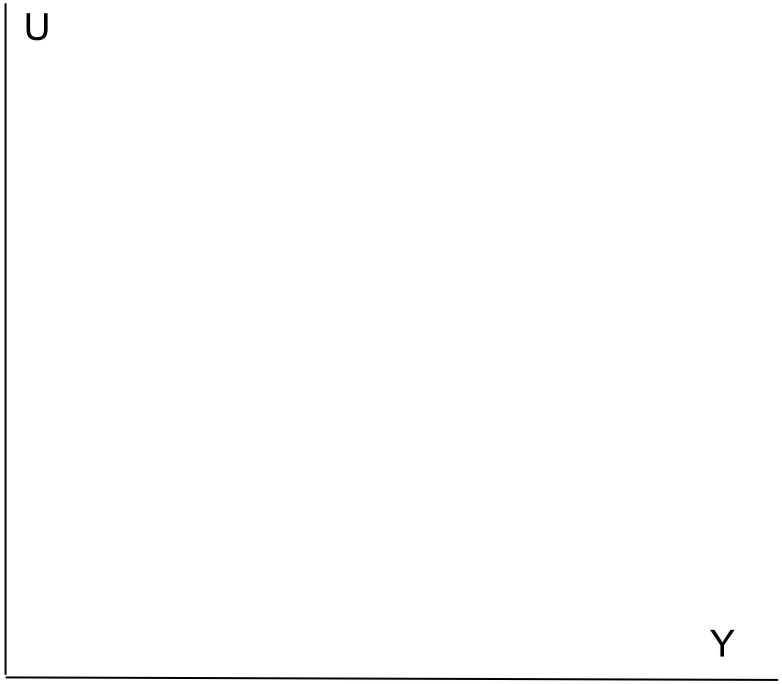
    - Often referred to as an indirect utility function
- We always assume it is increasing
- We almost always assume it is concave



Decreasing marginal utility of income

- Concavity means that <u>the marginal utility of income</u> is decreasing
    - Each dollar brings a little less utility than the one before it



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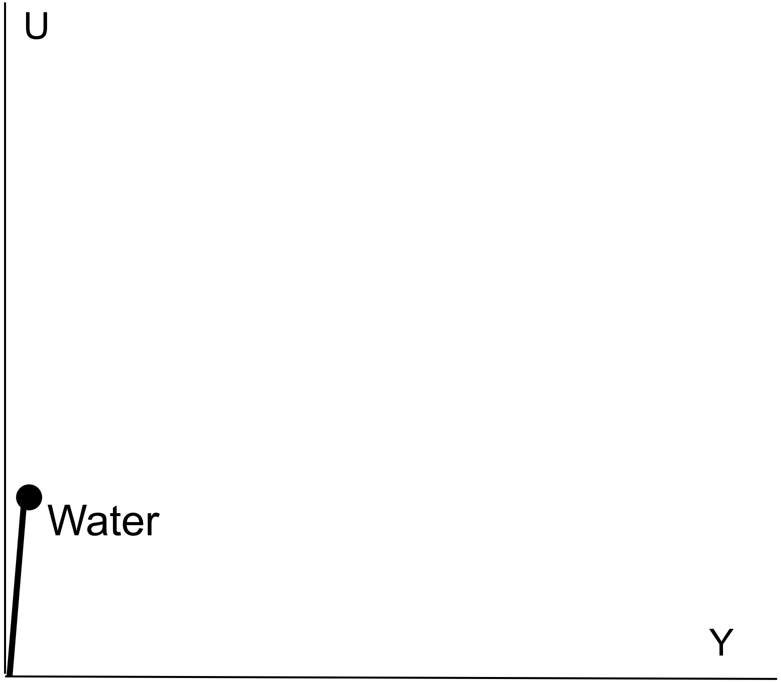

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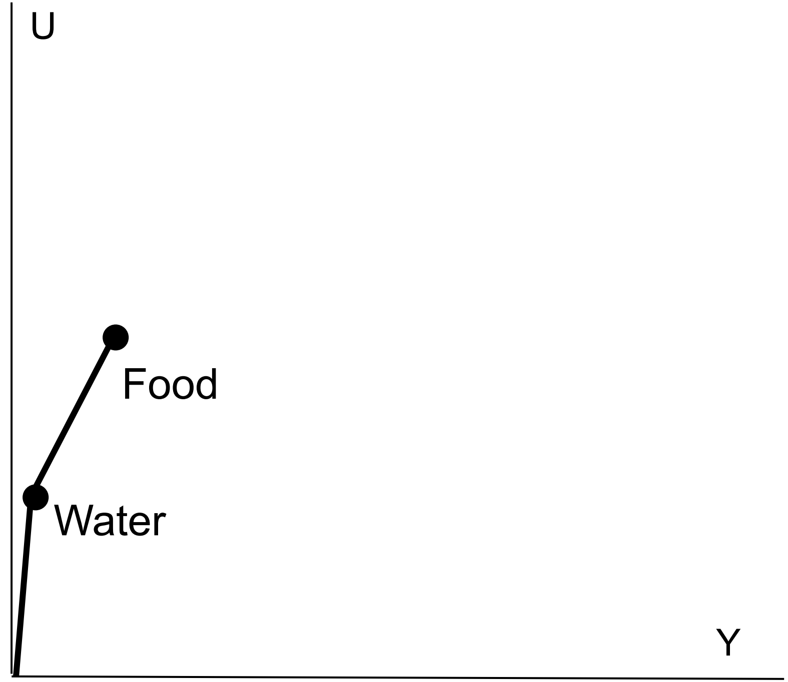
- Then food



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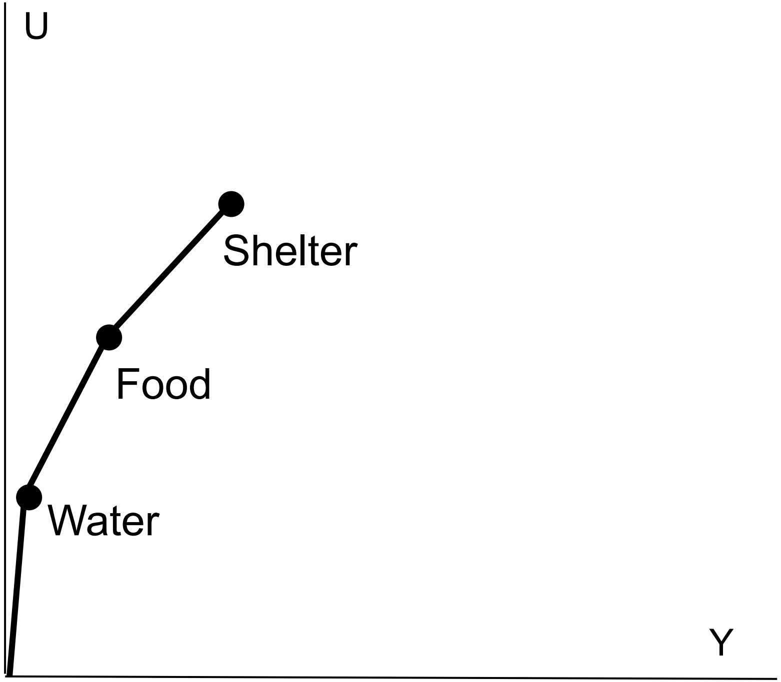
- Then shelter



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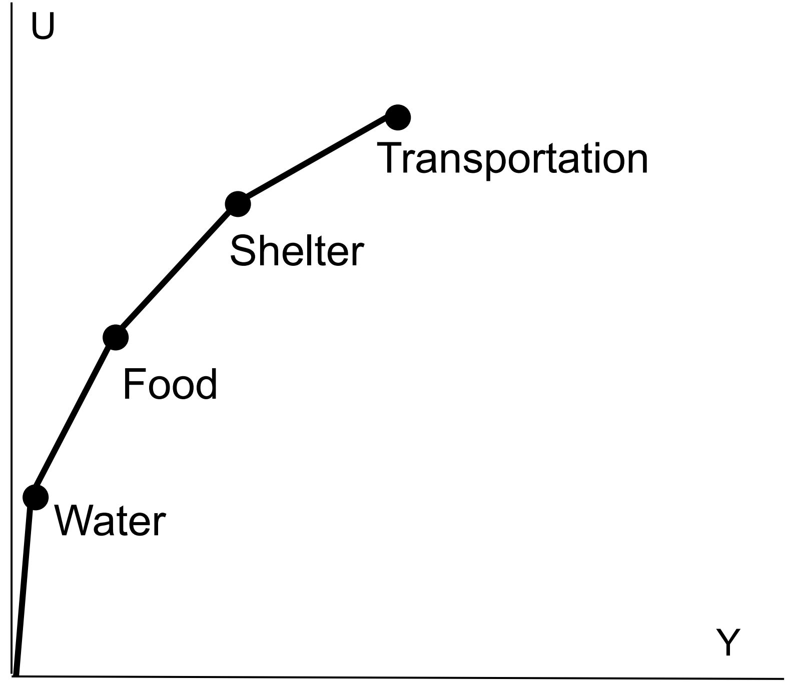

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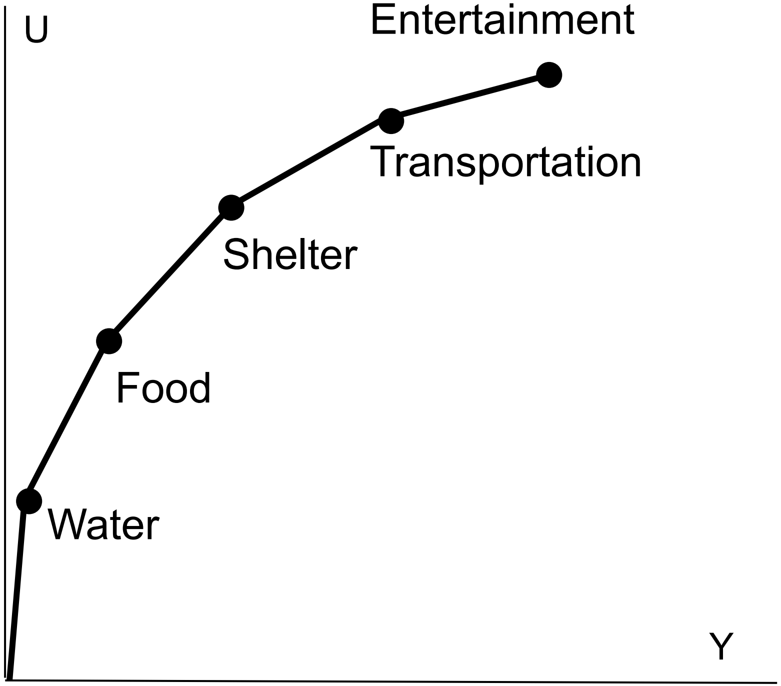

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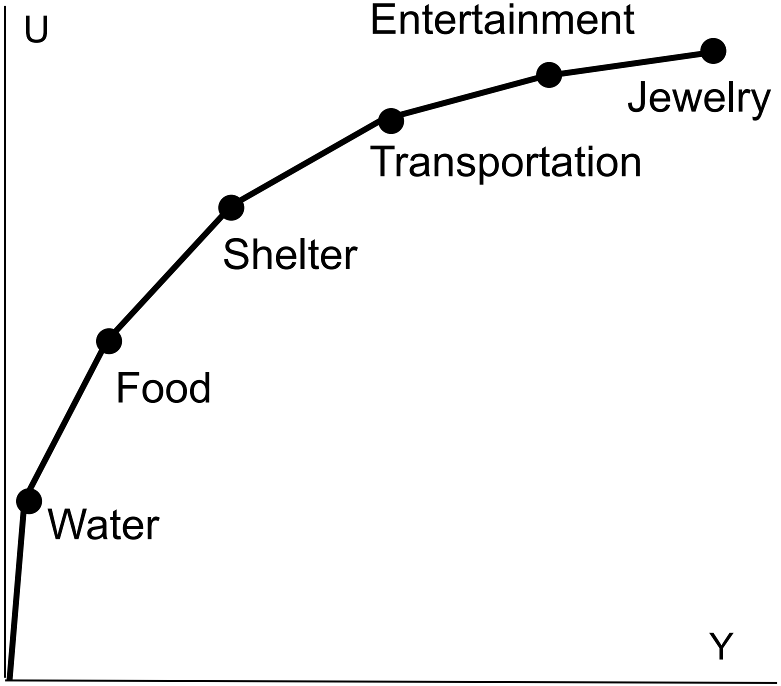

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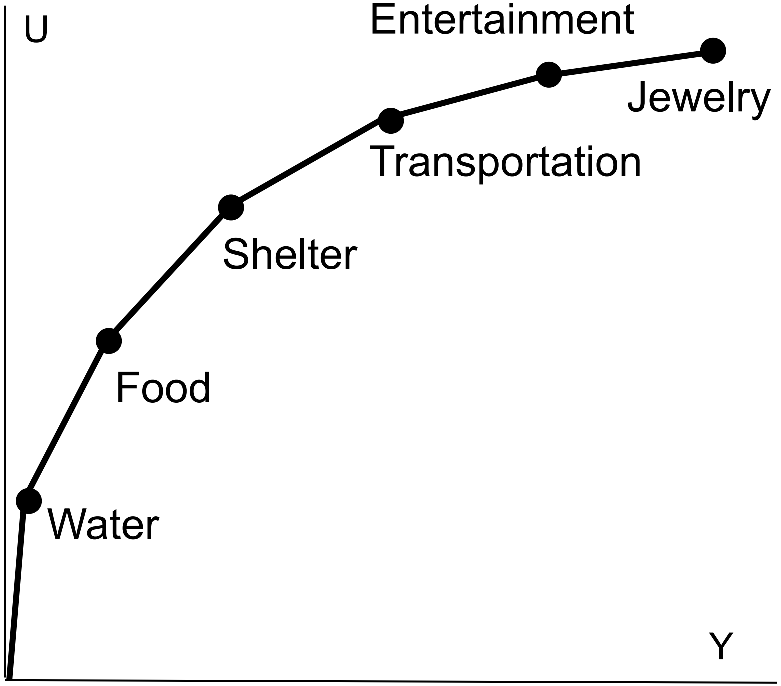
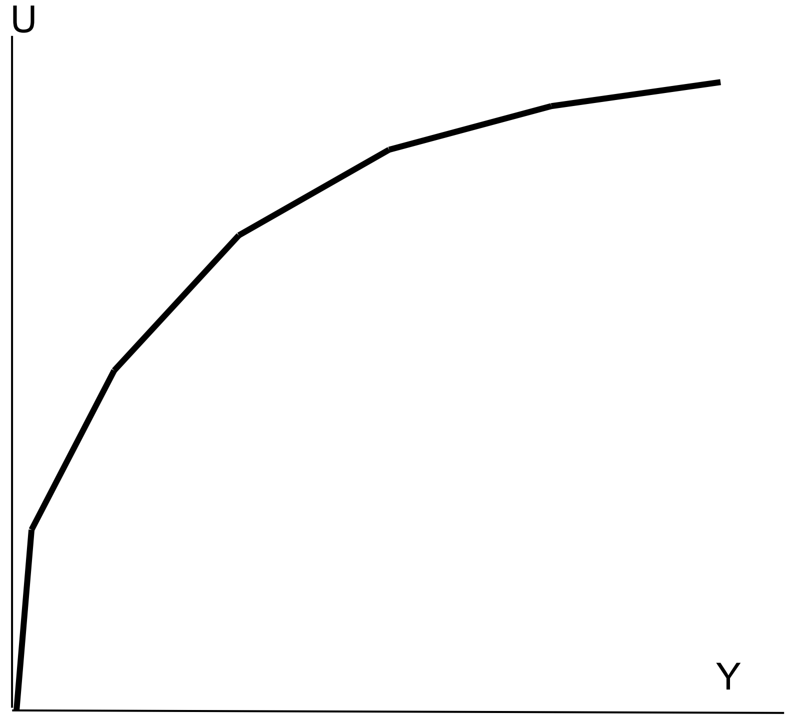
- Each bit of additional income makes you better off, but less so than previous dollars, which bought more important things



Preference for certainty

- Decreasing marginal utility of income is why people are risk averse



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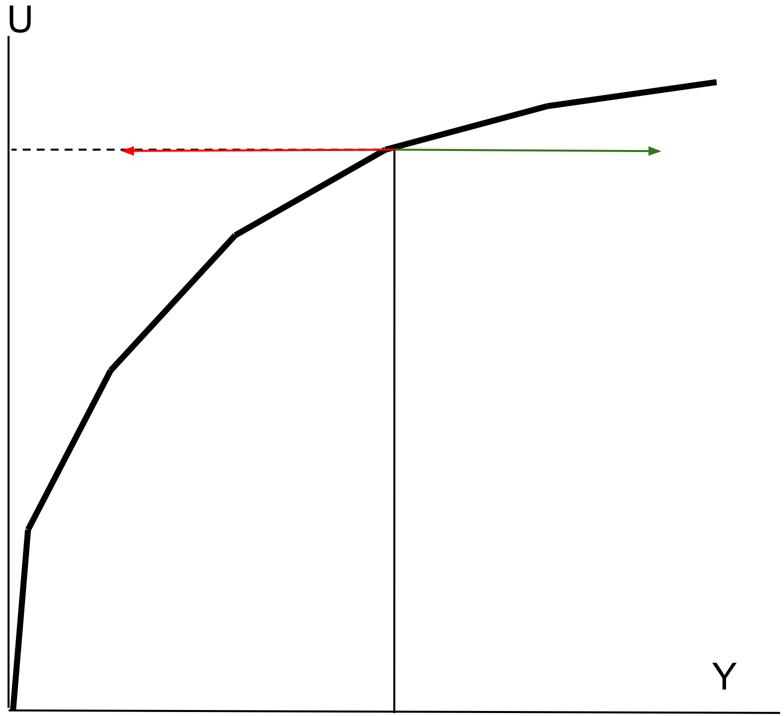

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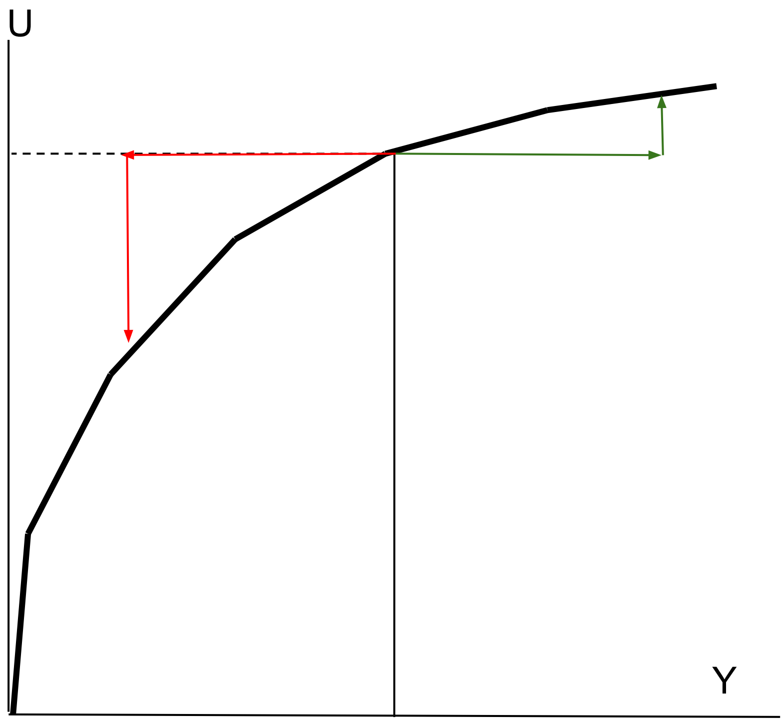

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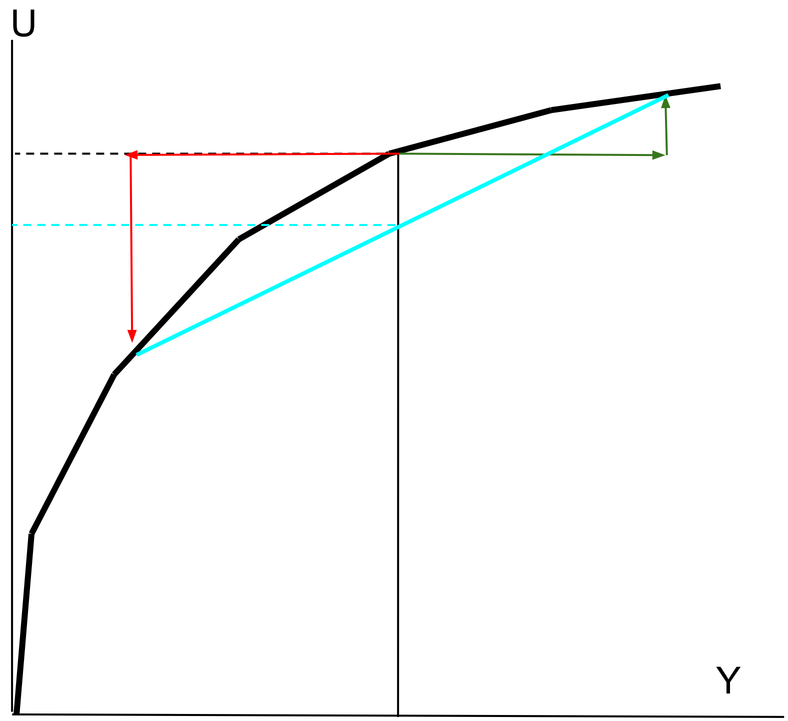
- <u>Expected utility</u> is the midpoint of the "chord" connecting the 2 potential outcomes



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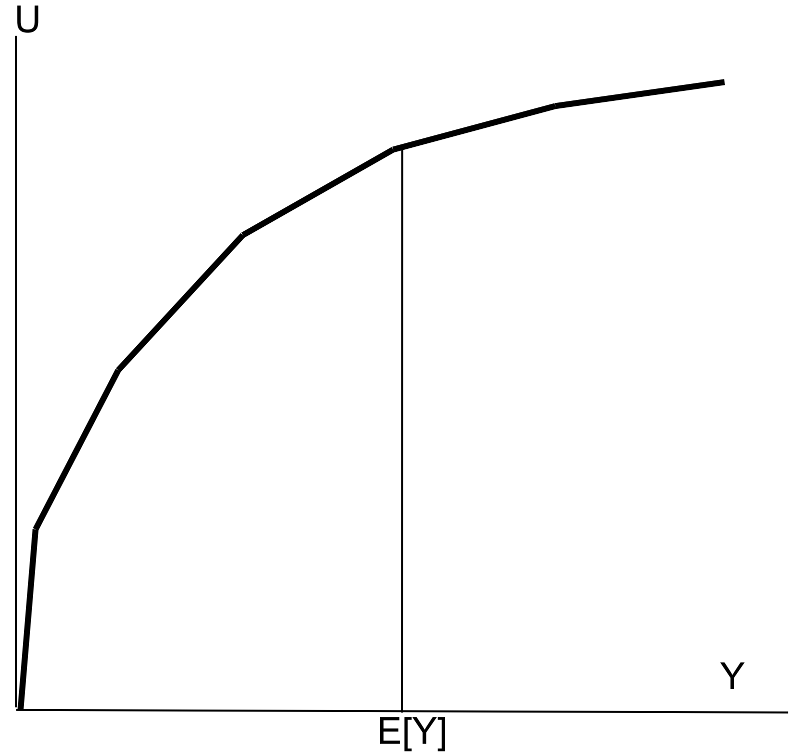

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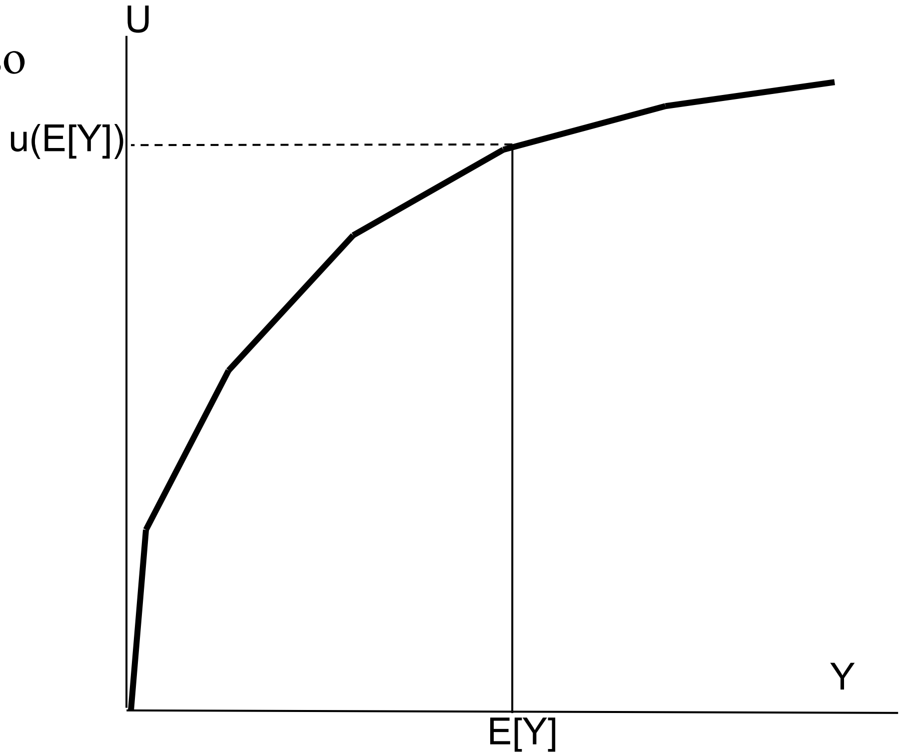
- But the sure thing gives it with certainty, so E[U] = 100%*u(E[Y])



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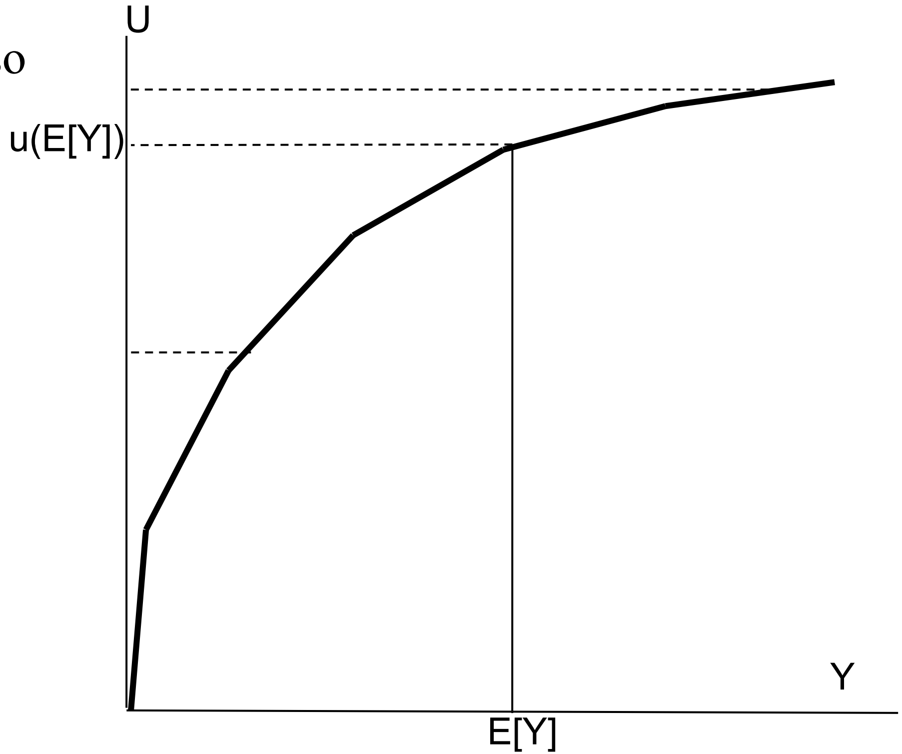

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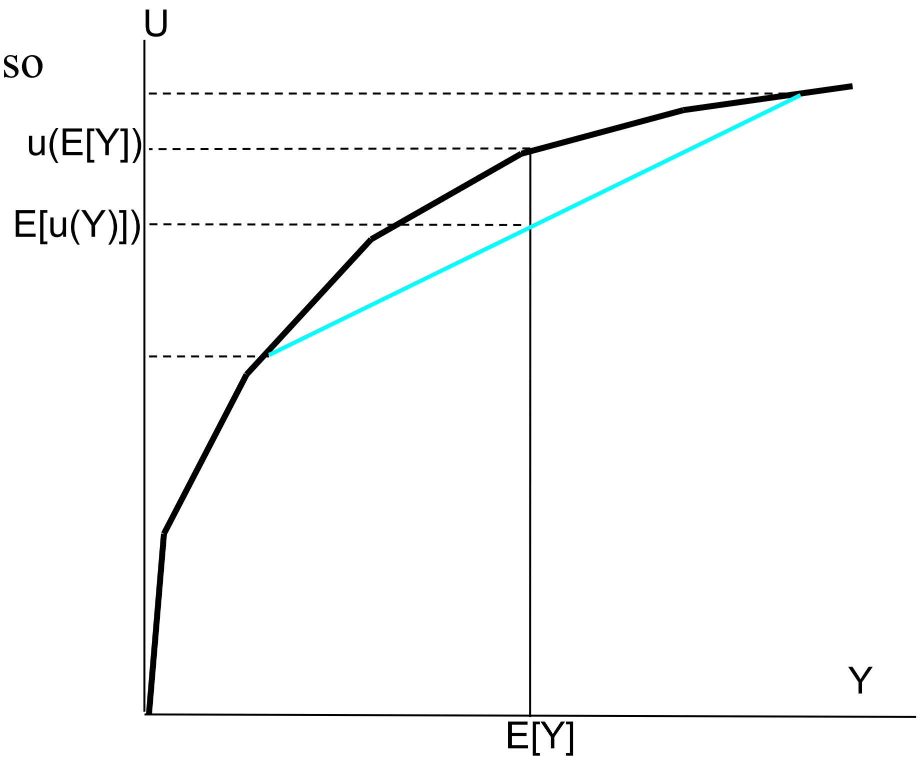

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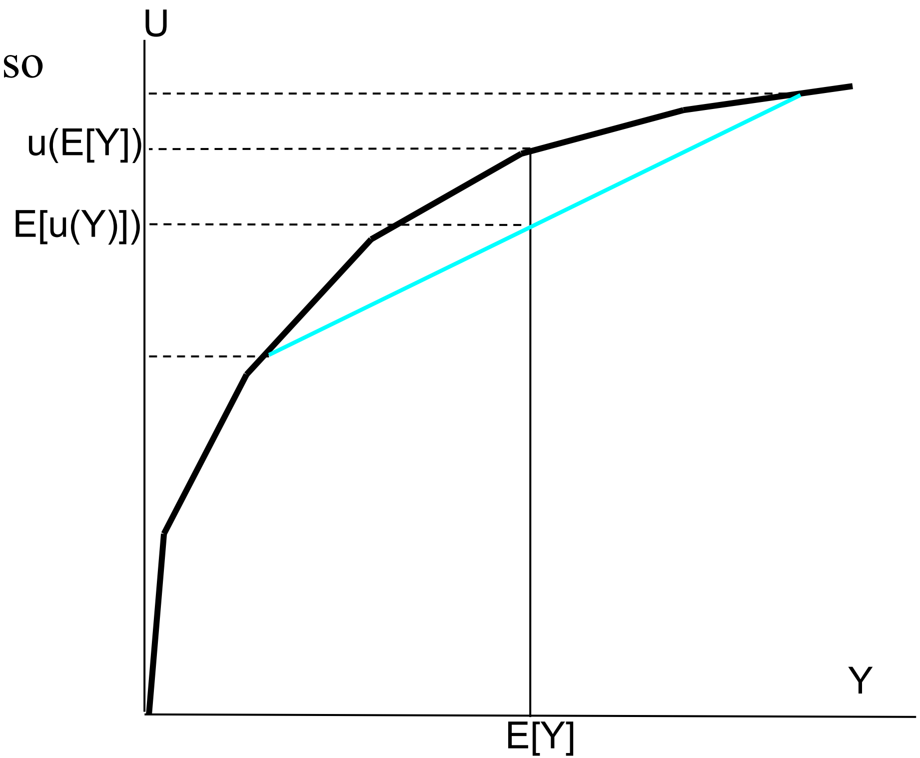
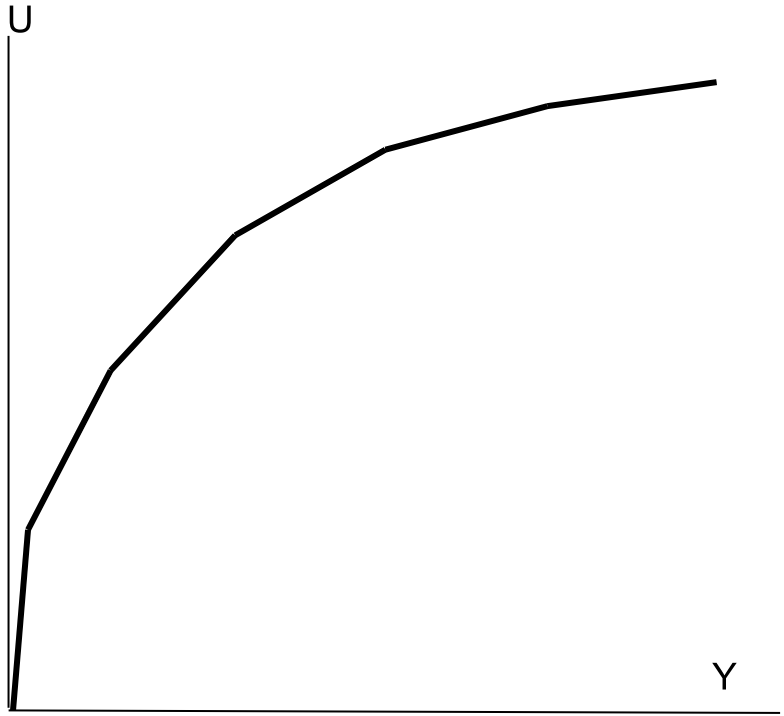
  - $E[U(Y)] = 50\%*u(Y_{low}) + 50\%*u(Y_{high})$
  - $E[U(Y)] < u(E[Y])$

While their expected incomes are the same, agent will prefer the sure thing because the gamble has a higher variance.



Value of insurance

- <u>Actuarially fair</u> insurance
  - On average, payouts balance with premiums, so you break even.



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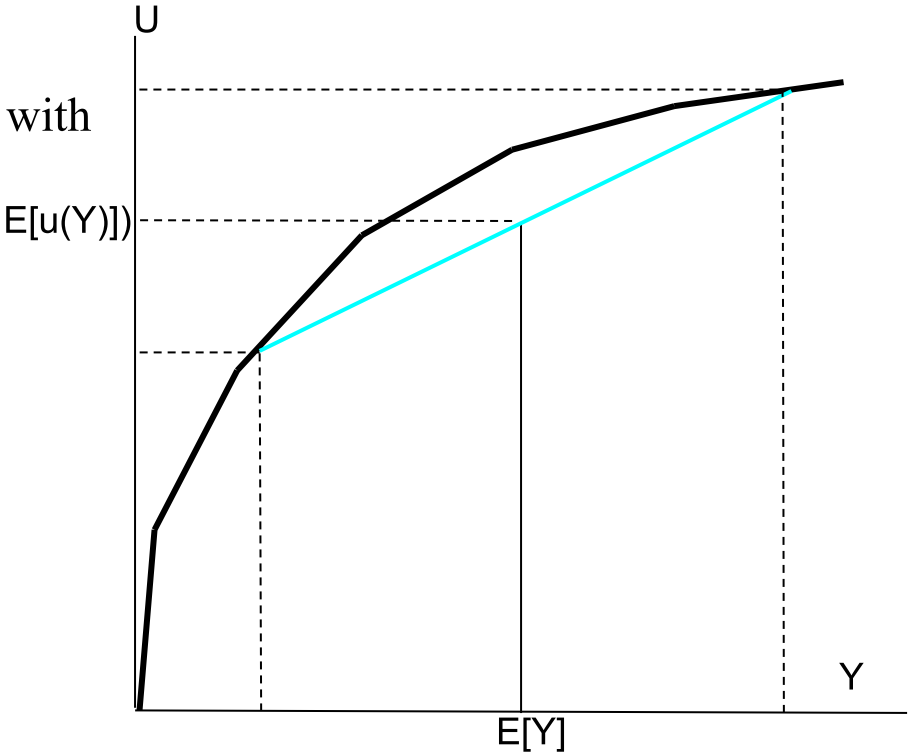
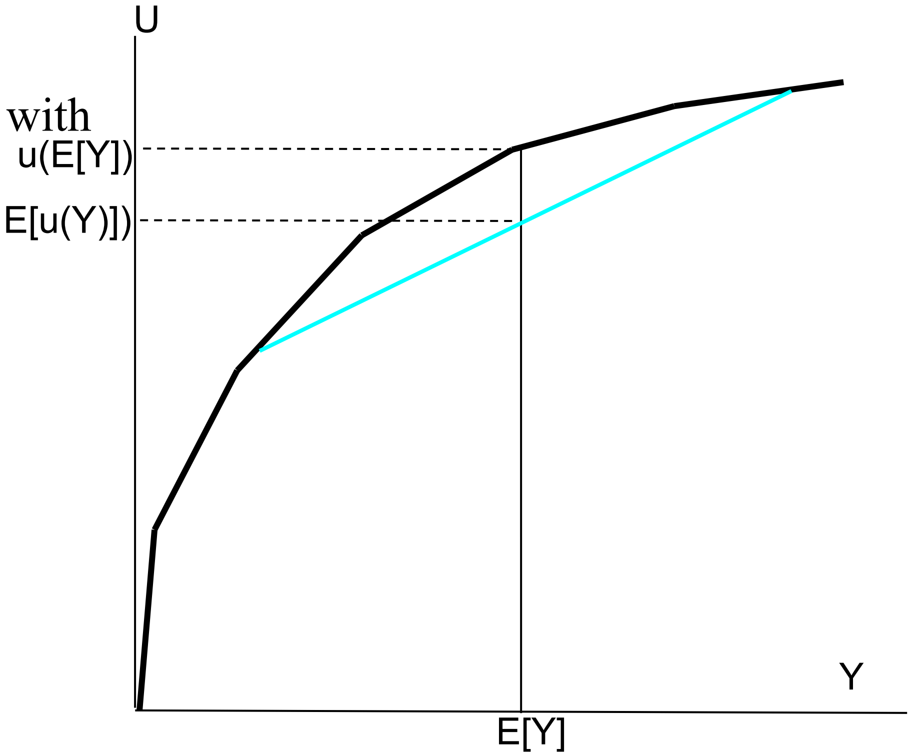


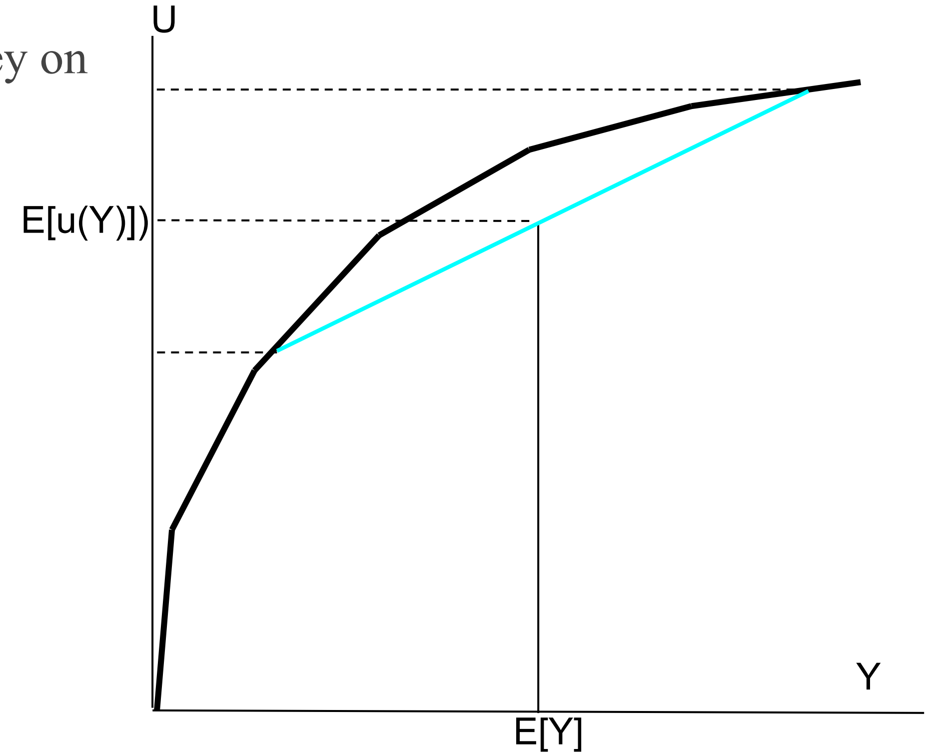
Value of insurance

- <u>Actuarially fair</u> insurance
  - On average, payouts balance with premiums, so you break even.
- Moves you from getting E[Y] on average with positive variance…
- …to getting E[Y] with certainty!
- Increases expected utility
  - This is why insurance exists



Risk premium

- But maybe you don’t get an actuarially fair offer.
- Would you be willing to lose some money on average to reduce uncertainty?



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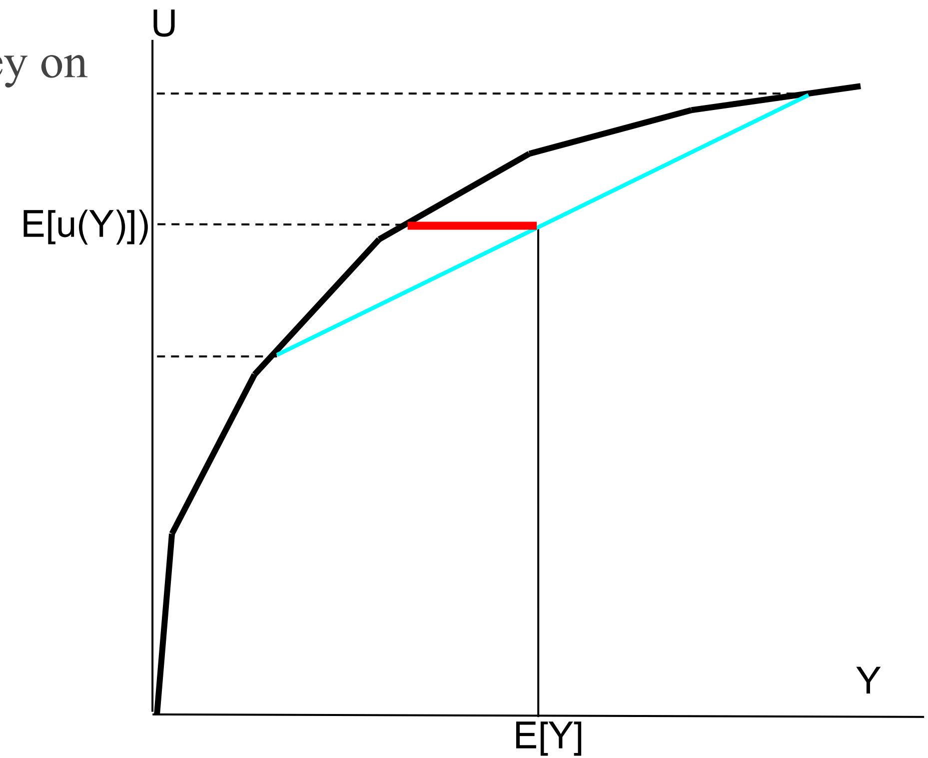
- Length of red line is <u>risk premium</u>



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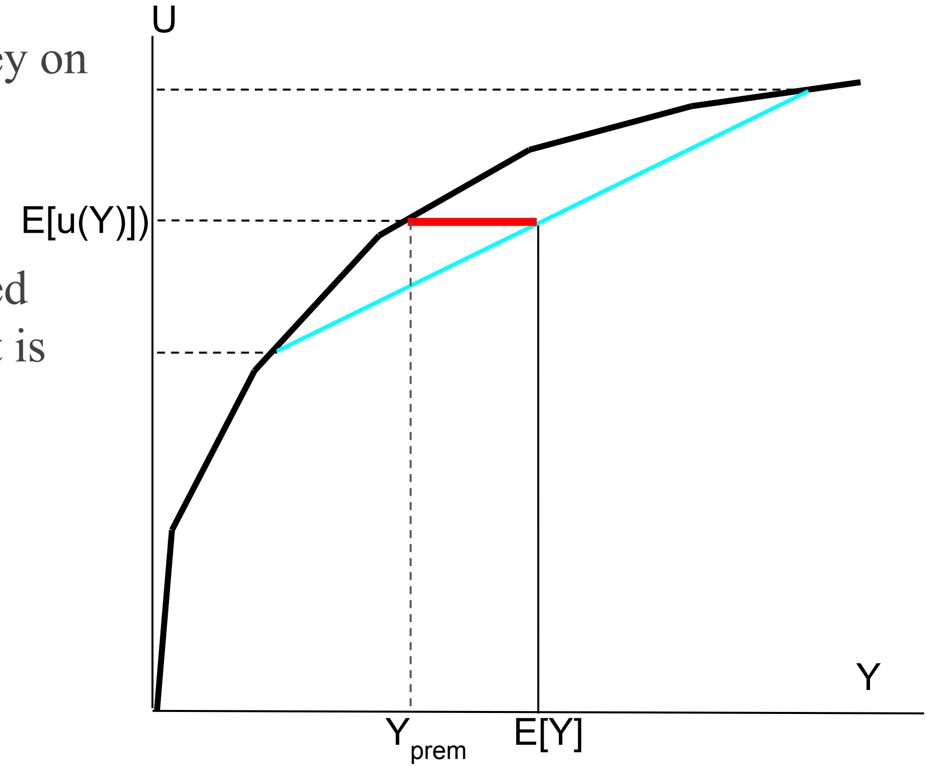

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- Would you be willing to lose some money on average to reduce uncertainty?
- Yes!
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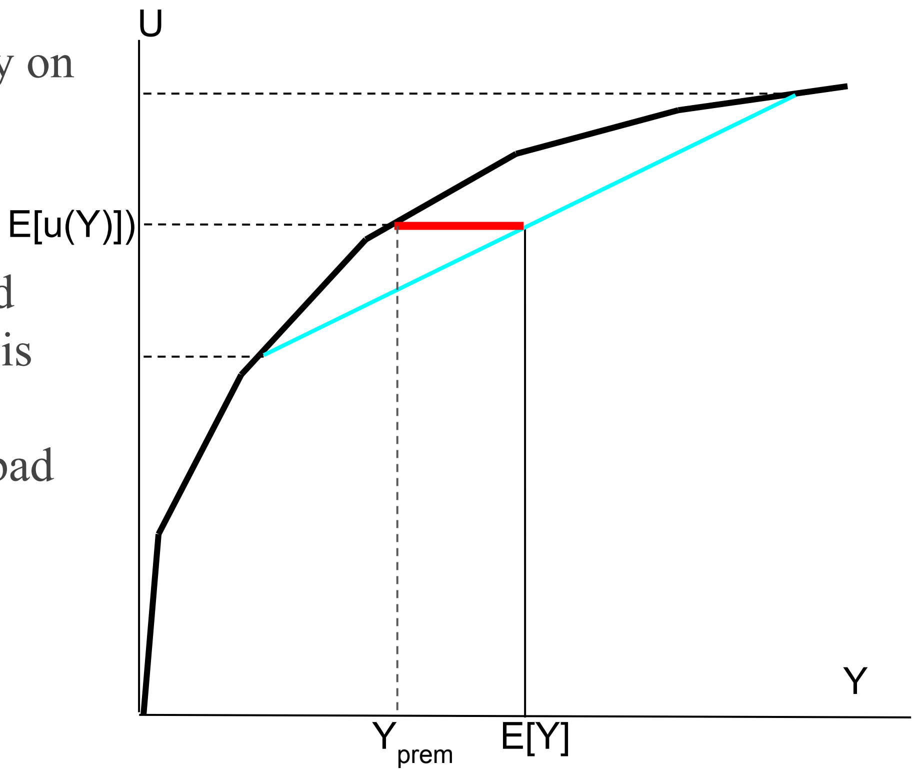
  - Y_{prem}, if certain, gives same expected utility as the gamble, even though it is lower average income
- Give up some money to avoid the really bad outcome



Portfolio selection

- Let’s now return to considering financial assets
- We typically assume investors want to maximize expected utility
- So they want their overall portfolio to have a high average value and low variance
  - This allows them to minimize the risk of a very low outcome

Portfolio selection: numerical example

- Suppose your indirect utility function is $U(Y) = Y^{1/2}$
- Suppose your main source of income is from labor, but you face a 50% risk of recession.

	Income		Statistical measures			Portfolio
Income type	Boom	Recession	Expected Value	Variance	Covariance with labor income	Expected utility when added to labor
Labor	81	16				6.50
Unemployment insurance						
Bond						
Stock						

- We will consider some assets you could use to supplement your income.

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- Unemployment insurance costs money in a boom and pays out during recession

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  - It has a negative covariance with income

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  - It increases utility by shifting income from the boom to the recession

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Bond	5	5				
Stock						

- A bond makes a constant payment

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- Which of these assets is most valuable? I.e. which would you pay most for?

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- Unemployment insurance, even though it has:
  - The lowest average payout…
  - …and the highest variance!

Covariance covariance covariance

- Even though you want a high mean and a low variance of your ***portfolio***…
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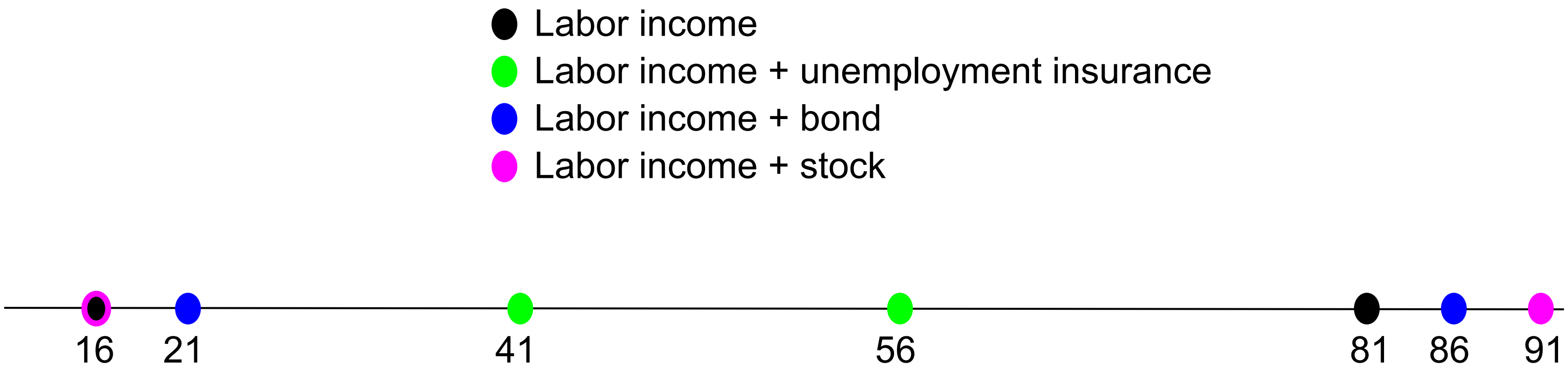

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- Without risk, we said asset returns should all be equal
  - With risk, *risk-adjusted returns* must be equal across all assets

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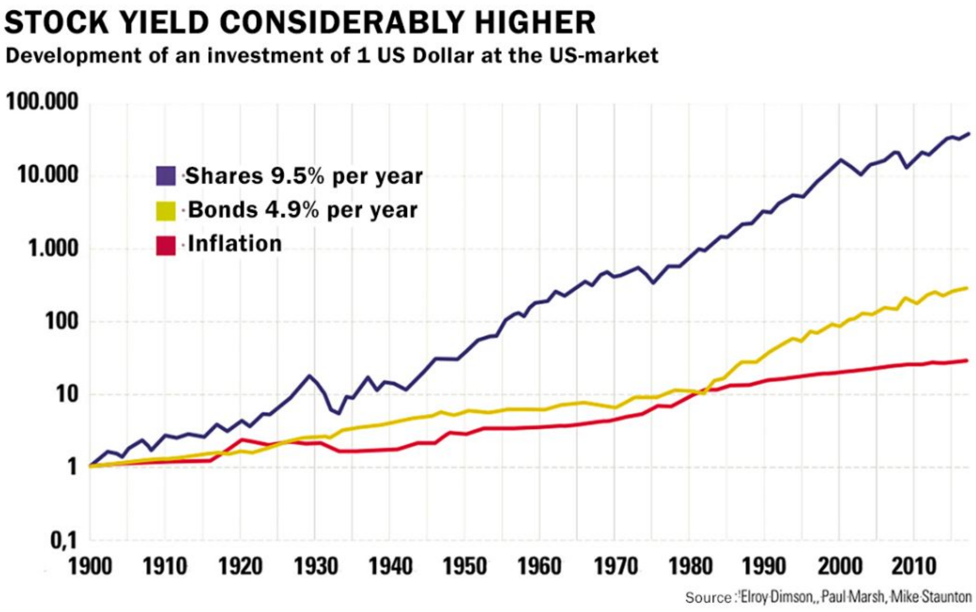

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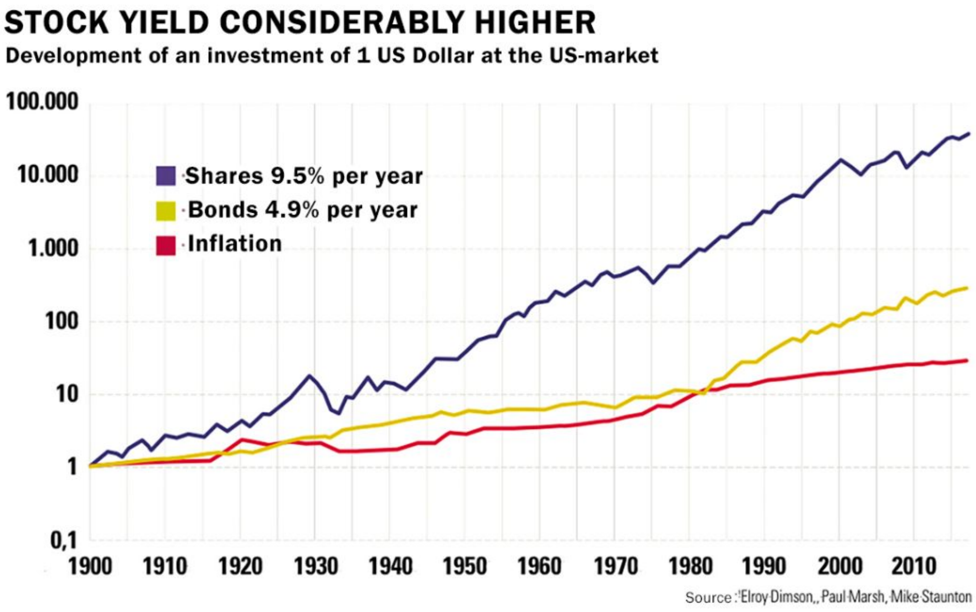
  - Referred to as the <u>equity premium</u>
- The equity premium exists because bond returns are more stable and, more importantly, co-move less with overall income
  - So investors are willing to hold them with lower returns, i.e. pay a higher price for them



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    - Young family can’t afford to buy a house with cash
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- Fix misallocation of risk
  - Pension funds can buy low-risk, low-yield assets while hedge funds by opposite
  - Patient bears all risk of health costs
    - Buys insurance – spreads risk across many patients