# Consumer Theory

Fall 2023
Econ 2316, Northeastern University
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P&R: chapters 3 (especially 3.1-3.5), 4 (especially 4.1-4.3), and 16.2-16.3
Emerson: 3, 2, 1 (can skip 1.1), 4, 5 (especially 5.3 and 5.6), 14 (skip 14.4)

# Introduction

# Why study consumer choice in more depth?

- Market demand curves result from individual decisions
- Better understanding individual decisions allows for more complete and nuanced understanding of market outcomes

# Goal of consumer choice theory in economics

- Not a complete description of how people make decisions
- Psychologists have a deeper understanding of all the various determinants of individuals' decisions
- Economists need something tractable that fits into a larger model
- We want to make sharp predictions about market-level phenomena
  - Can't have consumers who are too complicated
  - The art is to keep just the features that are needed for your purposes, ignore the rest
- Much of economics centers on how changes in price or income affect choices, so our model will focus on prices and income, while ignoring much else

# Economists' approach to consumer choice

- 2 fundamental components
  a. Budget set
     - Set of all available options for the consumer to choose from
     - Typically determined by an "endowed" income and prices of goods
  b. Method for selecting a single "choice" from the budget set
     - Economists' workhorse approach is "utility maximization"

# Budget Set

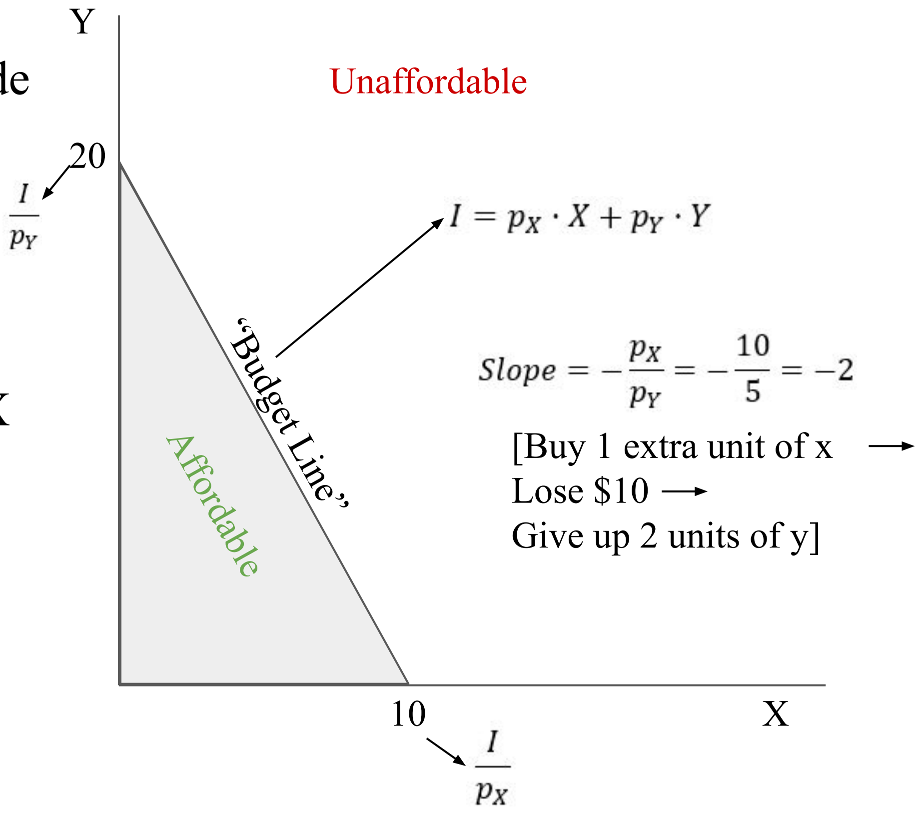
# Budget set

- A budget set delineates options – or “bundles” – that a consumer can afford
- Typical assumptions are:
  - Has an endowment to spend
  - Chooses a bundle made up of multiple goods
  - Each good has a constant price
- In the case of 2 goods, X and Y, the budget set is:

$$(X,Y)\ s.t.\ I \geq p_X \cdot X + p_Y \cdot Y$$

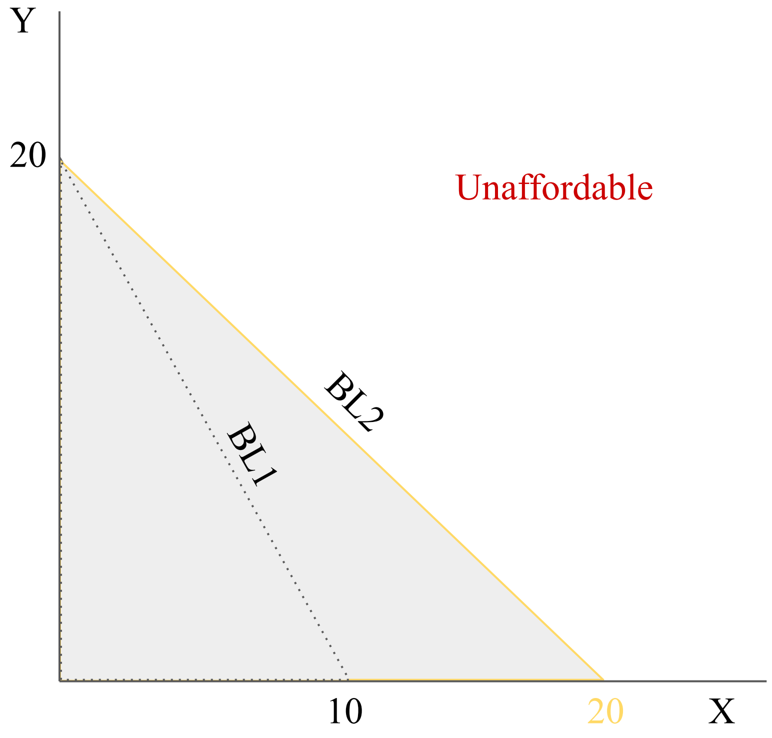
# Visual representation of a budget set

- Suppose a consumer has \$100 to divide between two goods.
  - Good X costs \$10 per unit
  - Good Y costs \$5 per unit
- She can buy 10 units of good X, or 20 units of good Y, or a combination of X and Y that does not exceed \$100.
- A change in price or income changes the choice set



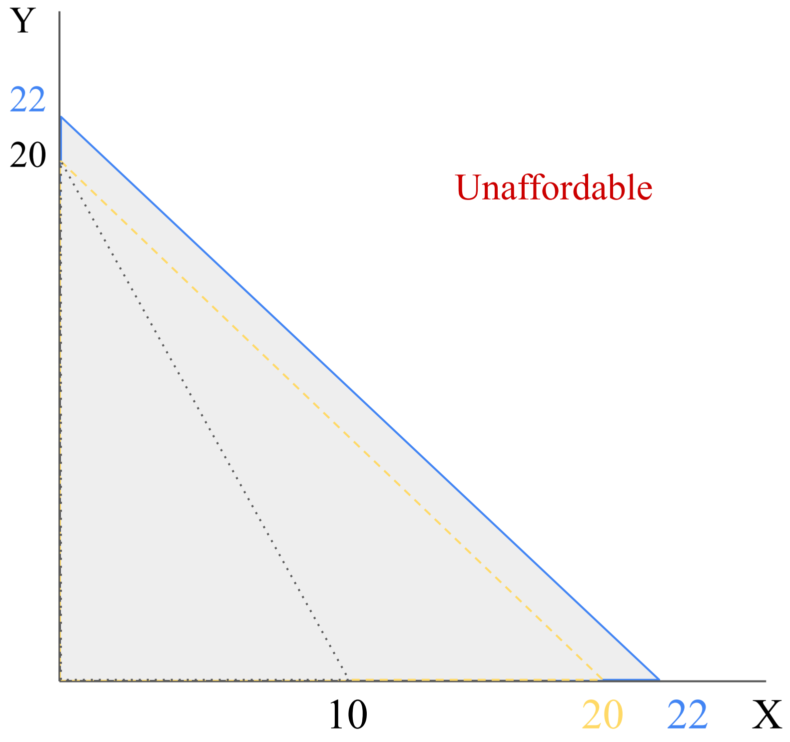
# Visual representation of a budget set

- Suppose a consumer has $100 to divide between two goods.
  - Good X costs $5 per unit
  - Good Y costs $5 per unit
- She can buy 20 units of good X, or 20 units of good Y, or a combination of X and Y that does not exceed $100.
- A change in price or income changes the choice set



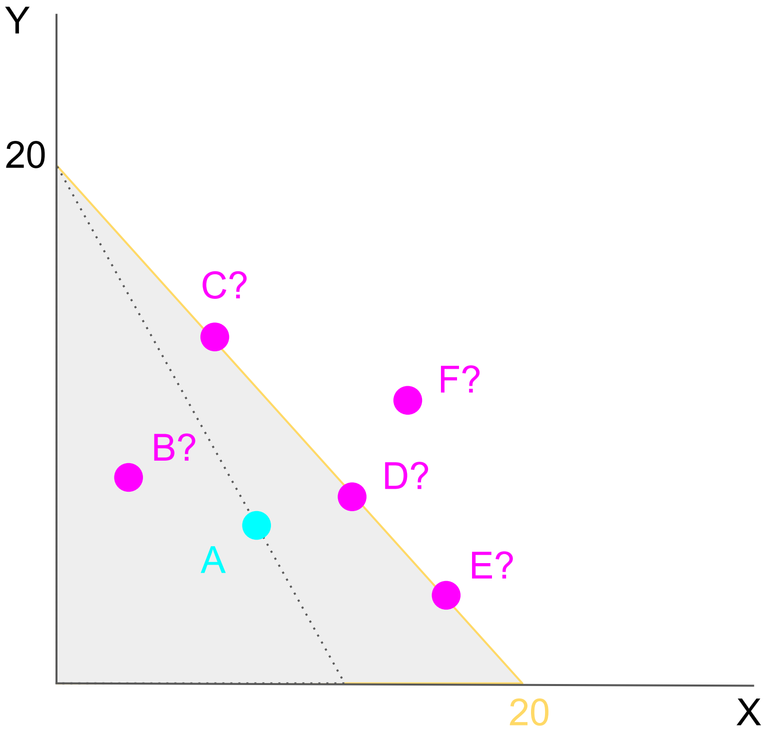
# Visual representation of a budget set

- Suppose a consumer has $110 to divide between two goods.
  - Good X costs $5 per unit
  - Good Y costs $5 per unit
- She can buy 22 units of good X, or 22 units of good Y, or a combination of X and Y that does not exceed $110.
- A change in price or income changes the choice set



# “Comparative static” based only on choice set

- Suppose the customer chose point A and then the price of X fell.




Based purely on the choice set, what can we say about how this affects the consumer and her choice?

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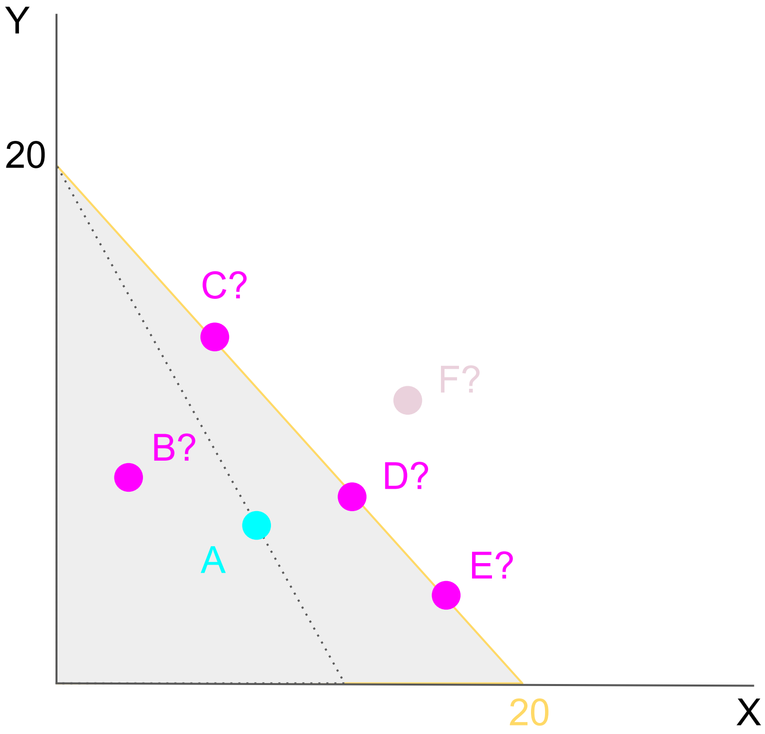


Based purely on the choice set, what can we say about how this affects the consumer and her choice?

- F is still unaffordable

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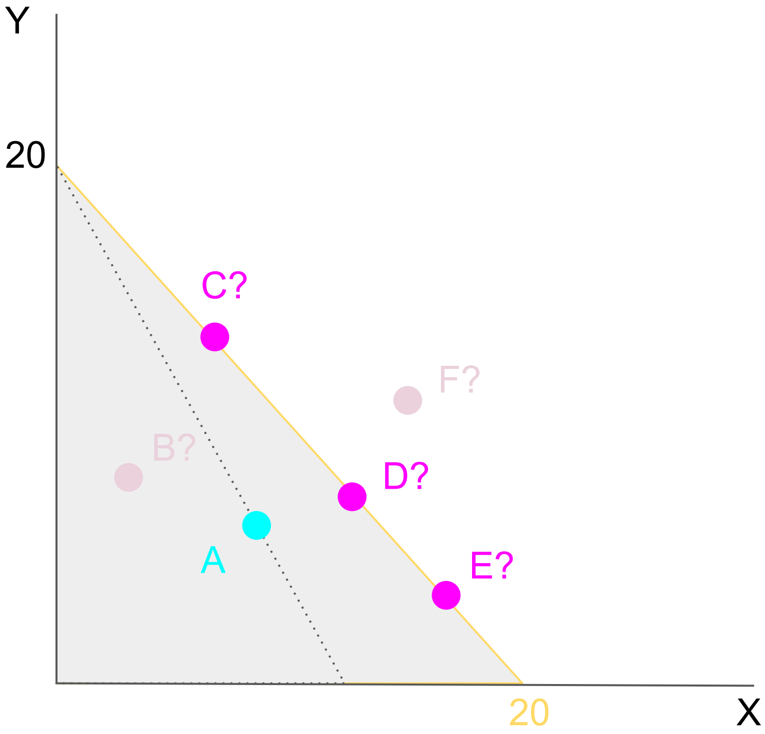


Based purely on the choice set, what can we say about how this affects the consumer and her choice?

- F is still unaffordable
- B is less desirable than A (“revealed preference”) and A is still affordable, so B will not be chosen

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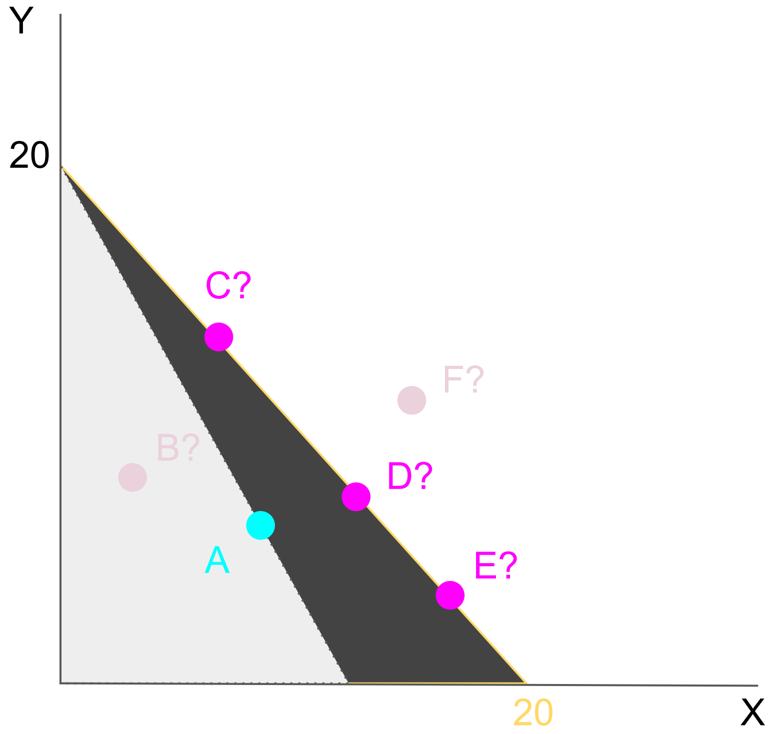


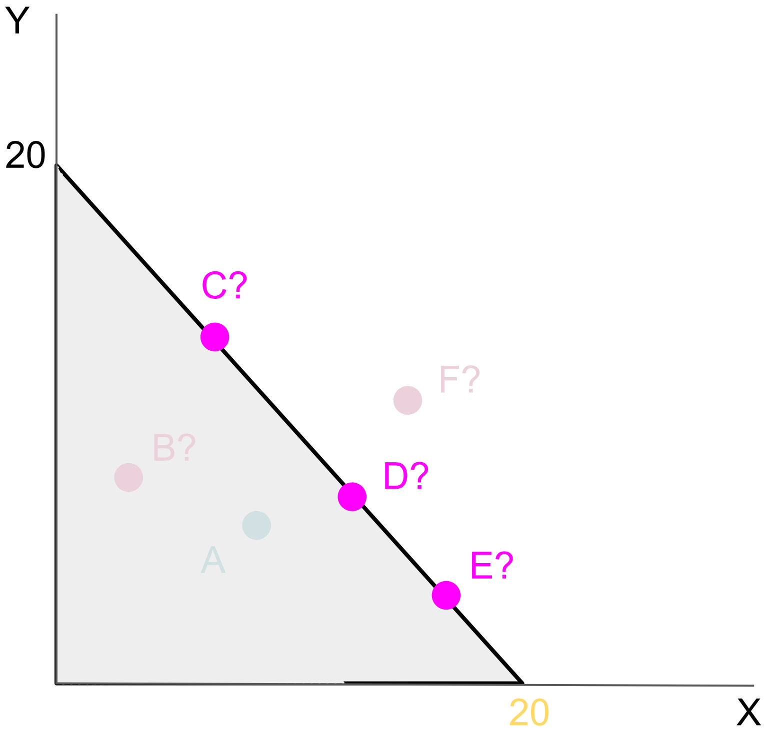

Based purely on the choice set, what can we say about how this affects the consumer and her choice?

- F is still unaffordable
- B is less desirable than A (“revealed preference”) and A is still affordable, so B will not be chosen

This leaves a lot of possibilities…

# Comparative static with “non-satiation”

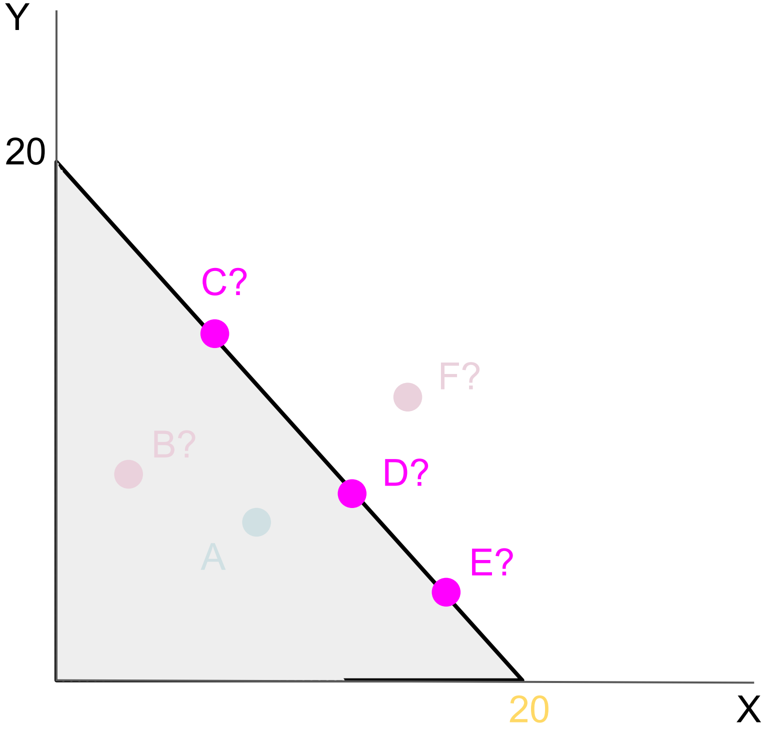
- Suppose the customer chose point A and then the price of X fell.




If we assume that more is better than less (“non-satiation”), we will know that the new choice will be on the new budget line.

# Comparative static with "non-satiation"

- Suppose the customer chose point A and then the price of X fell.

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C, D, and E are all possibilities, but they're quite different

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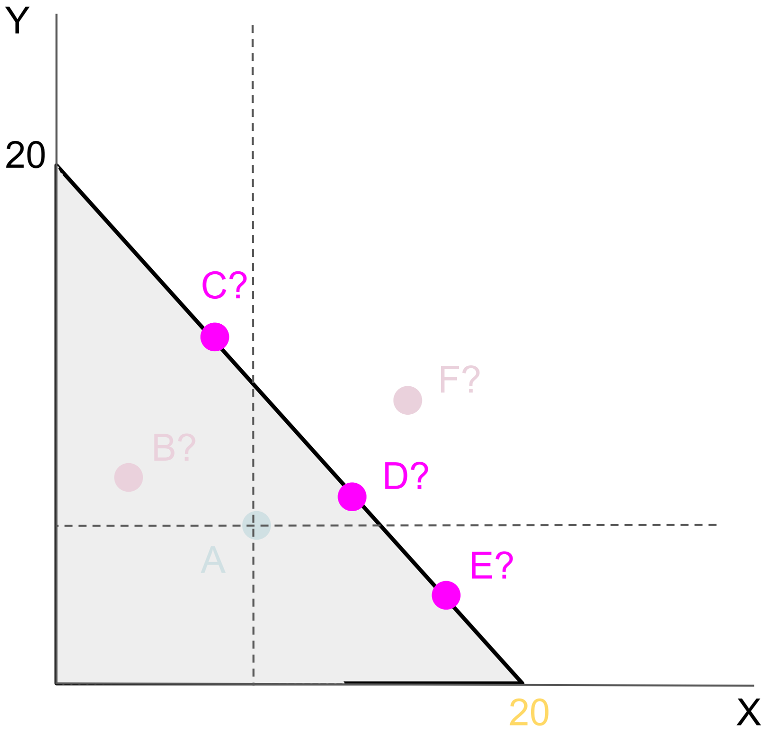



If we assume that more is better than less (“non-satiation”), we will know that the new choice will be on the new budget line.

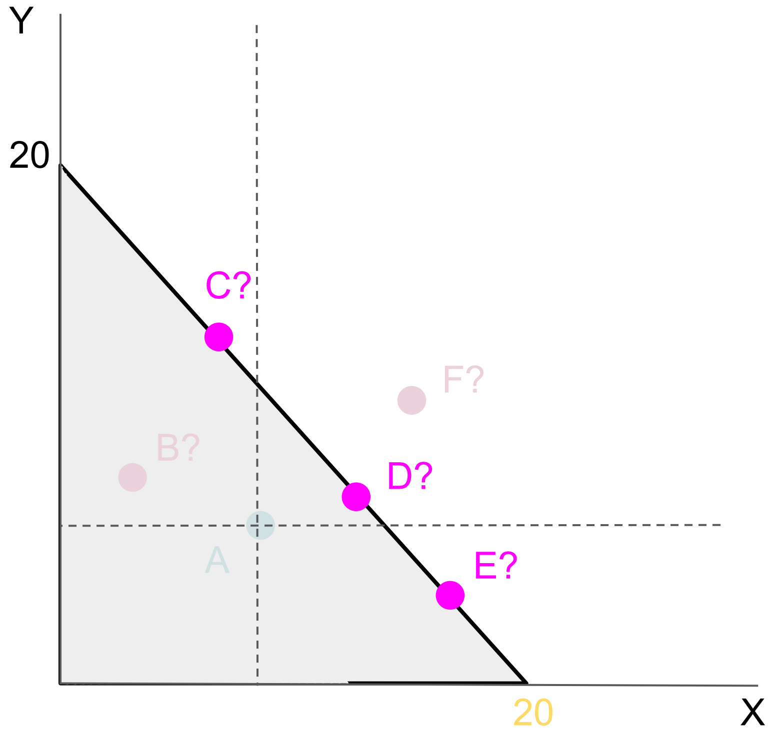
C, D, and E are all possibilities, but they’re quite different

Compared to A…

- C: X↑, Y↓
- D: X↑, Y↓
- E: X↑, Y↓

# Comparative static with “non-satiation”

- Suppose the customer chose point A and then the price of X fell.




If we assume that more is better than less (“non-satiation”), we will know that the new choice will be on the new budget line.

C, D, and E are all possibilities, but they’re quite different

All we can really say is that she’s better off because she can buy more. Sharper analysis requires more structure/assumptions.

# Utility Function

# Introduction to utility functions

- A utility function assigns a <u>number</u> to every choice in a budget set
- Consumer choice then amounts to “simply” choosing the option that produced the highest number (“utility maximization”)

# Example utility function

$u = U(X,Y) = X^{0.5}Y^{0.5}$

Can compare bundle A = (X=1, Y=9) to bundle B = (X=4, Y=4).

- U(X=1, Y=9) = 3
- U(X=4, Y=4) = 4

A consumer with preferences represented by this utility function will prefer B to A

# More examples

$u = U(X,Y) = X^{0.8}Y^{0.2}$

$u = U(X,Y) = X^{0.8}Y^{0.3}$

$u = U(X,Y) = 3X^{0.8} + 4Y^{0.2}$

$u = U(X,Y) = 2X + 4Y$

$u = U(X,Y) = 2X + 4Y^{0.5}$

$u = U(X,Y) = \min\{X, Y\}$

All of these represent different preferences a consumer might have

# Marginal utility

Marginal utility is one of the most important concepts in consumer theory

The marginal utility (of good X) is the additional utility the consumer gets from consuming one more unit (of X).

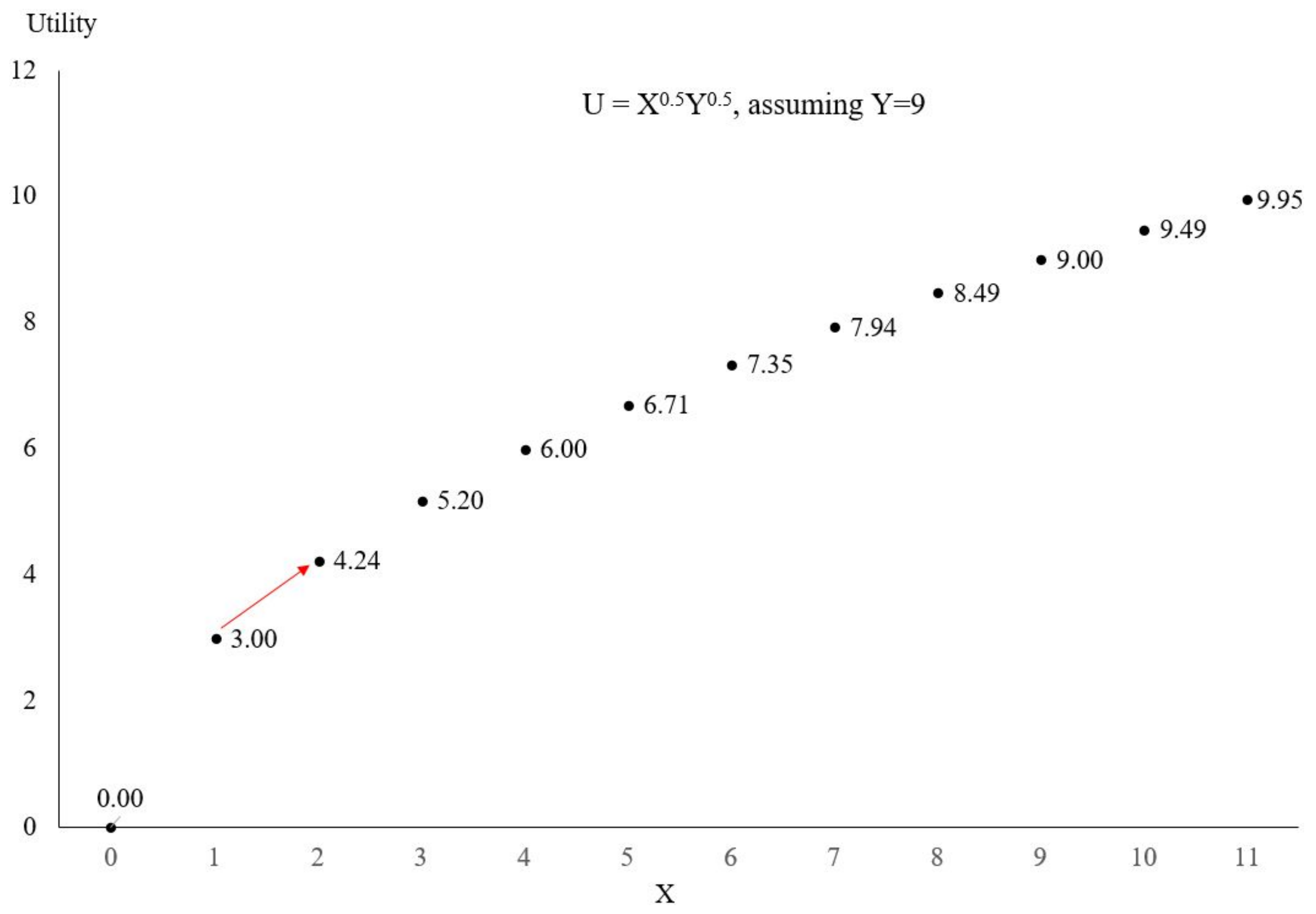
Marginal utility is critical both for the intuition of consumer choice and for deriving solutions mathematically

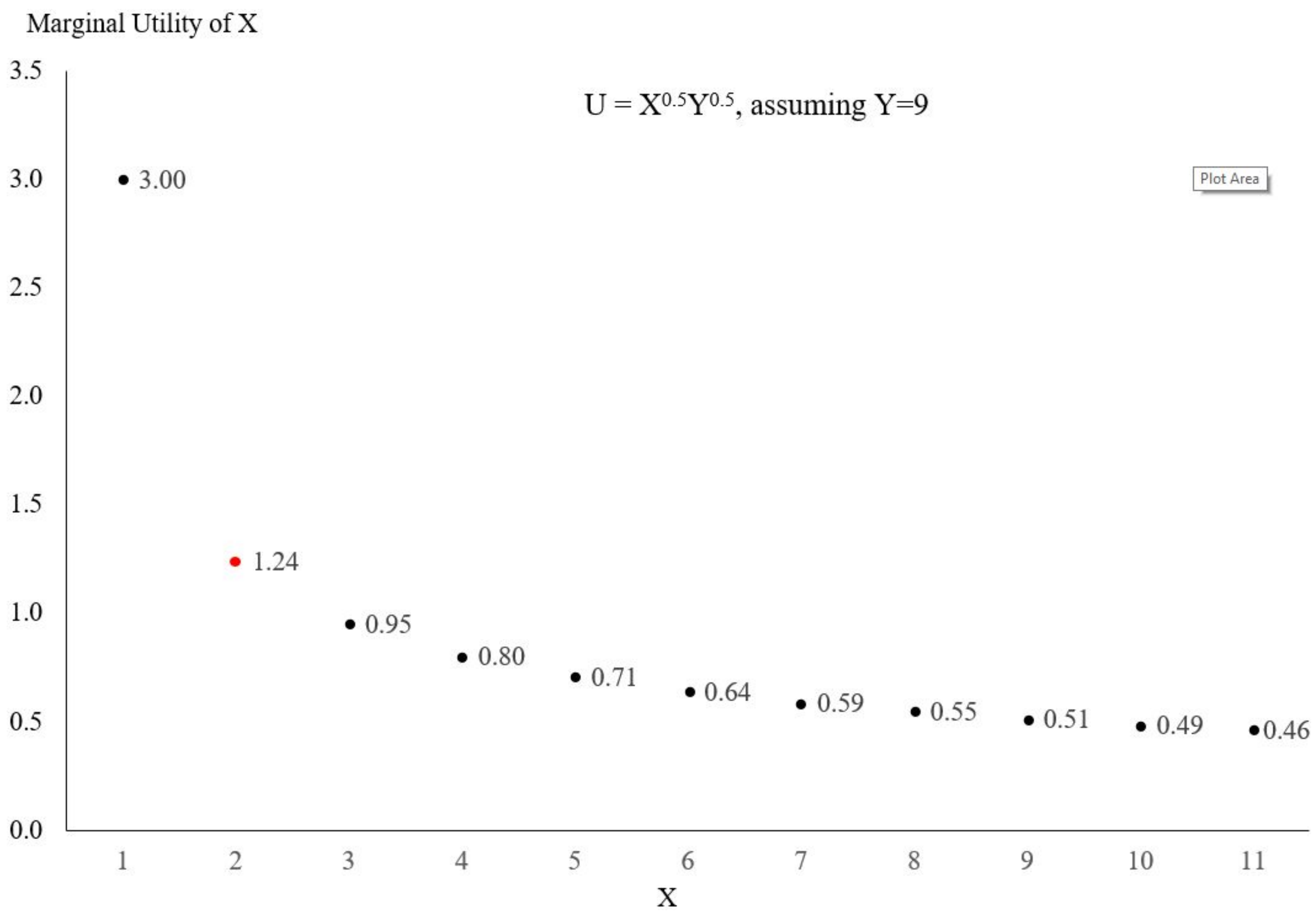
- Ultimately, the consumer must compare the benefit of additional consumption (marginal utility) to its cost (price) to determine optimal bundle

# Marginal utility example

If $U(X,Y) = X^{0.5}Y^{0.5}$ and current bundle is (X=1, Y=9), then u = 3.

- Increasing X from 1 to 2 brings utility to 4.24, so the marginal utility of X is 1.24 (4.24-3)
- Increasing Y from 9 to 10 brings utility to 3.16, so the marginal utility of Y is 0.16 (3.16-3)


Utility
U = X^0.5Y^0.5, assuming Y=9
0.00
3.00
4.24
5.20
6.00
6.71
7.35
7.94
8.49
9.00
9.49
9.95
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
11
X

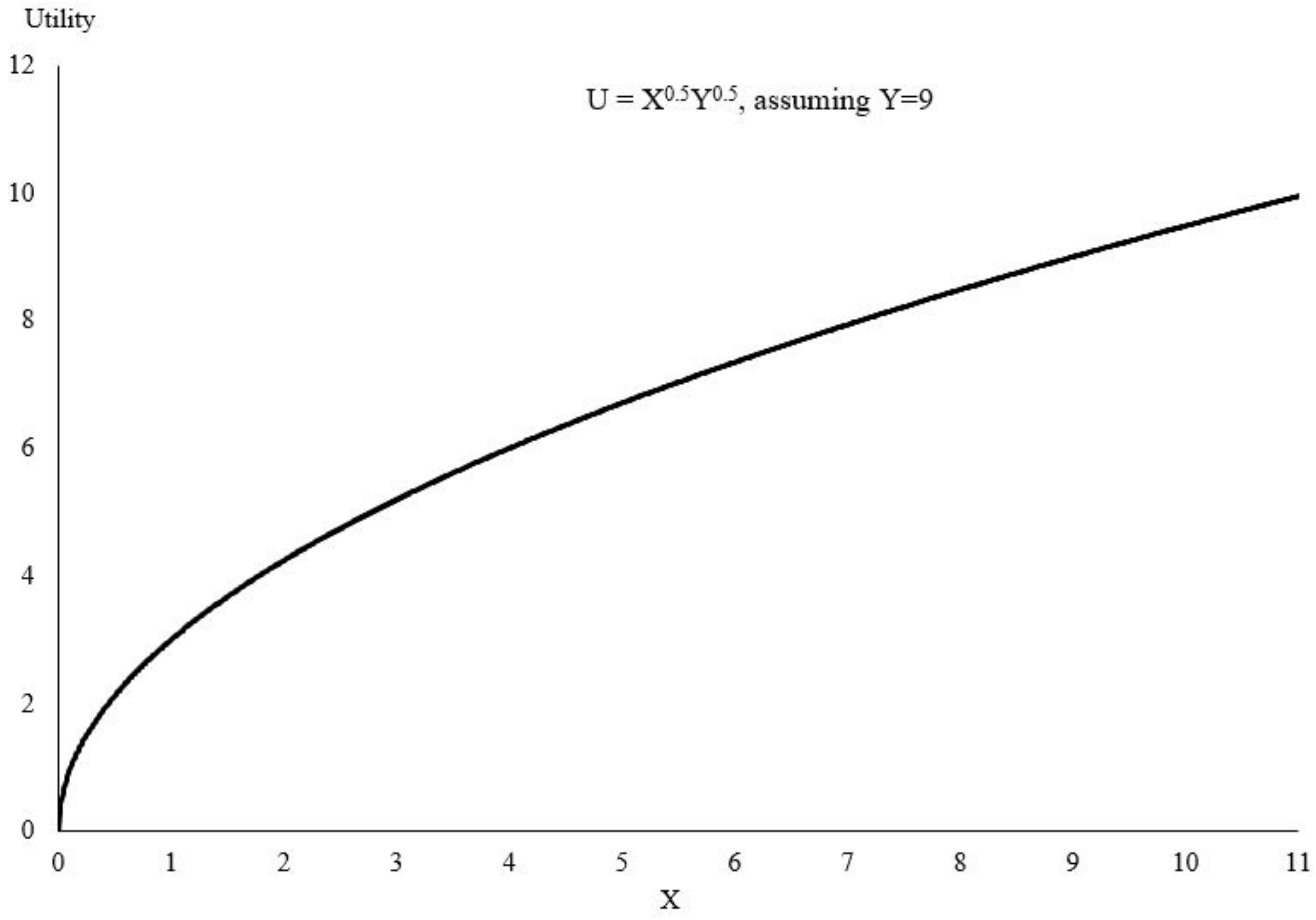

Marginal Utility of X
U = X^0.5Y^0.5, assuming Y=9
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
3.00
1.24
0.95
0.80
0.71
0.64
0.59
0.55
0.51
0.49
0.46
Plot Area
1
2
3
4
5
6
7
8
9
10
11
X

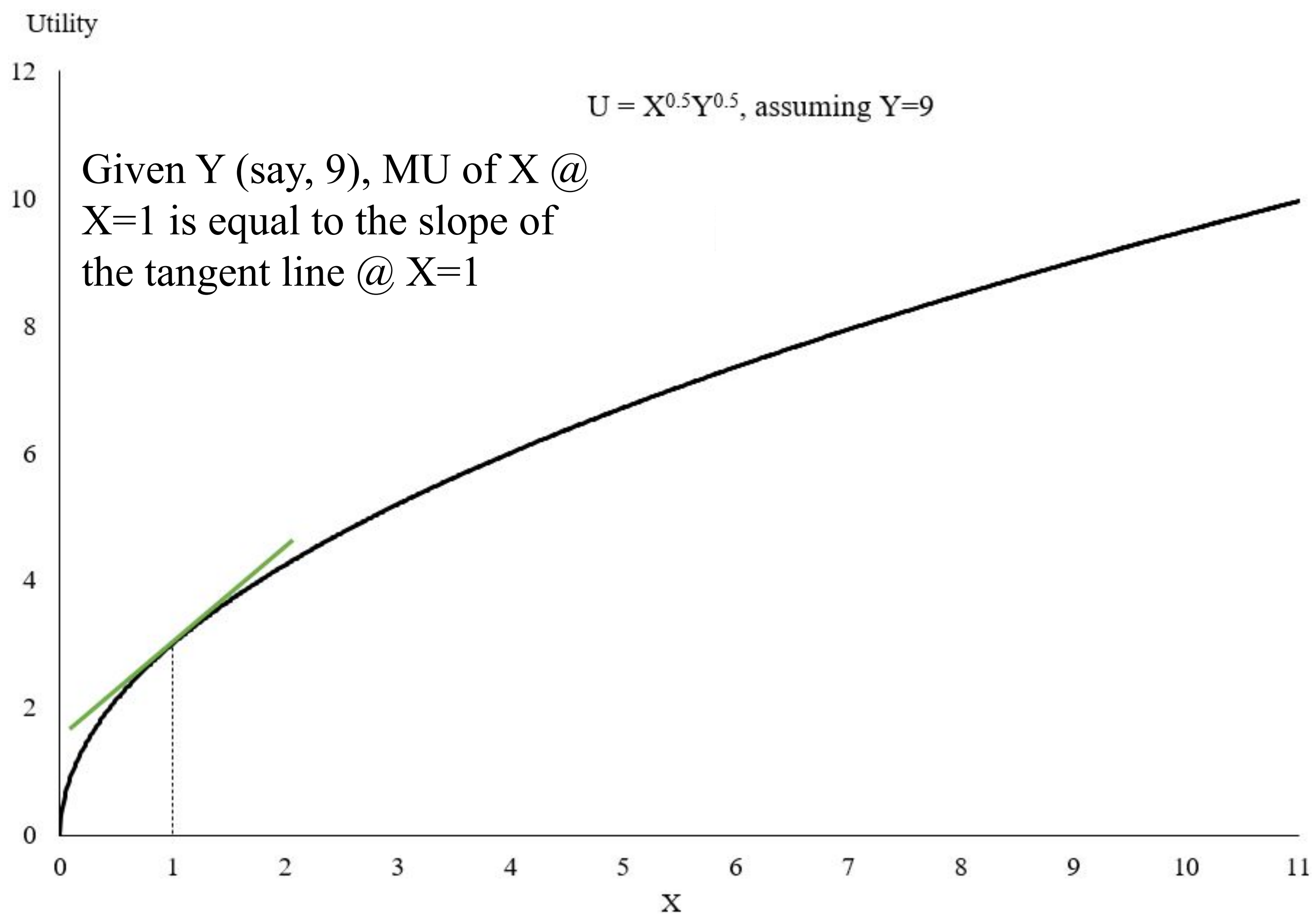
# Decreasing marginal utility

- Previous example demonstrated “decreasing marginal utility”
  - The more X she has, the less additional utility she gets from more X
- This is a standard feature of utility functions
  - Consider food. How much would you value food if you:
    - Haven’t eaten in multiple days?
    - Haven’t eaten all day?
    - Haven’t eaten in a few hours?
    - Just finished a large meal?
- This is the water-vs.-diamonds paradox
  - Overall, water is much more valuable to us, but the marginal glass is not so important
  - Diamonds are much less important, but we have so few that we place high value on the margin
- We will discuss some exceptions to the decreasing marginal utility principle

# Continuous functions

- We will typically assume that the consumer can choose non-integer values
  - I.e. not just numbers like 1, 2, and 3, but also numbers like 4.3190501341354.
- This doesn’t really affect the substance of the analysis, but it makes our lives easier.
  - Easier to represent graphically
  - Easier to solve with calculus


Utility
U = X0.5Y0.5, assuming Y=9
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
9
10
11
X



Utility
12
10
8
6
4
2
0
U = X0.5Y0.5, assuming Y=9
Given Y (say, 9), MU of X @ X=1 is equal to the slope of the tangent line @ X=1
0
1
2
3
4
5
6
7
8
9
10
11
X

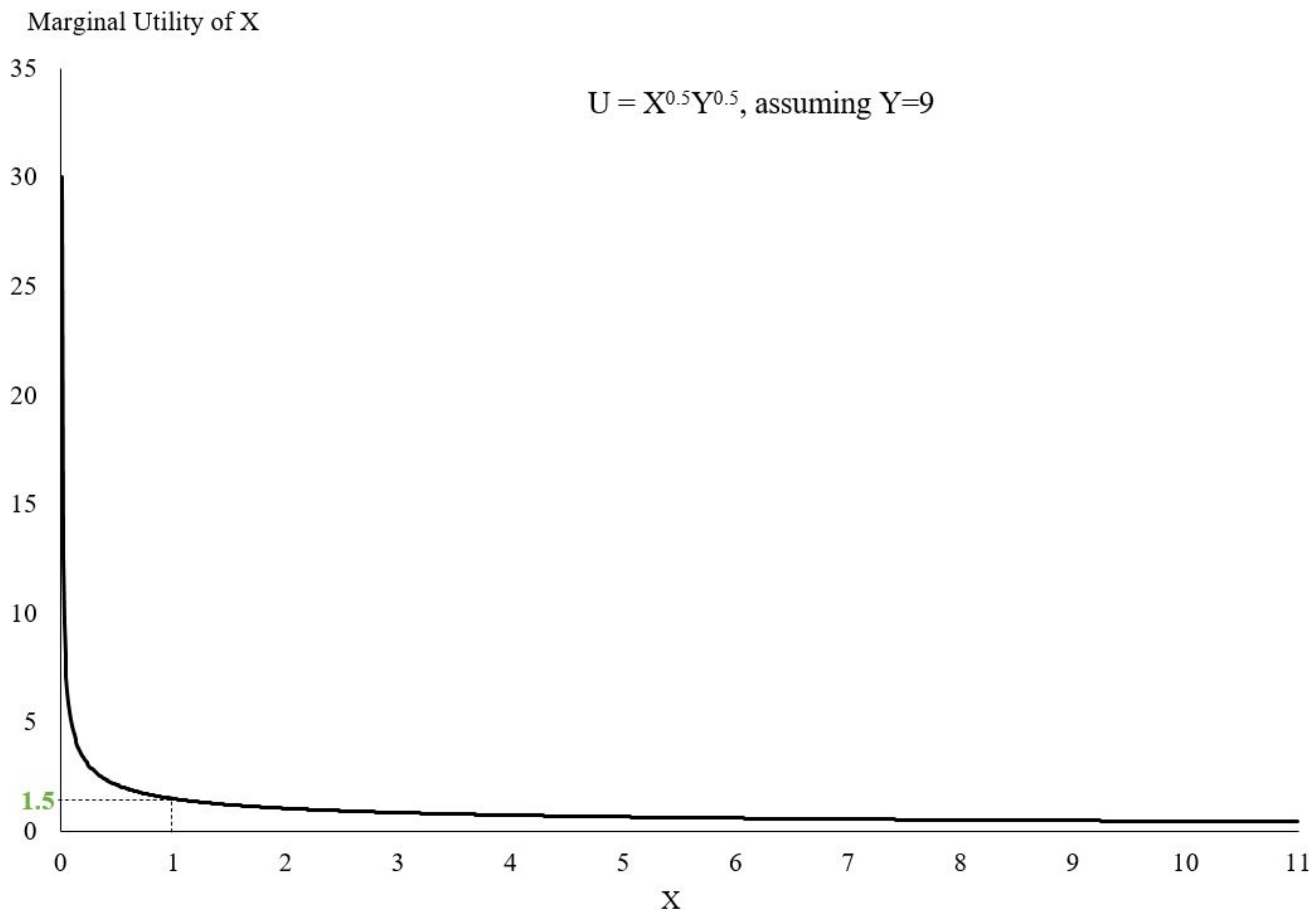
# Partial derivatives

To mathematically derive the marginal utility of U with respect to X, take the “partial derivative” of with respect to X, holding Y constant.

$$\frac{\partial U}{\partial X} = \frac{\partial (X^{0.5} \cdot Y^{0.5})}{\partial X} = Y^{0.5} \cdot 0.5 \cdot X^{-0.5}$$

If Y=9 and X=1, then

$$\frac{\partial U}{\partial X} = 1.5$$



Marginal Utility of X
$U = X^{0.5}Y^{0.5}$, assuming Y=9
35
30
25
20
15
10
5
1.5
0
0
1
2
3
4
5
6
7
8
9
10
11
X

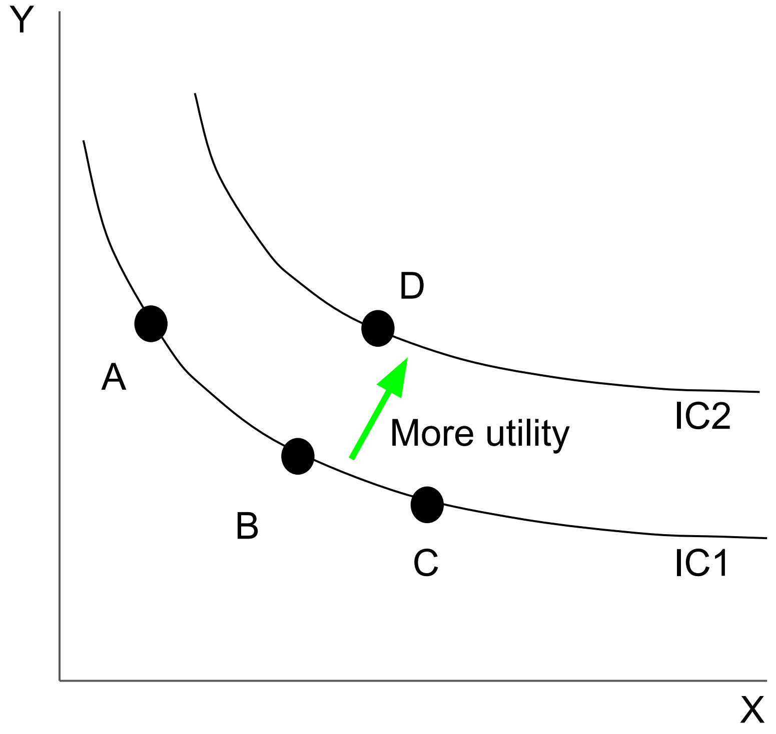
# Indifference Curves

# Introduction to indifference curves

- Utility functions are hard to draw
  - Even in a model with just 2 goods, a 3-D graph would be required
    - Good 1, Good 2, Utility
- Economists deal with this by drawing a “contour map” made up of <u>indifference curves</u>
  - A contour map is a way to show a 3-dimensional object in 2 dimensions

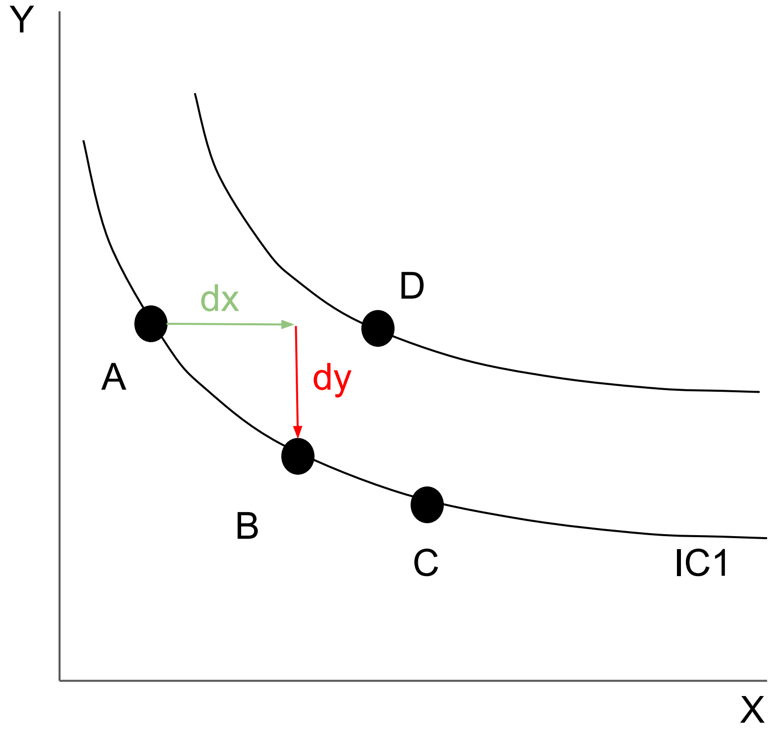
# A typical set of indifference curves

- The consumer gets the same utility from A, B, C, and all other points on the IC1 curve (i.e. indifferent between the points).
- D is on a higher indifference curve and so gives more utility than any point on IC1.
- *Every* point in the plane is on some indifference curve!
    - We just choose to show a select few as a way to view "slices" of this 3-D object on a 2-D page



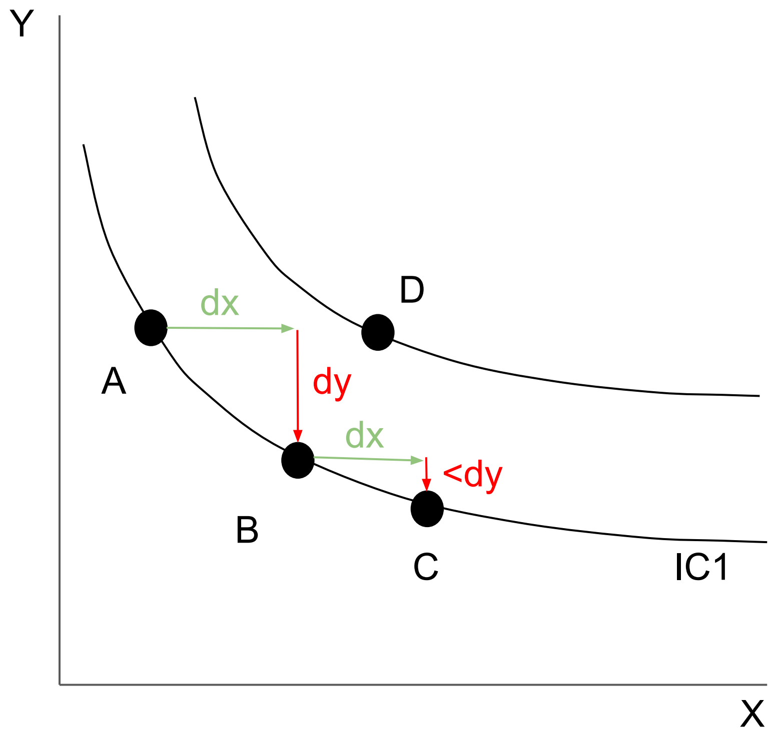
# A typical set of indifference curves

- The ICs are <u>decreasing</u> because she likes both goods
    - If she gets more X, take away Y to make her indifferent



# A typical set of indifference curves

- IC1 is <u>convex</u> (slope flattens) because of decreasing marginal utility
  - At point A, she has much Y and little X, so she's willing to give up a lot of Y to increase X (and stay indifferent)
  - At point B, she has less Y and more X, so she will only give up a little Y to increase X (and stay indifferent)



# Marginal rate of substitution (MRS)

- The slope of an indifference curve is called the marginal rate of substitution (MRS)
- If she gains 1 unit of X, how much Y must she give up to be indifferent?
    - i.e. rise over run, or slope
- As the last slide suggested, this is closely tied to the concept of marginal utility. A little bit of math will make the connection clear…

# “Perturbation”/“Total Derivative”

- Suppose we allow X and Y to both vary a little bit; how will utility change?
  - This is a “total derivative,” as opposed to the partial derivative we looked at earlier
- Utility function is U(X,Y). Change in utility is:

$$dU = \frac{\partial U}{\partial X} \cdot dX + \frac{\partial U}{\partial Y} \cdot dY = MU_X \cdot dX + MU_Y \cdot dY$$

- X and Y both change, so to see how each one affects utility, you just multiply by the marginal utility of each one, then add them together.

# Finding the indifference curve's slope

- The premise of the indifference curve is that X and Y change but U does not
- So let's set the total derivative (dU) equal to 0

$$dU = MU_X \cdot dX + MU_Y \cdot dY = 0$$

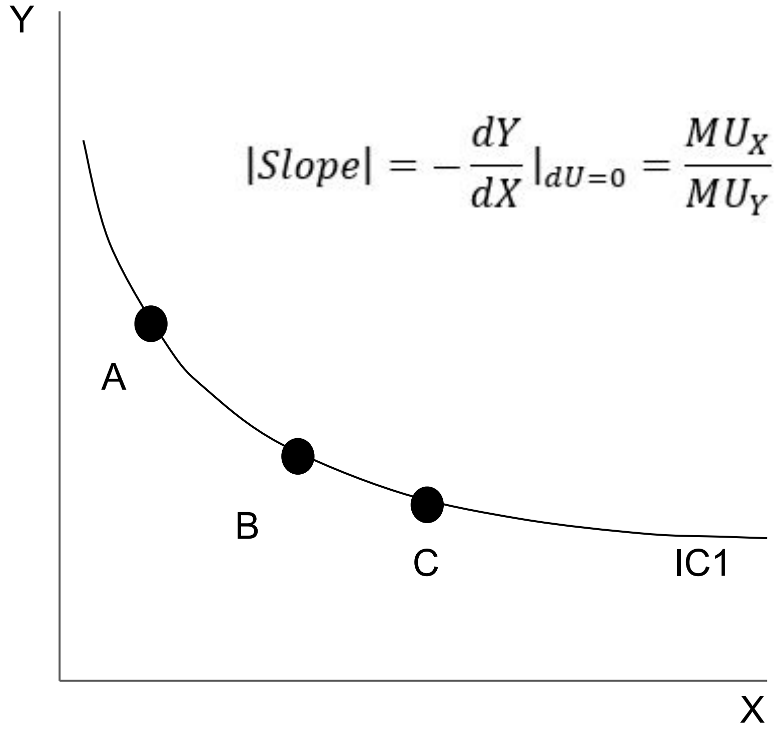
- And now we can find the slope: how much Y must change per unit of change of X to keep utility constant:

$$-\frac{dY}{dX}\bigg|_{dU=0} = \frac{MU_X}{MU_Y}$$

# Visualizing the MRS

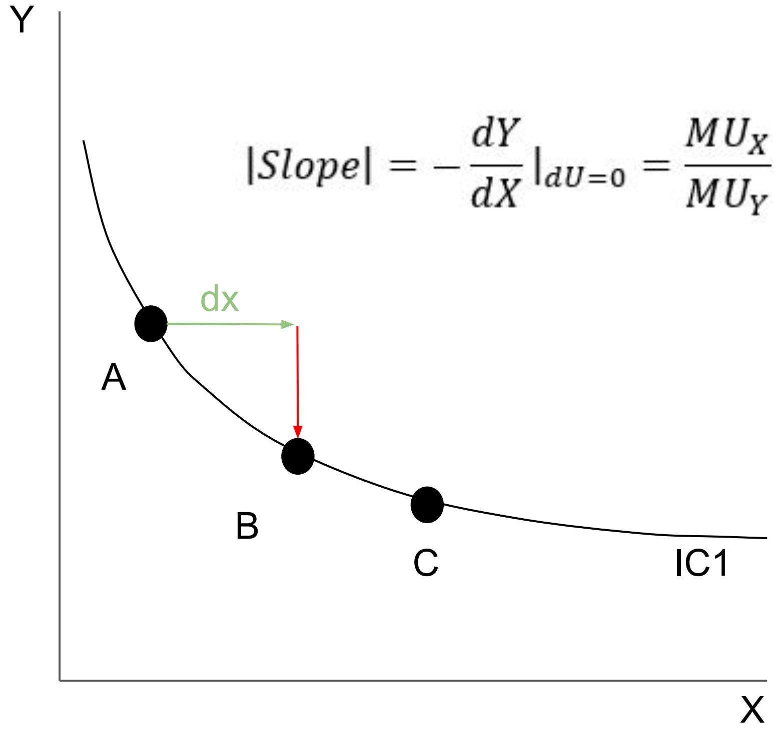
- (The magnitude of) the slope of an indifference curve is the ratio of the marginal utilities

$$|Slope| = -\frac{dY}{dX}\Big|_{dU=0} = \frac{MU_X}{MU_Y}$$



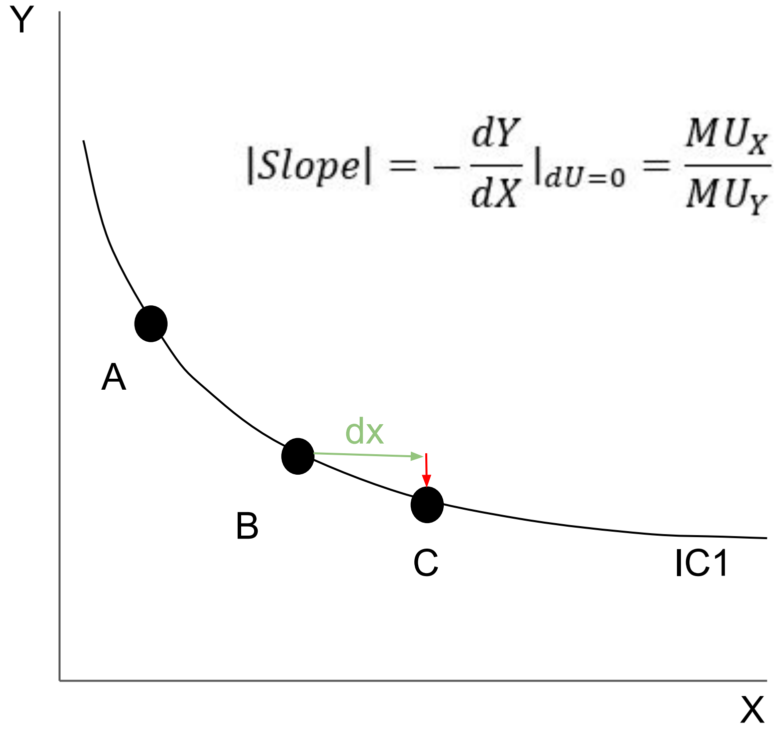
# Visualizing the MRS

- If the marginal utility of X is high relative to Y (because X is low and Y is high), she'd be willing to give up a lot of Y in order to get another unit of X.
  - Therefore, the indifference curve will be steep.



# Visualizing the MRS

- If the marginal utility of X is low relative to Y (because X is high and Y is low), she would not be willing to give up much Y in order to get another unit of X.
    - Therefore, the indifference curve will be flat.

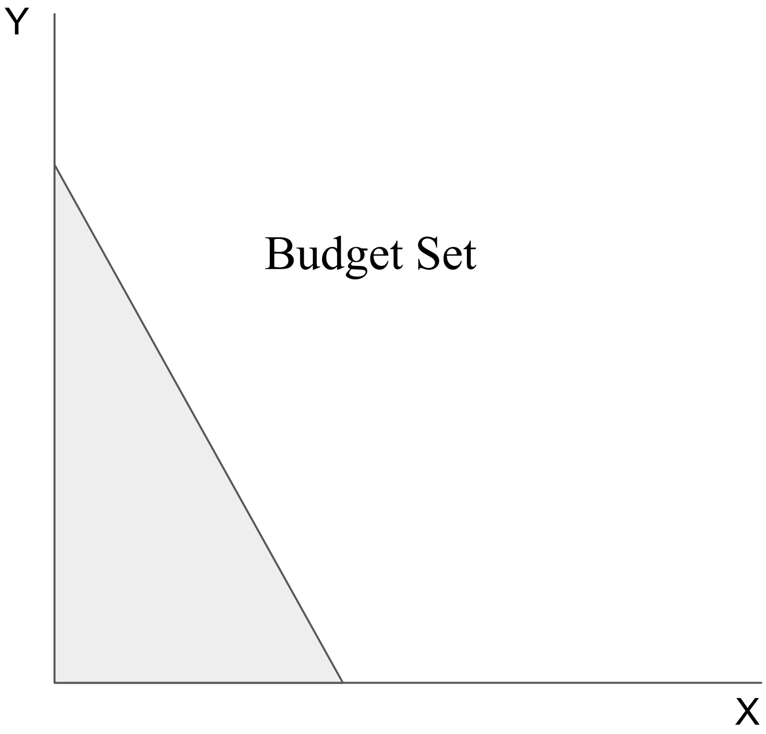


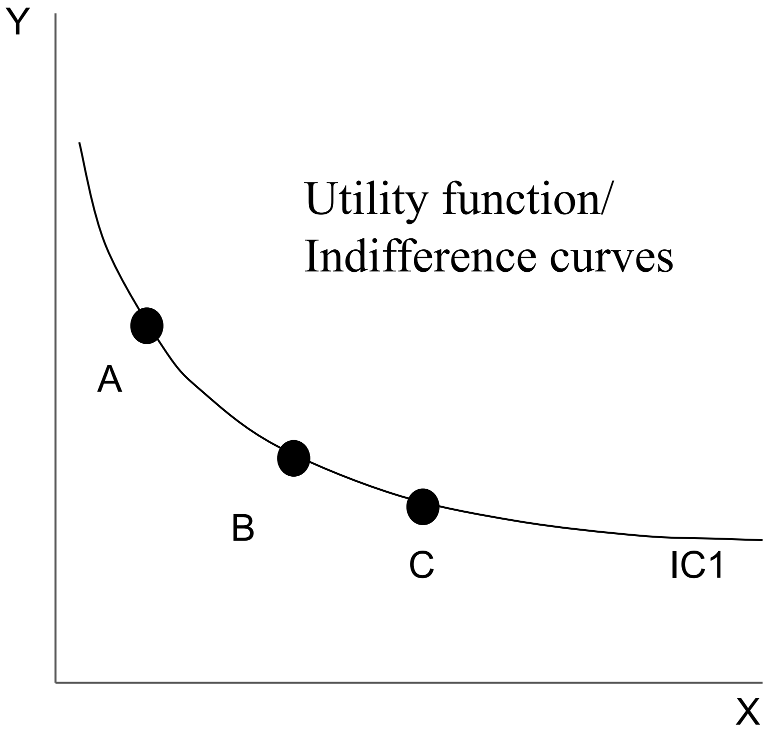

$$|Slope| = -\frac{dY}{dX}\Big|_{dU=0} = \frac{MU_X}{MU_Y}$$

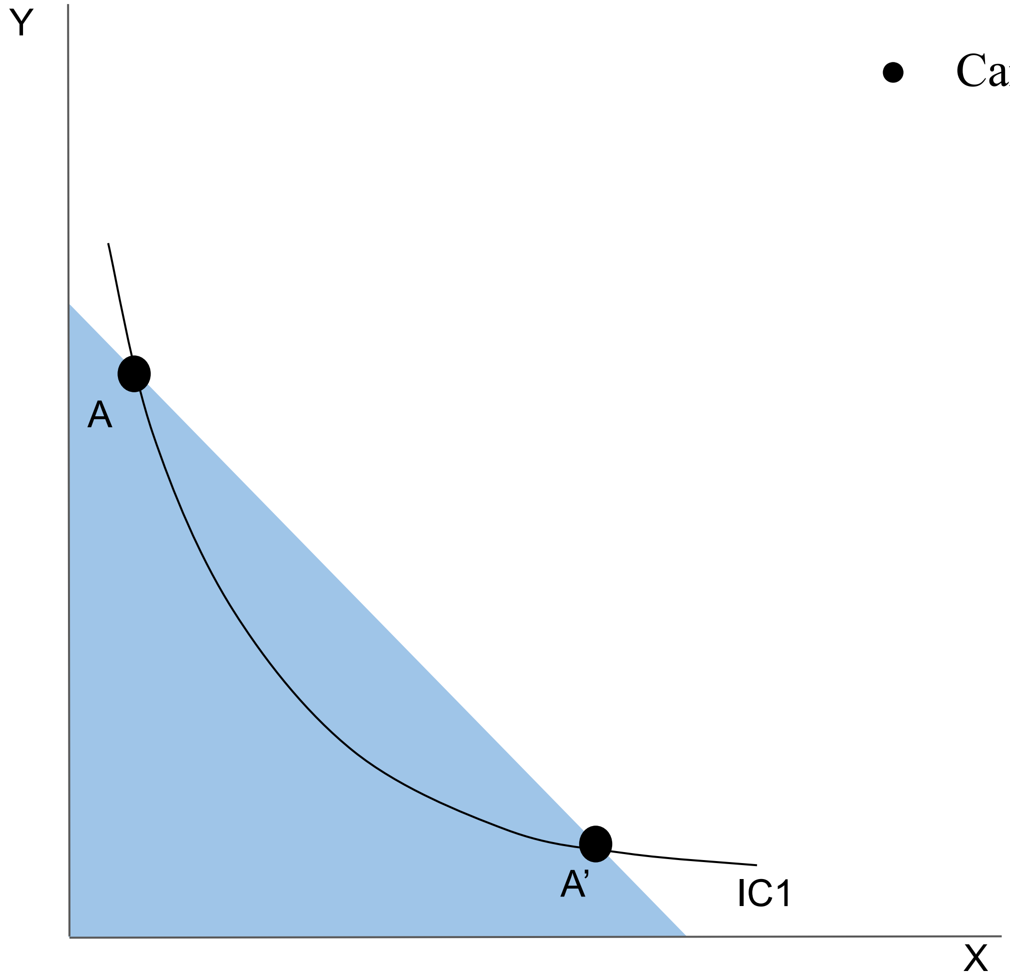
# Solving the Consumer Choice Problem

# Putting it all together

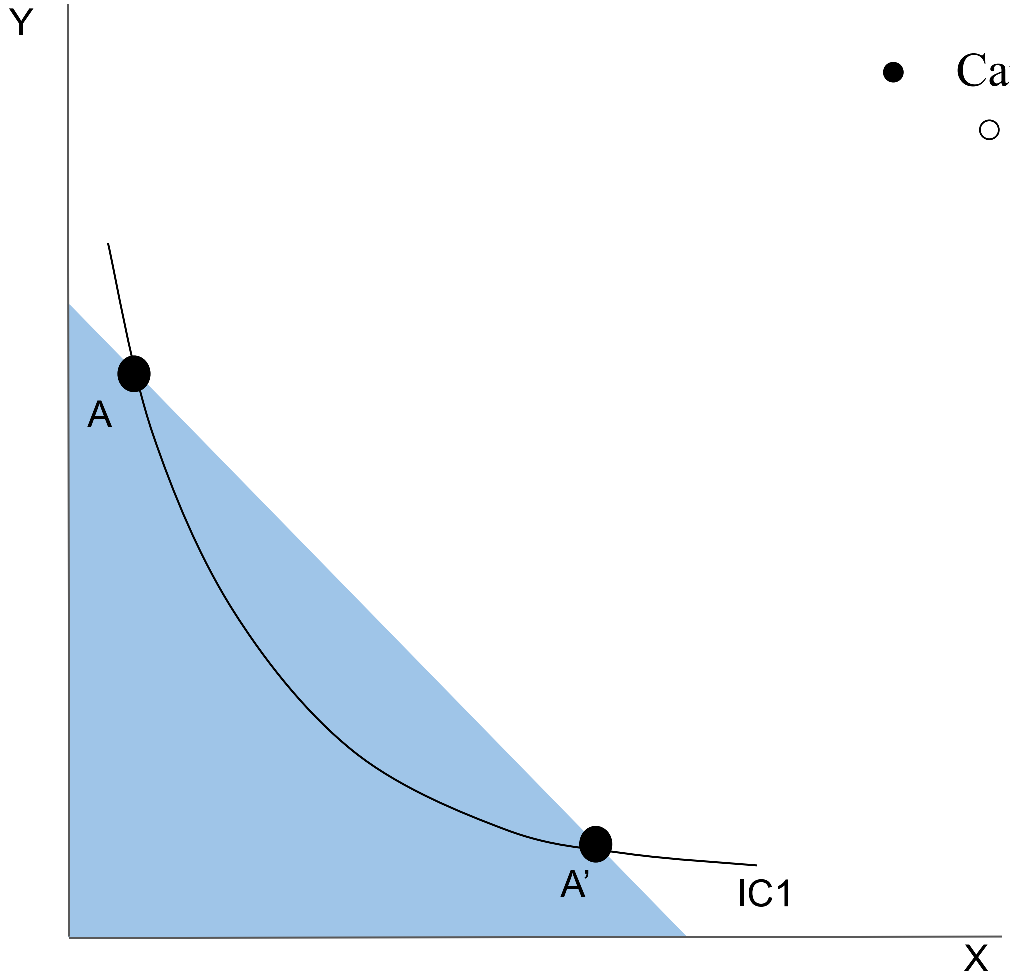
- We’ve now studied the two main ingredients to a consumer’s choice



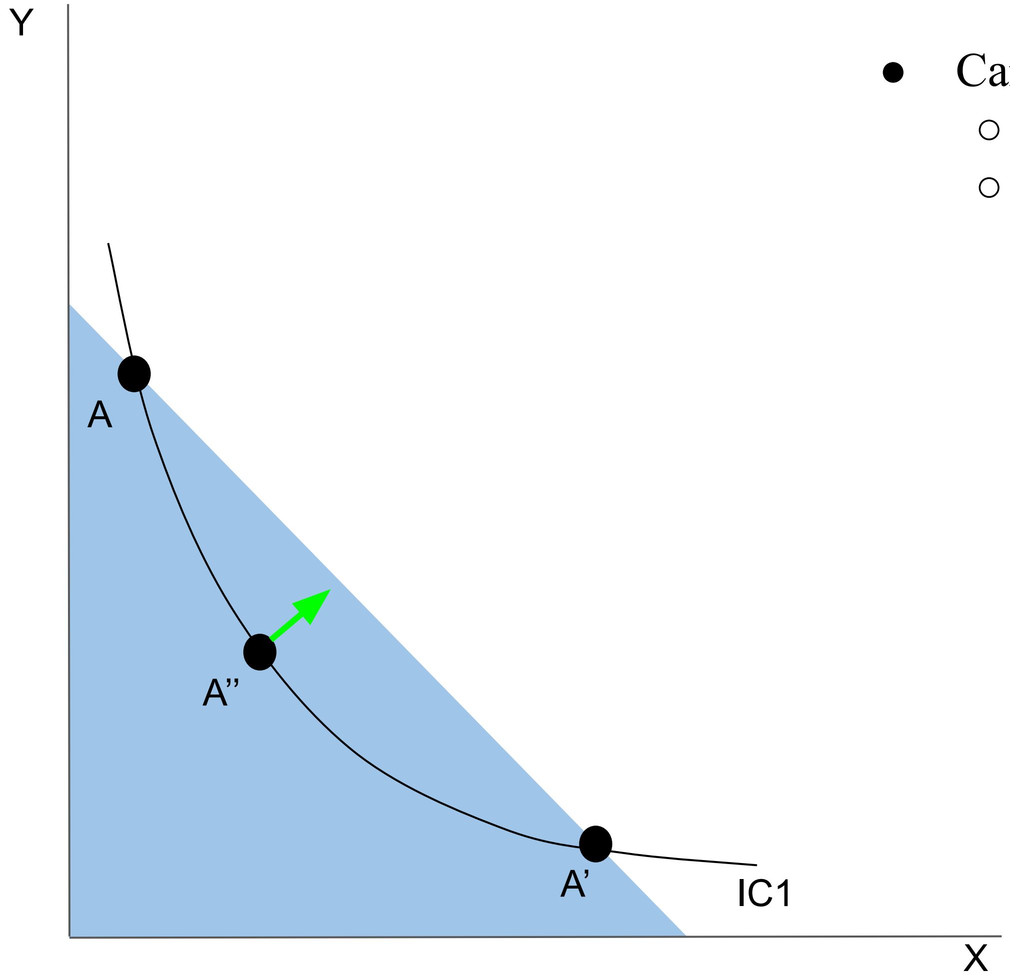





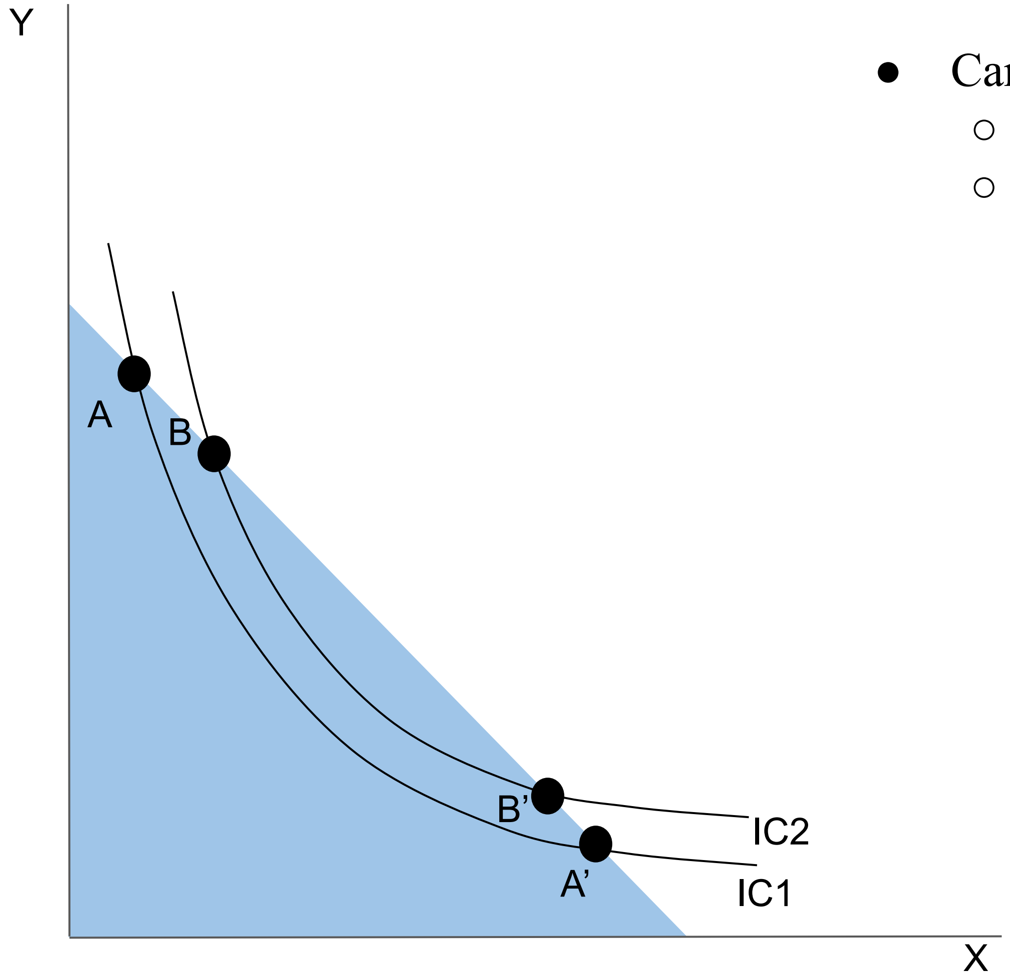

- Can A or A' be optimal?



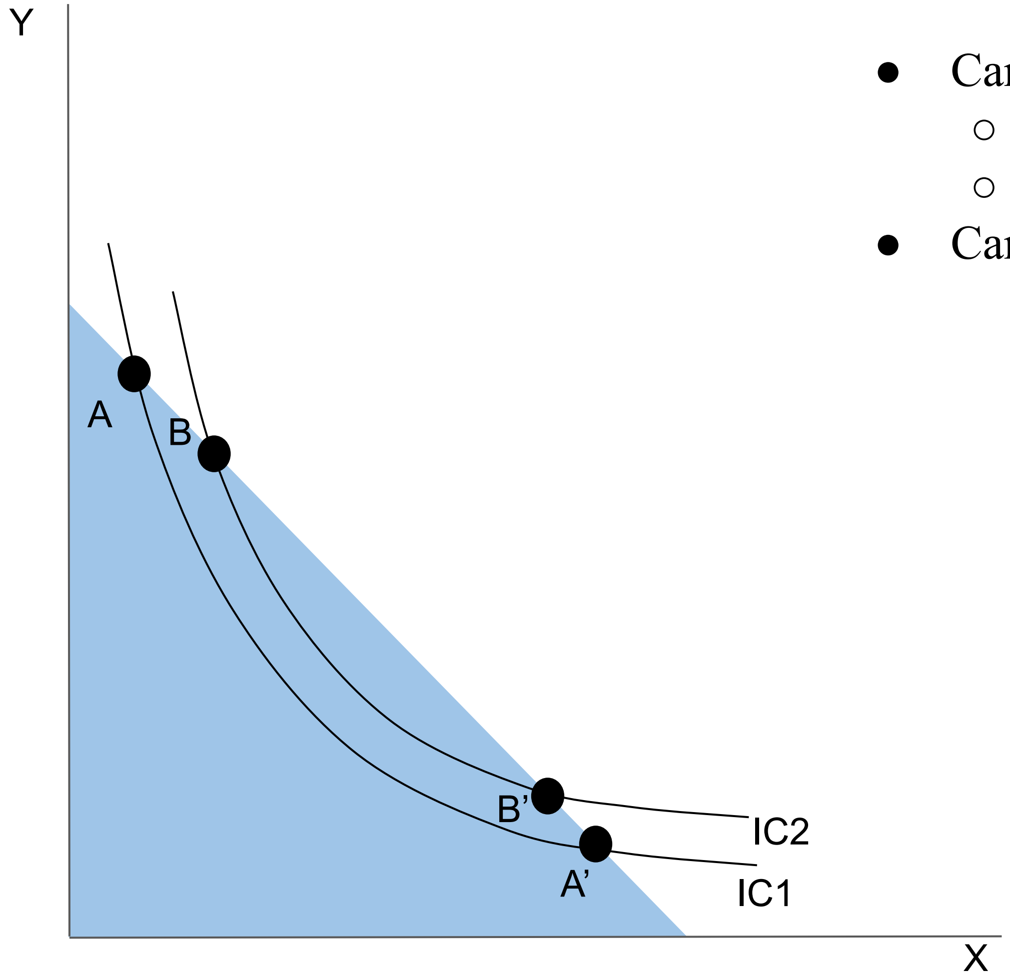

- Can A or A' be optimal?
  - No



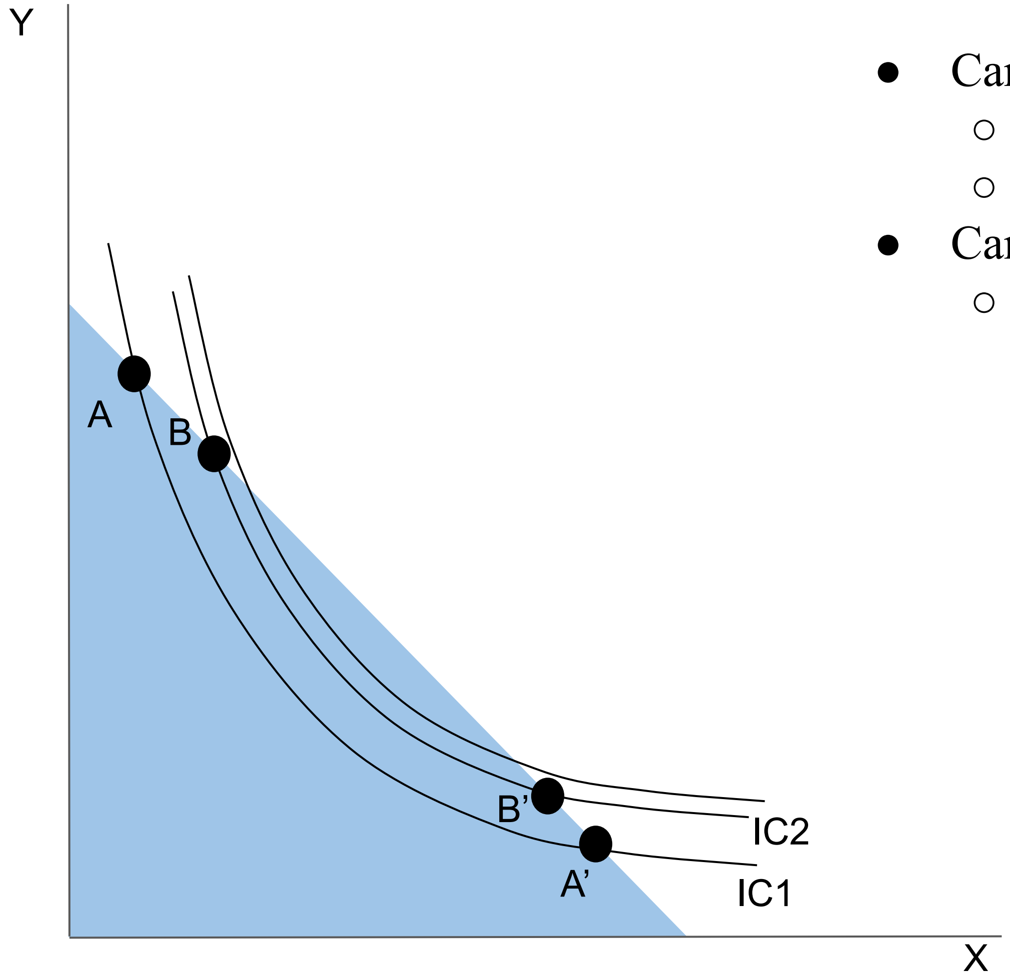

- Can A or A' be optimal?
    - No
    - A'' gives same utility, but clearly is not best



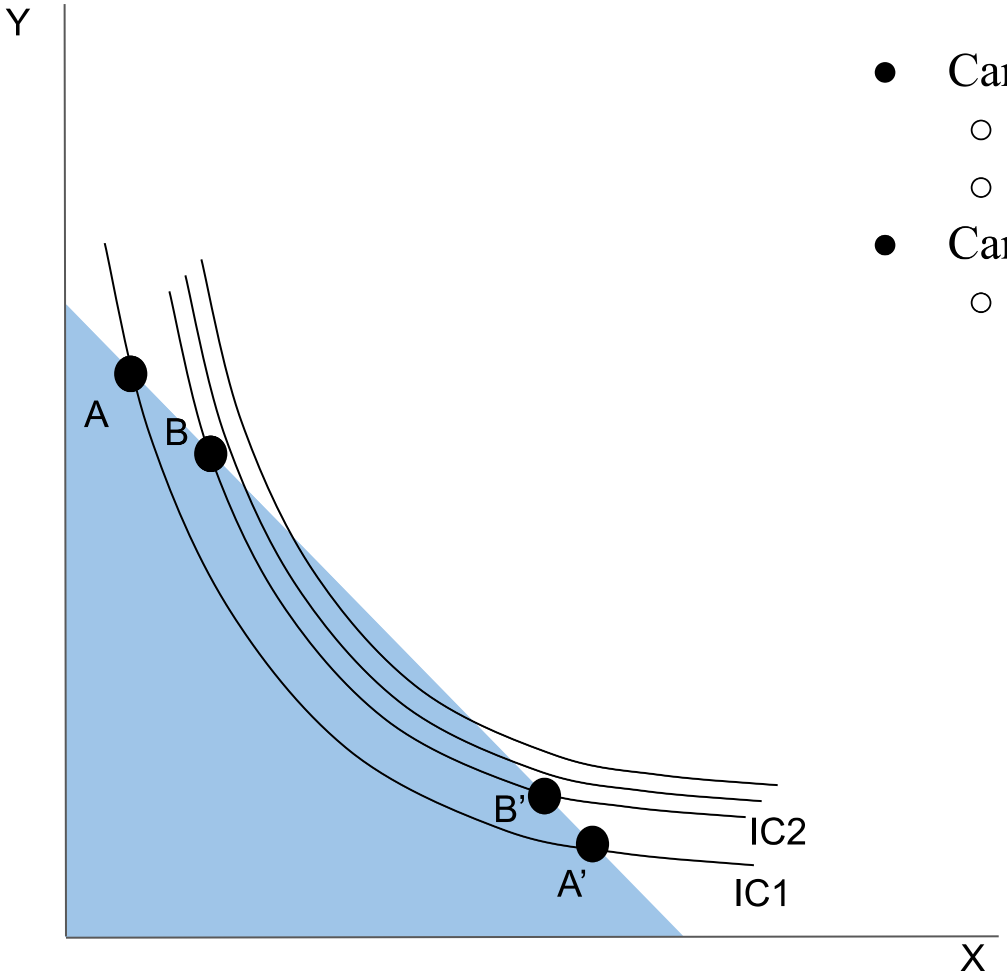

- Can A or A' be optimal?
  - No
  - B and B' are better than A and A'




- Can A or A’ be optimal?
  - No
  - B and B’ are better than A and A’
- Can B or B’ be optimal?




- Can A or A' be optimal?
  - No
  - B and B' are better than A and A'
- Can B or B' be optimal?
  - No

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  - No
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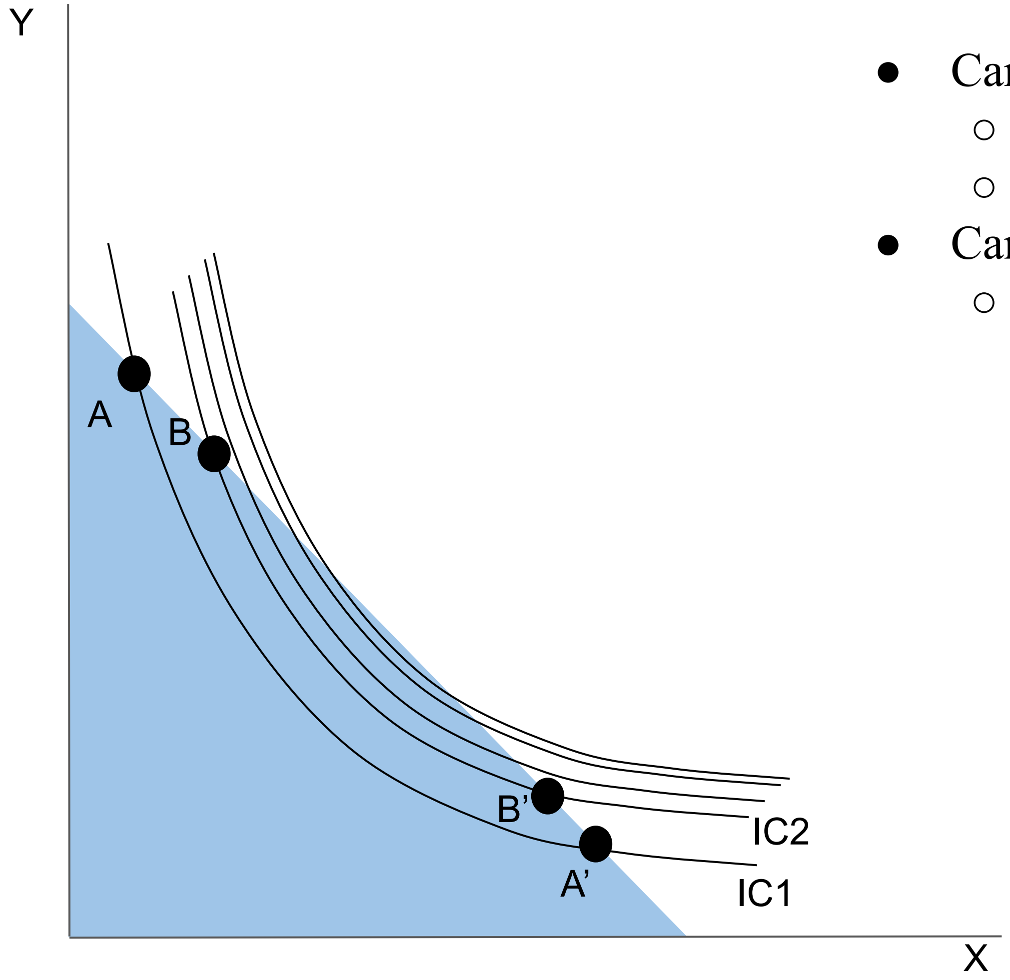
  - No

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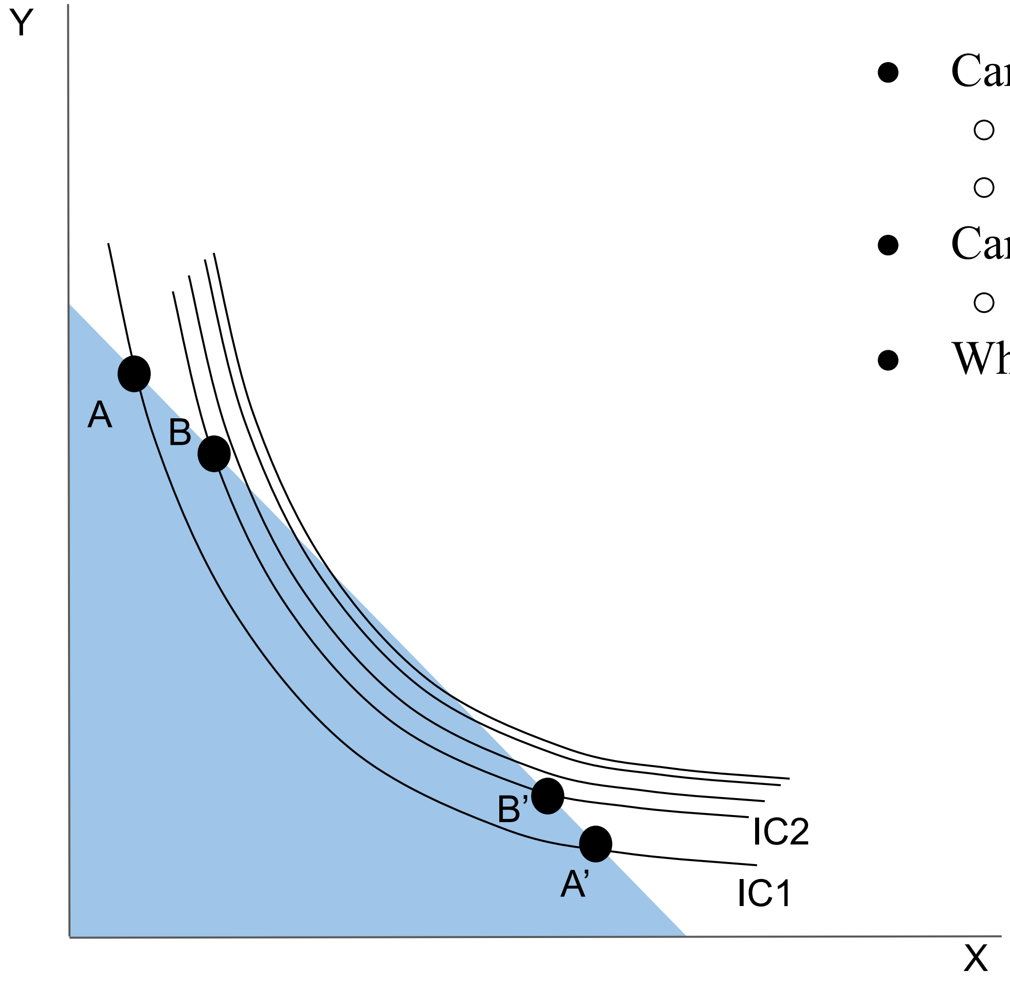
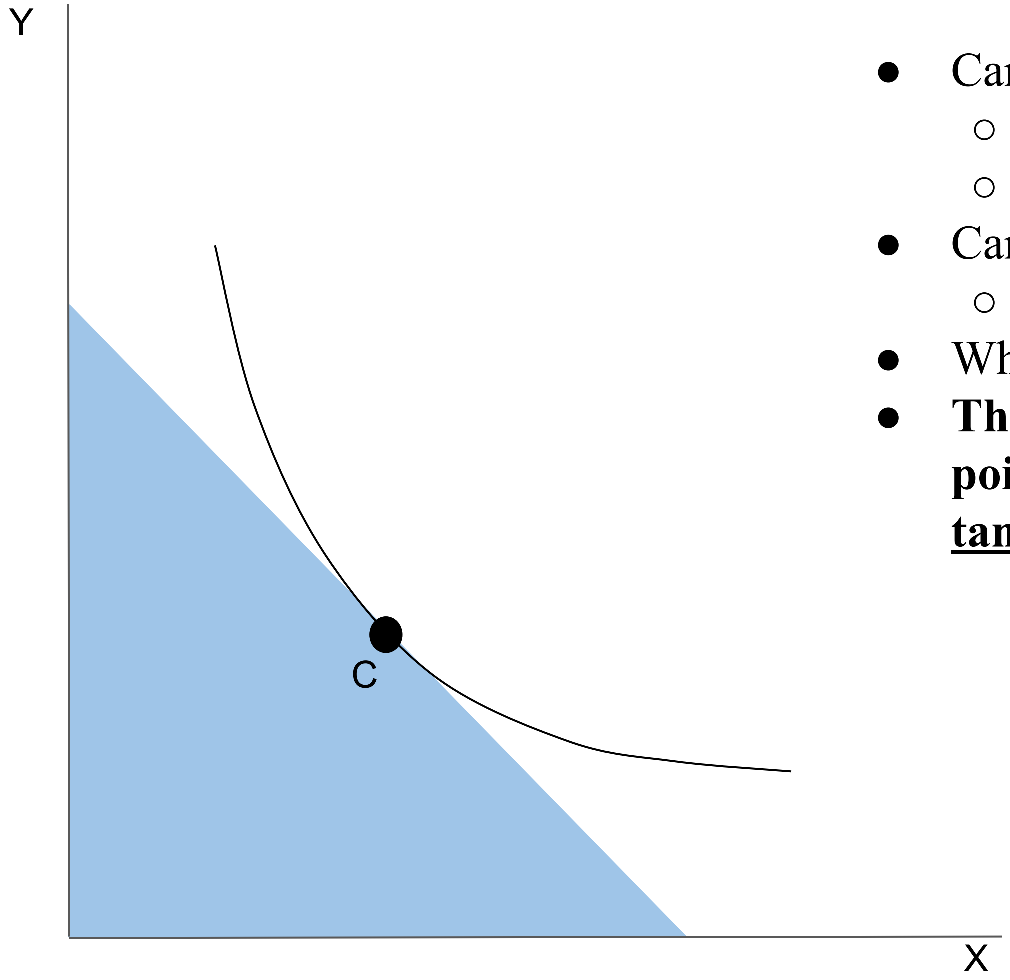
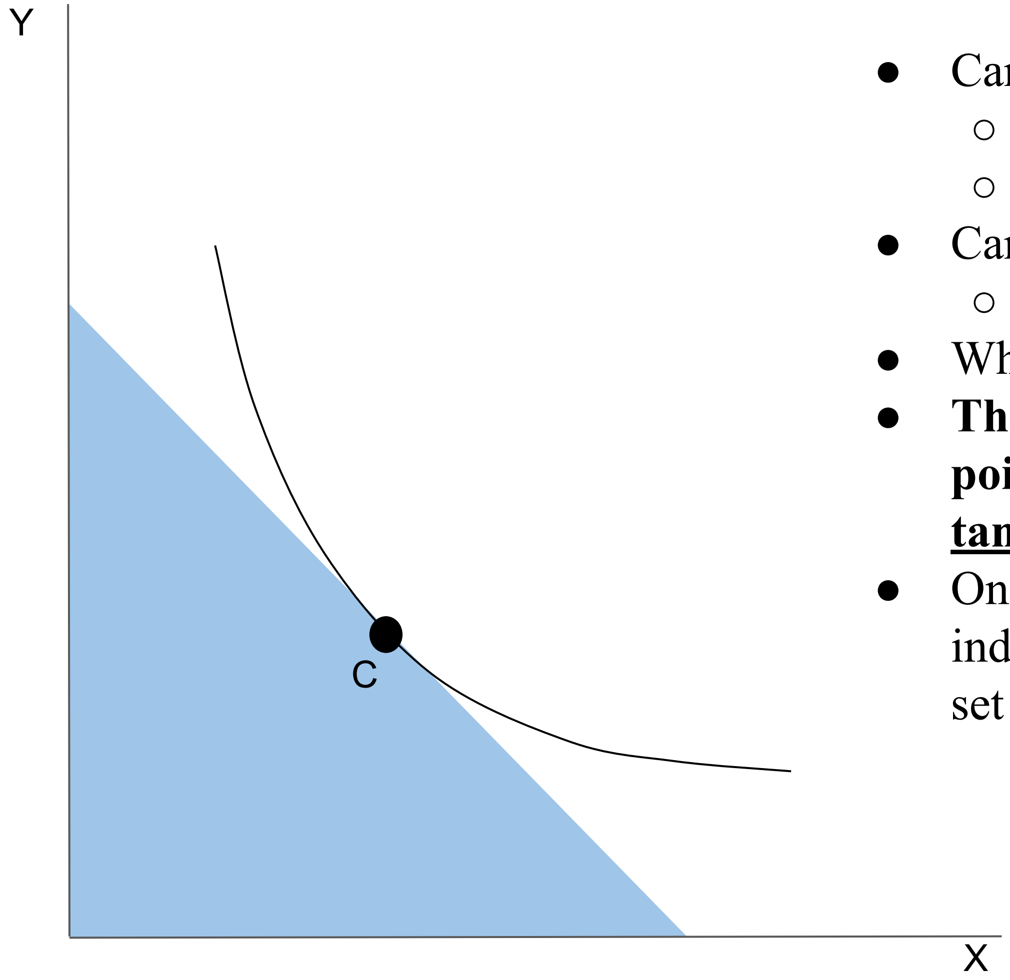
  - No



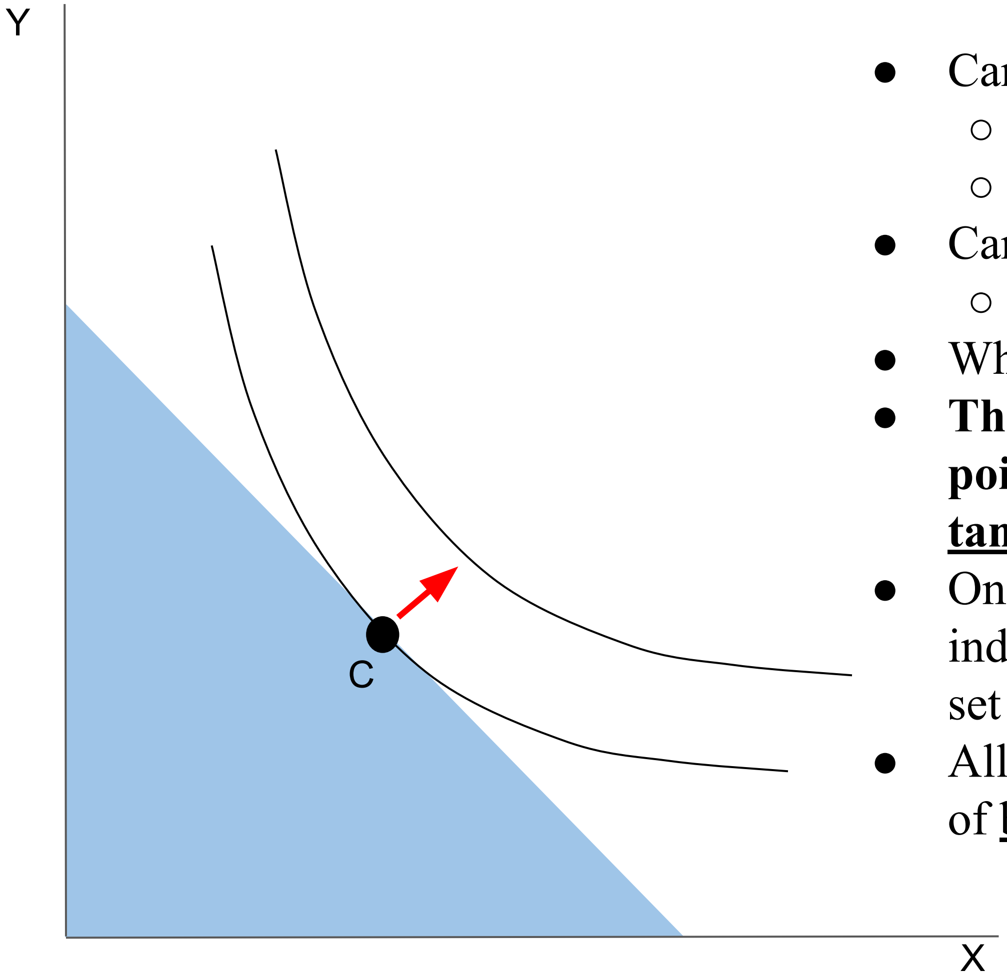

- Can A or A' be optimal?
  - No
  - B and B' are better than A and A'
- Can B or B' be optimal?
  - No
- Where will it end?




- Can A or A’ be optimal?
  - No
  - B and B’ are better than A and A’
- Can B or B’ be optimal?
  - No
- Where will it end?
- **The utility maximizing choice is C, the point at which an indifference curve is <u>tangent</u> to the budget line**




- Can A or A’ be optimal?
  - No
  - B and B’ are better than A and A’
- Can B or B’ be optimal?
  - No
- Where will it end?
- **The utility maximizing choice is C, the point at which an indifference curve is <u>tangent</u> to the budget line**
- Only at C can she not slide over to a higher indifference curve that intersects the budget set




- Can A or A' be optimal?
  - No
  - B and B' are better than A and A'
- Can B or B' be optimal?
  - No
- Where will it end?
- **The utility maximizing choice is C, the point at which an indifference curve is tangent to the budget line**
- Only at C can she not slide over to a higher indifference curve that intersects the budget set
- All higher indifference curves require more of both X and Y, which is not affordable

# Characterizing optimality

- We just showed for the optimal choice, the slope of the indifference curve is equal to the slope of the budget line. Therefore:

$$|Slope\ of\ BL| = \frac{P_X}{P_Y} = \frac{MU_X}{MU_Y} = |Slope\ of\ IC|$$

- The ratio of prices is equal to the ratio of marginal utilities (MRS)
- What is the intuition for this?

# Intuition of optimal bundle (1)

$\frac{P_X}{P_Y}$ The tradeoff between goods the market is accepting: in exchange for 1 unit of X, the price ratio is the number of units of Y she can get.

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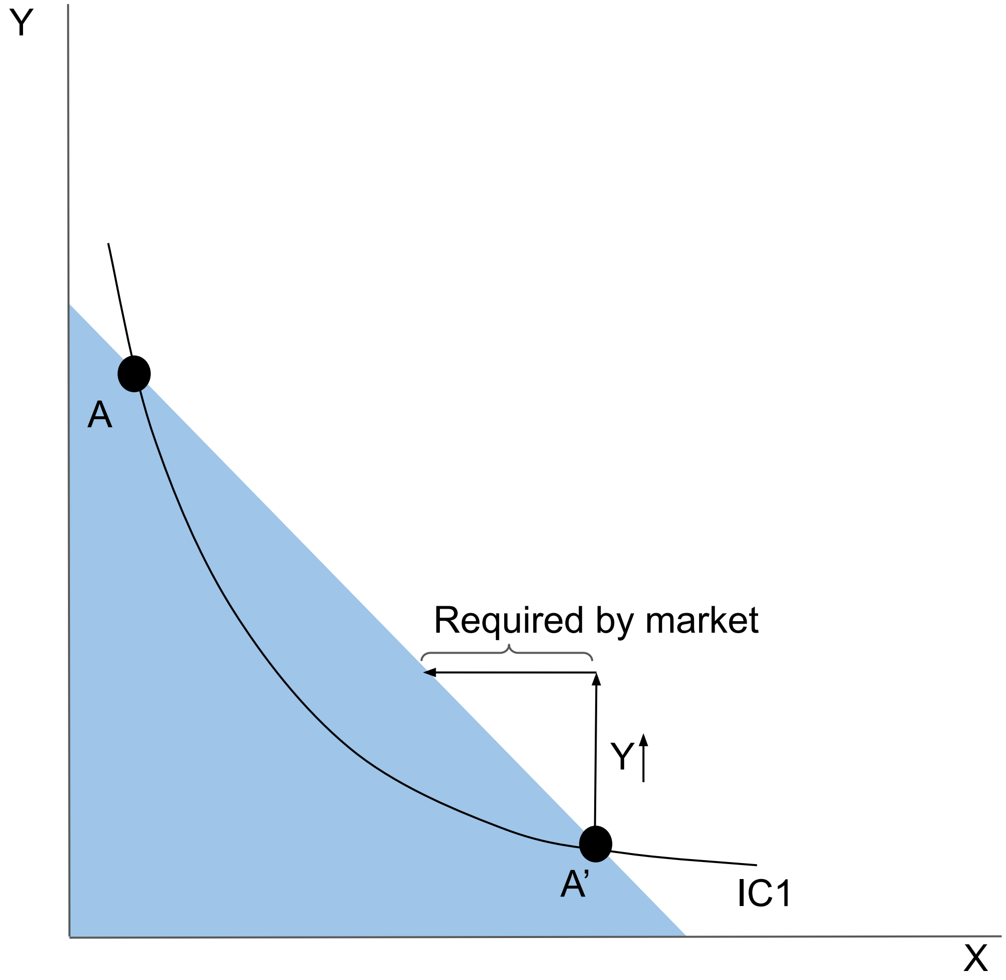
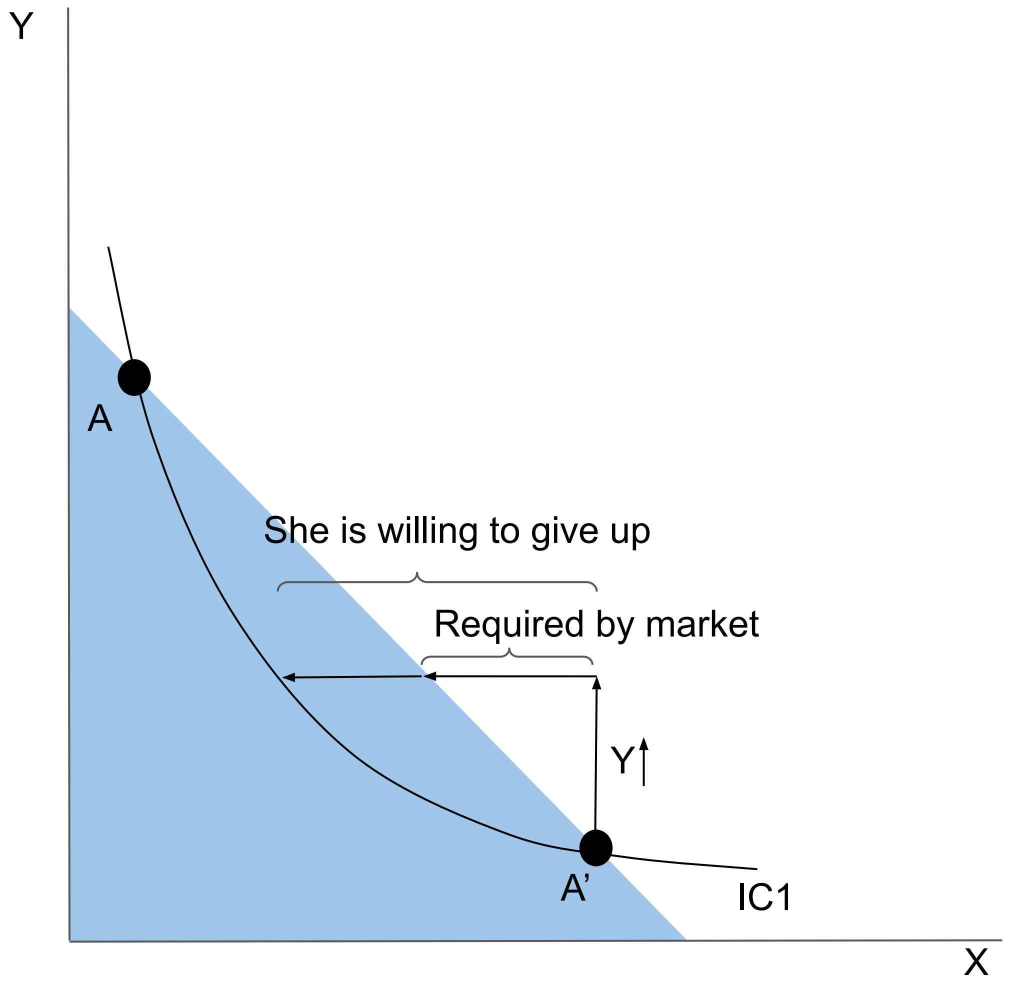
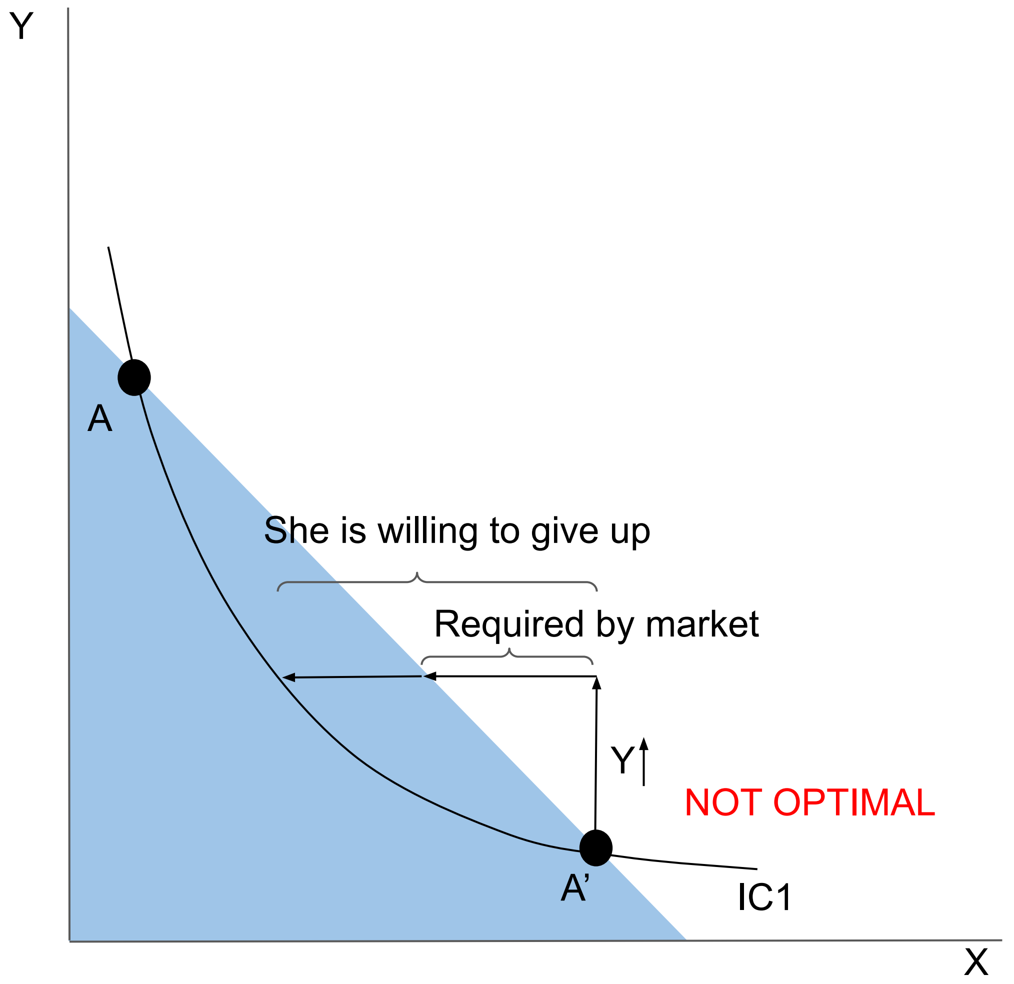

Y
A
Required by market
Y
A'
IC1
X


Y
A
She is willing to give up
Required by market
Y
A'
IC1
X


Y
A
She is willing to give up
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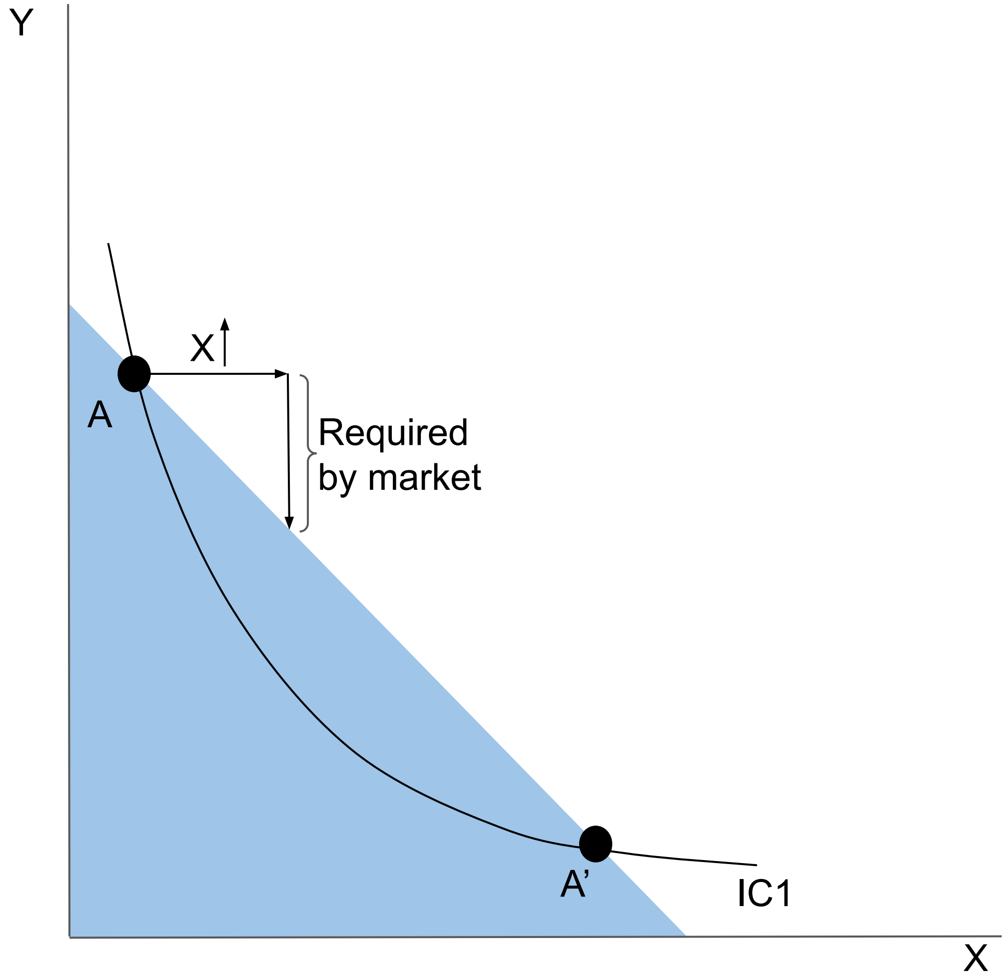
Y
X
Required
by market
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IC1
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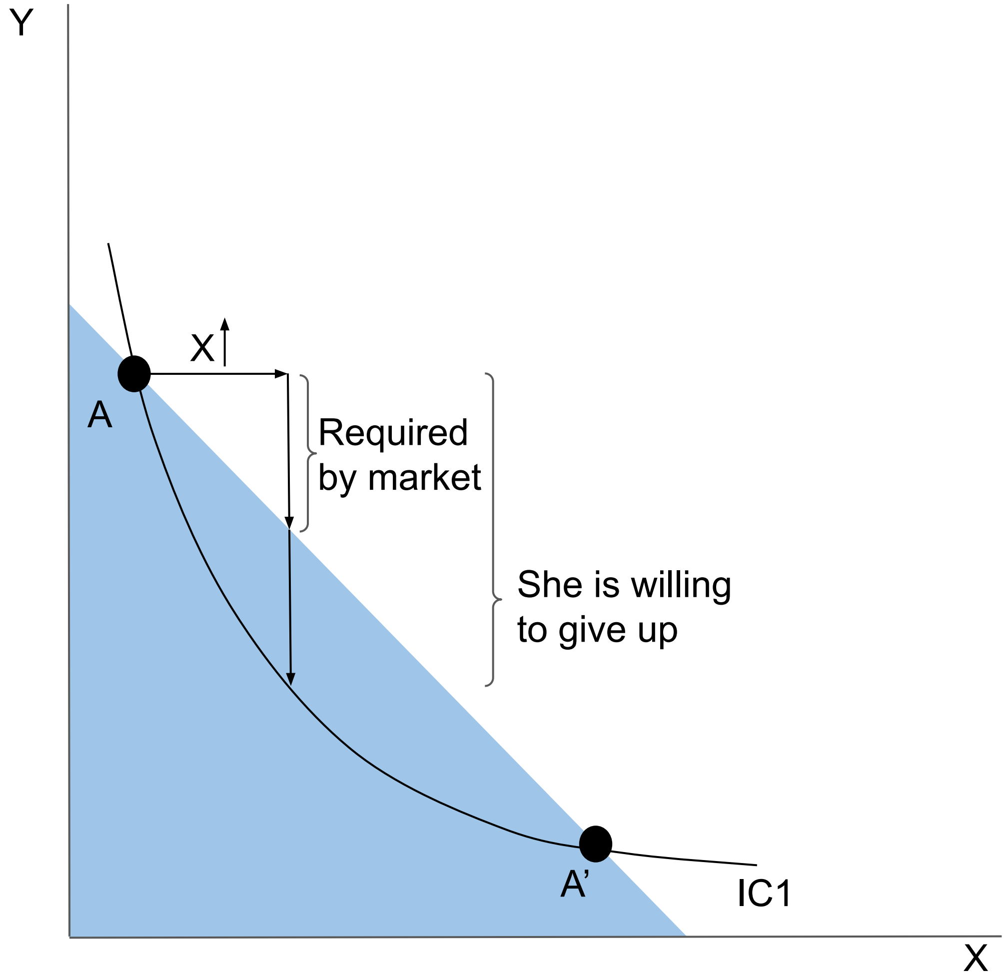
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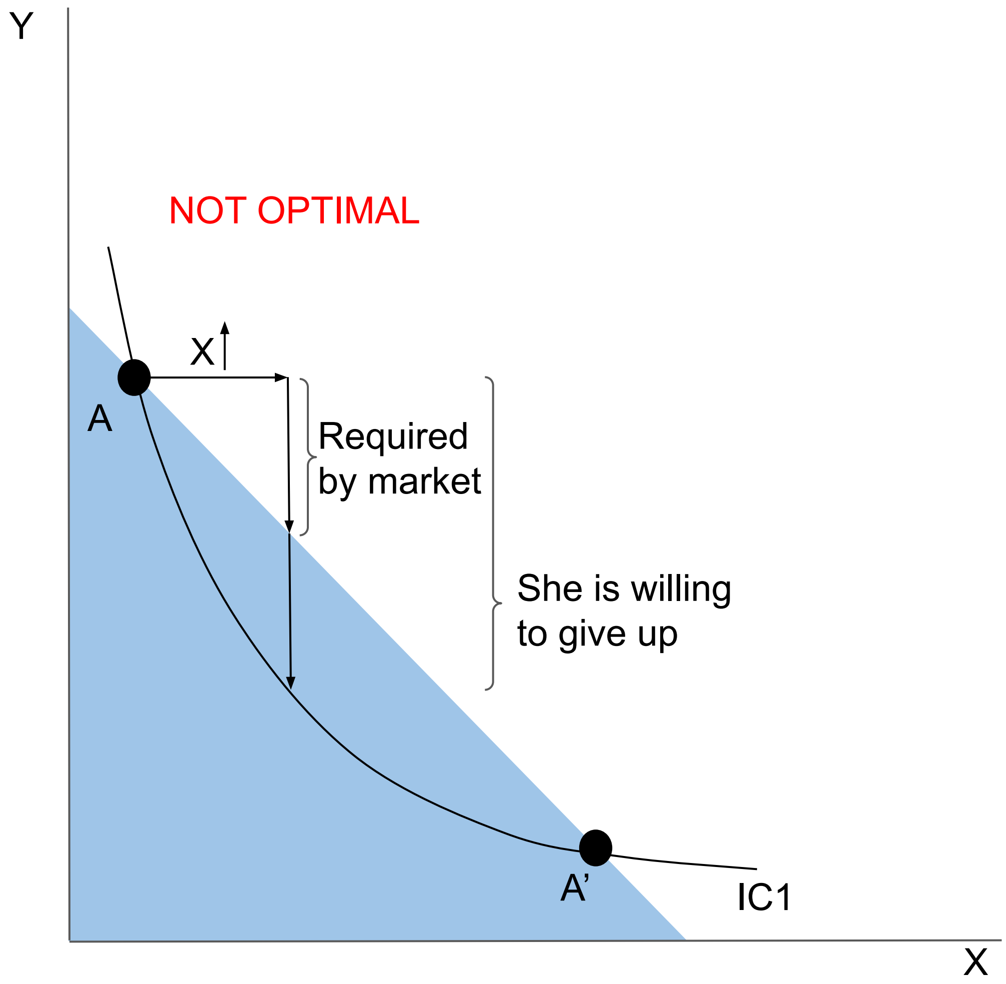
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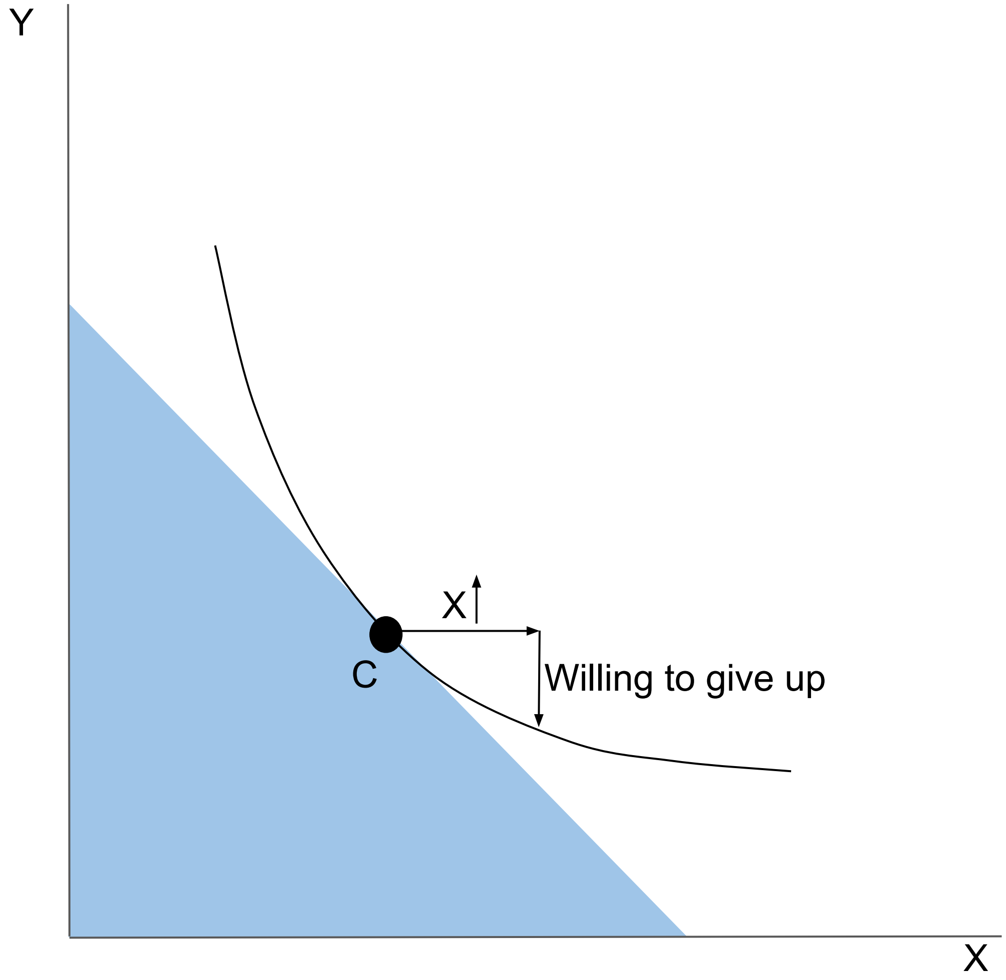
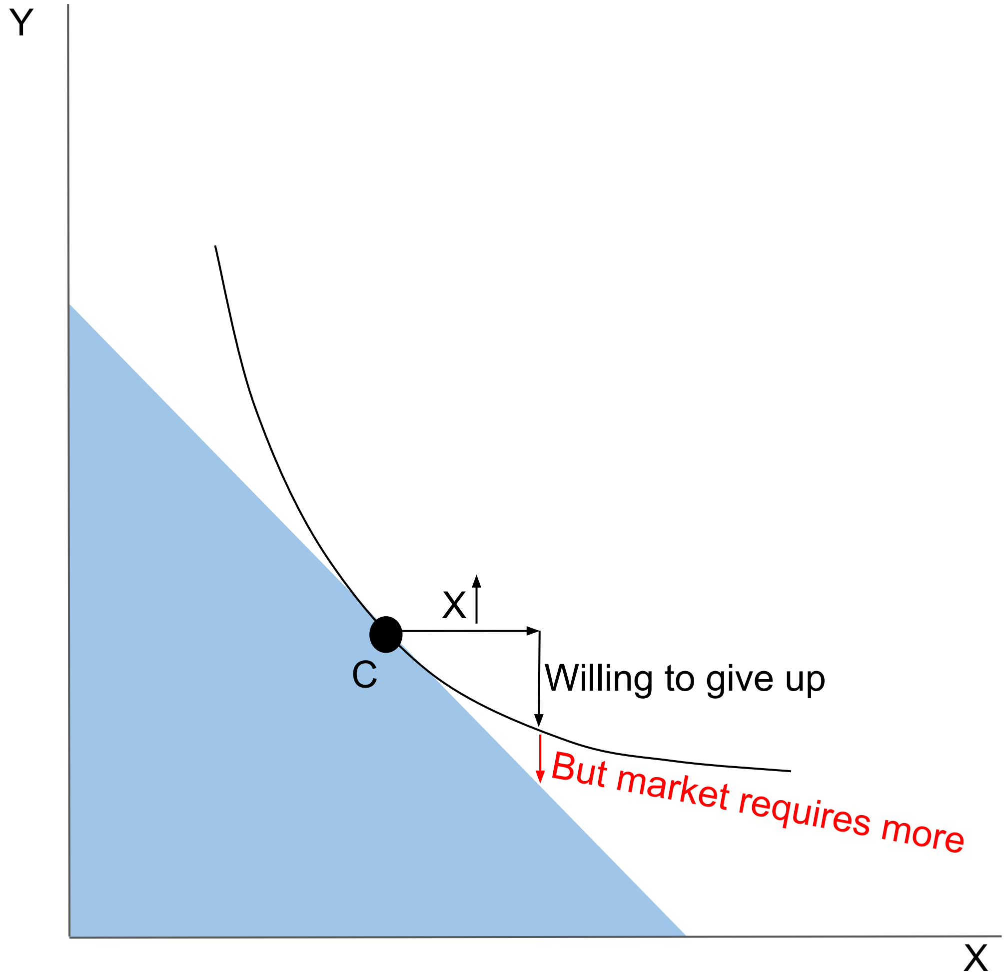
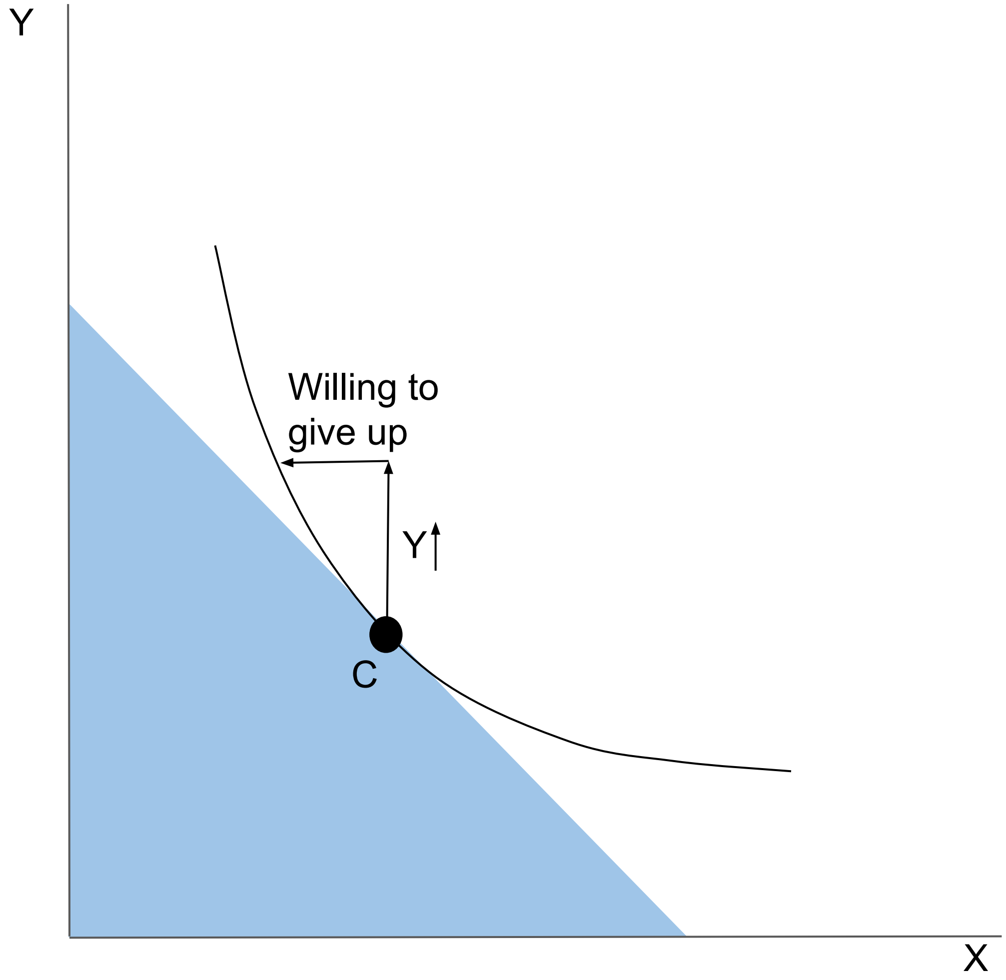
**Only when** $\frac{P_X}{P_Y} = \frac{MU_X}{MU_Y}$ **can no utility-improving trade be made – therefore, it's optimal.**


Y
X↑
C
Willing to give up
X



Y
X
C
Willing to give up
But market requires more
X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give up
Y
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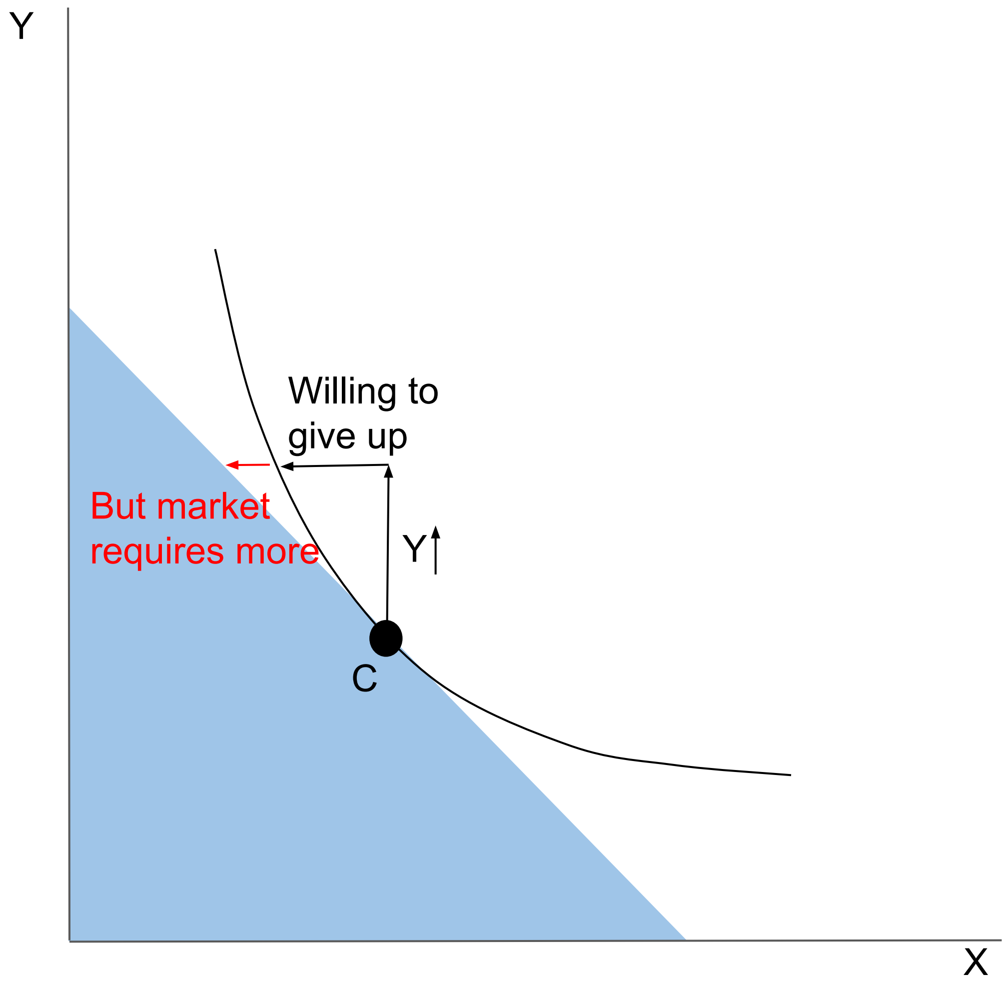
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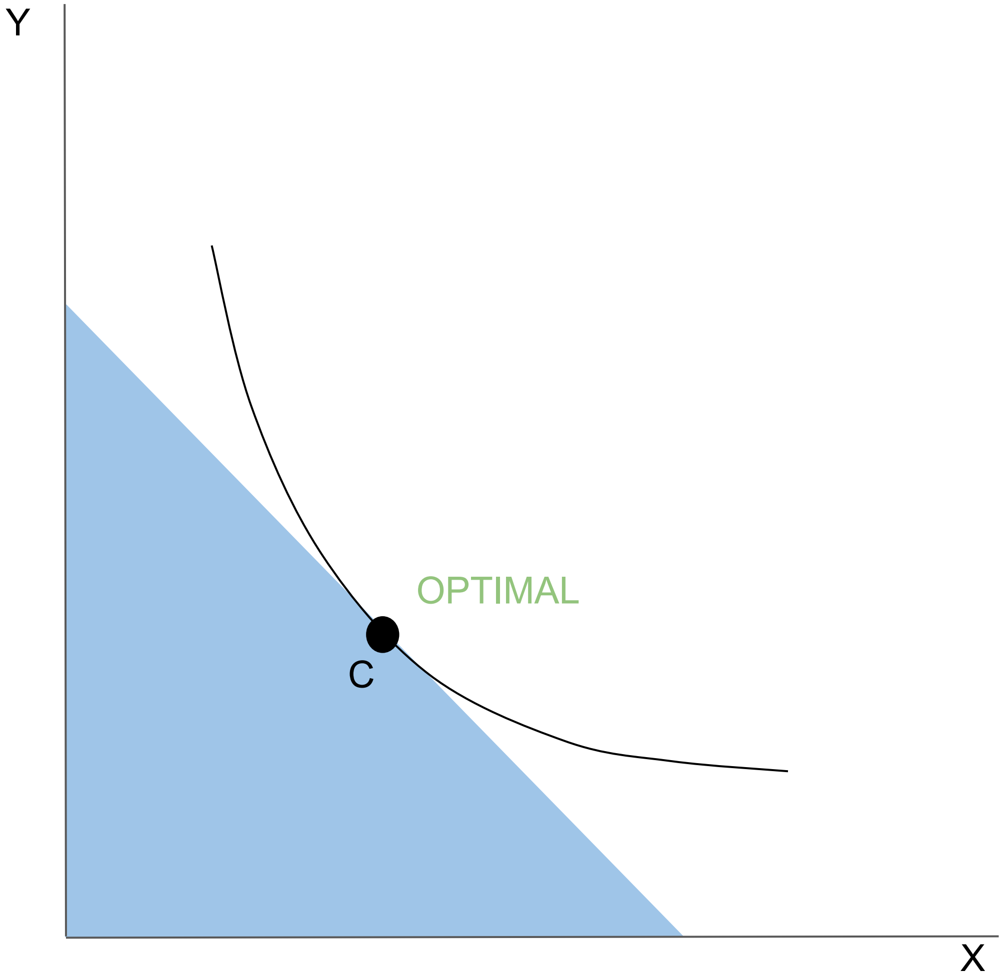
Y
OPTIMAL
C
X

# Intuition of optimal bundle (2)

$$\frac{MU_Y}{P_Y} = \frac{MU_X}{P_X}$$

This version says that spending an additional dollar on one good should yield the same utility as spending an additional dollar on the other good. If she is indifferent between the two options, she is optimizing.

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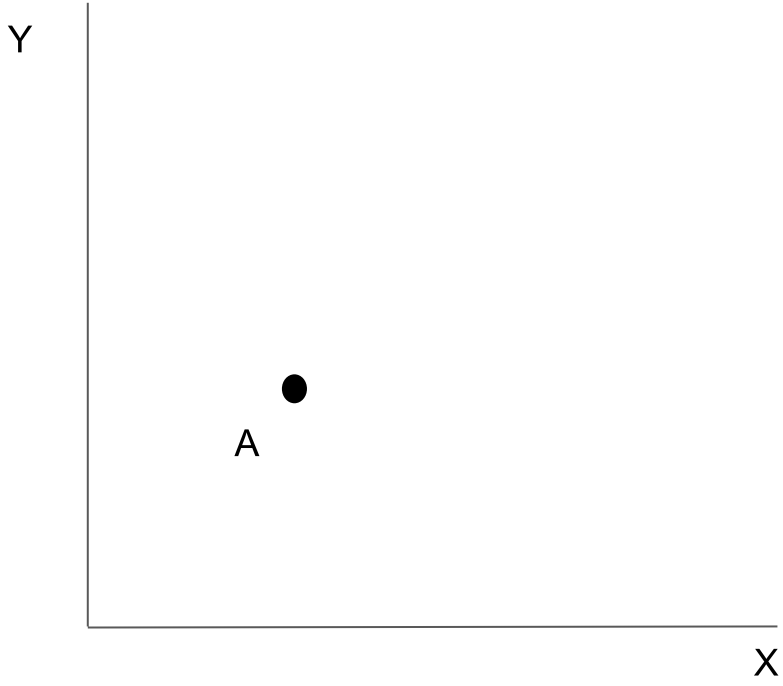
# “Non-Convex” Preferences

# “Non-convex” preferences

- So far we have studied preferences with decreasing MRS (ICs get flatter and flatter)
  - Economists refer to these as “well-behaved” because they can be solved with calculus
- In some instances, this is not an appropriate description of preferences
  - Indifference curves are different, and solutions are found in a different way

# “Bads”

- Suppose the consumer likes X (“good”) but dislikes Y (“bad”)




- Consider bundle A

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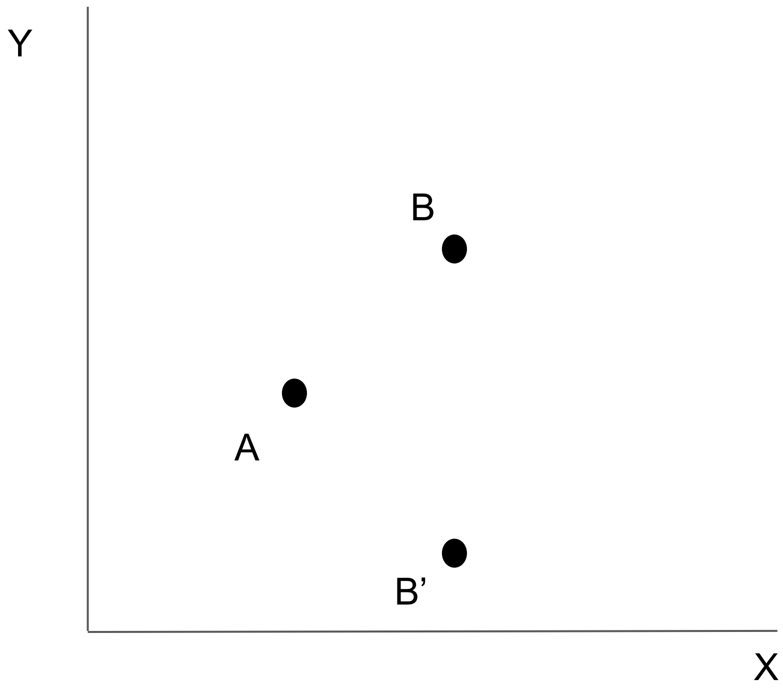


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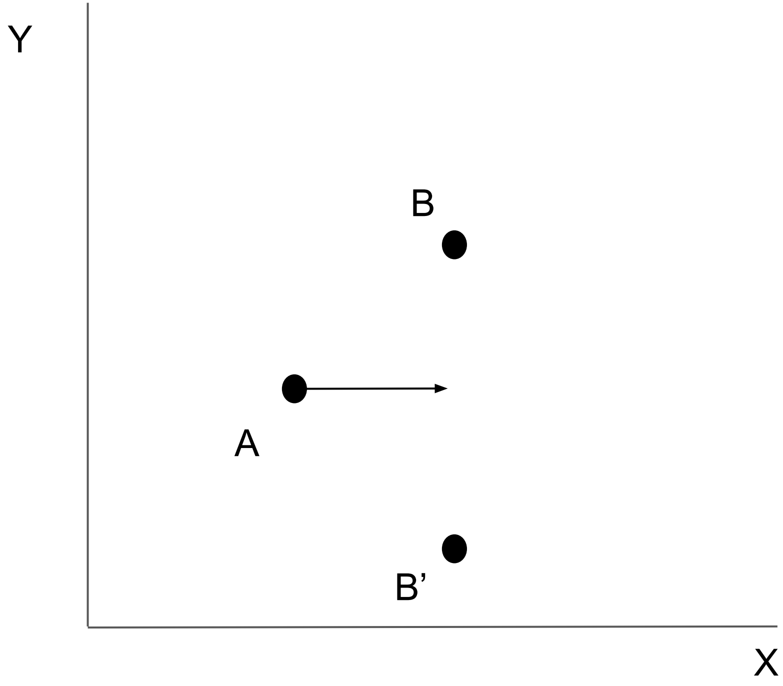


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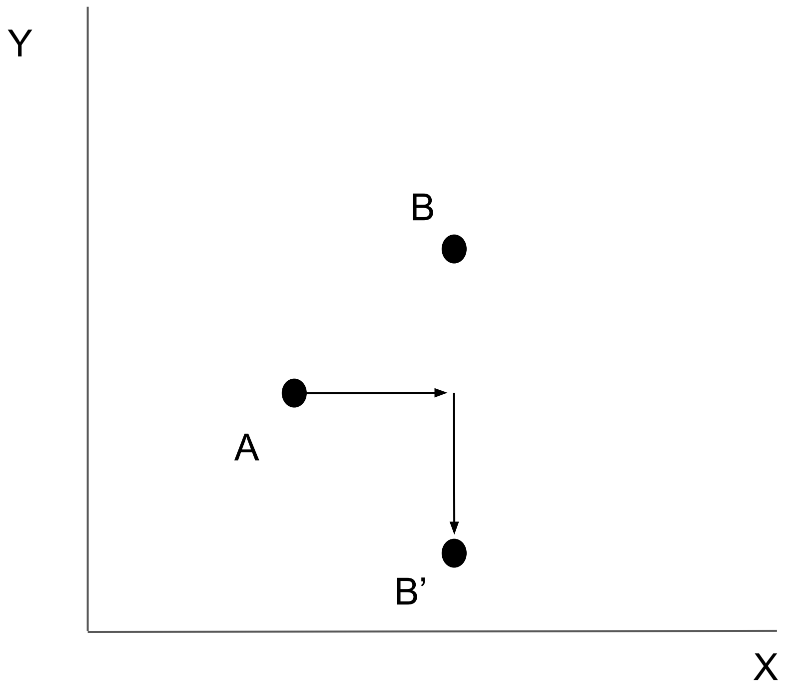


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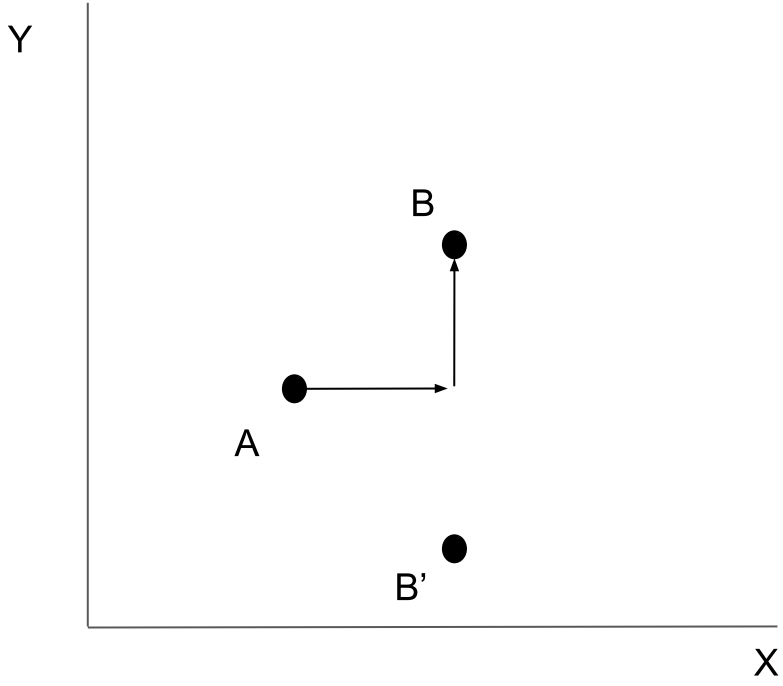


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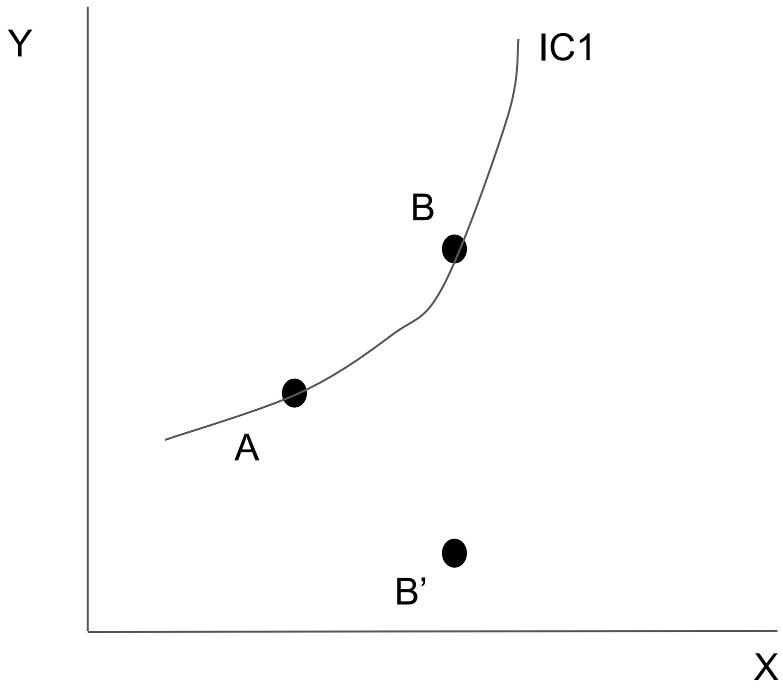


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# Perfect substitutes

- Suppose both goods have constant marginal utility
- E.g. $U(X,Y) = X + Y$
    - MU for both goods is 1, MRS = 1 – i.e. MRS is constant
- What does this imply about the slope of the consumer's ICs?

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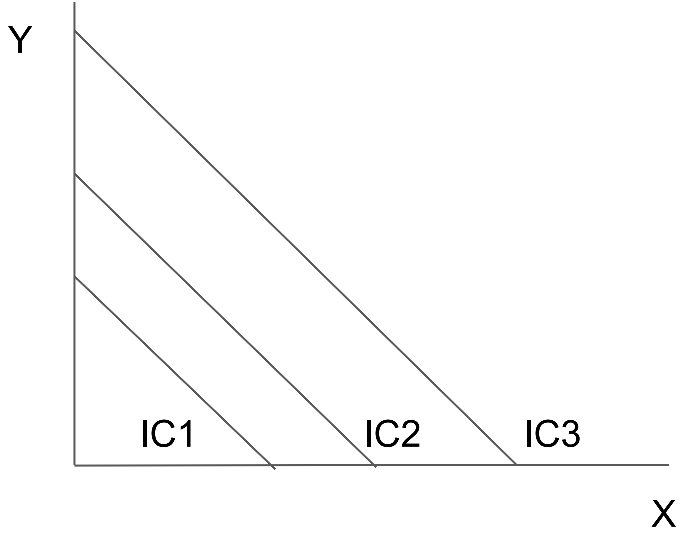
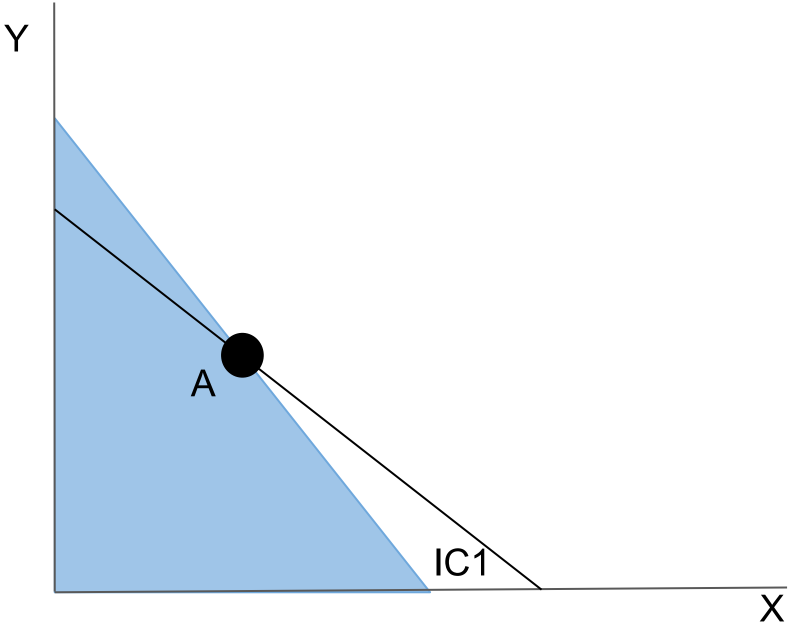


# Solution with perfect substitutes

- MRS is constant, so can't find a point where MRS is equal to price ratio
- Useful to think about graphically…

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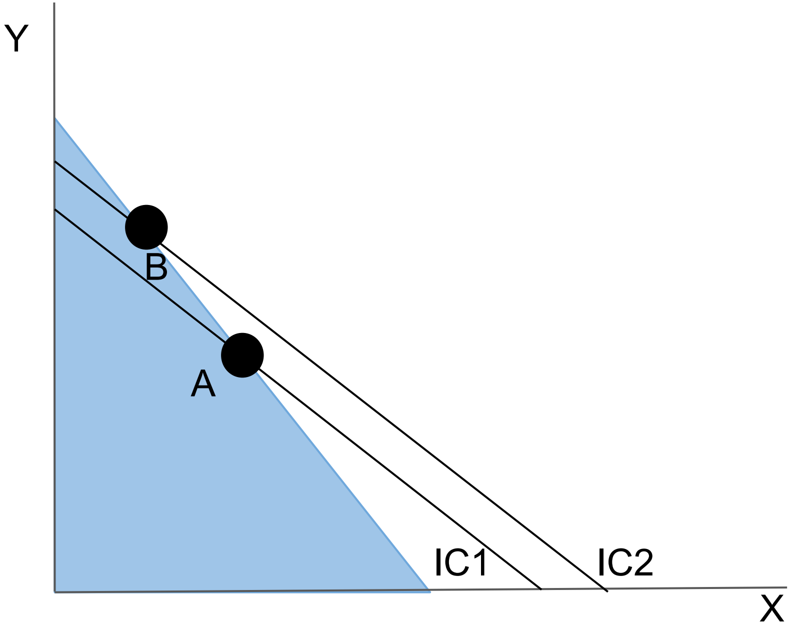


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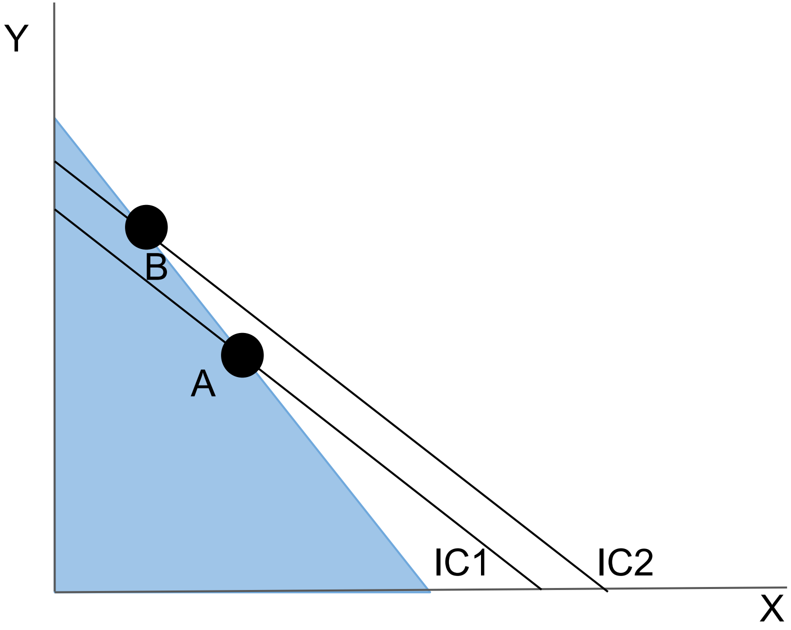


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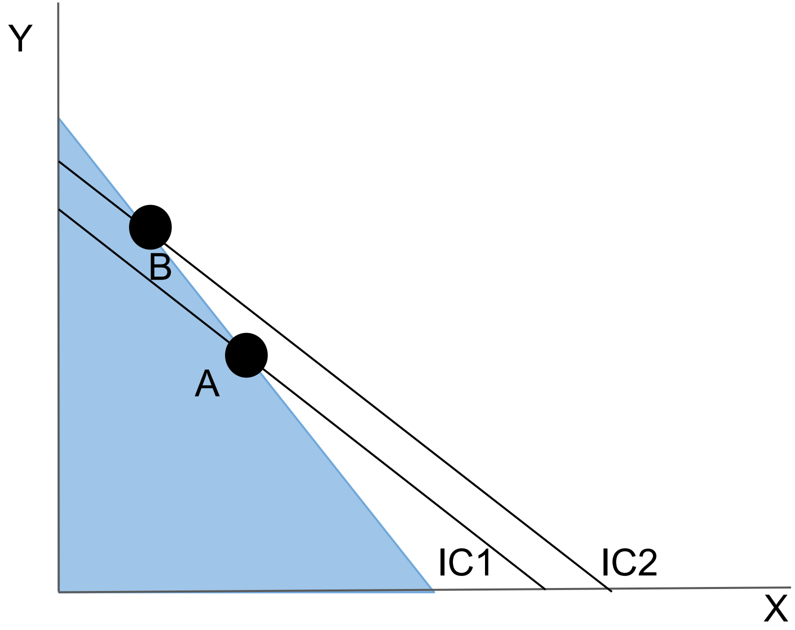


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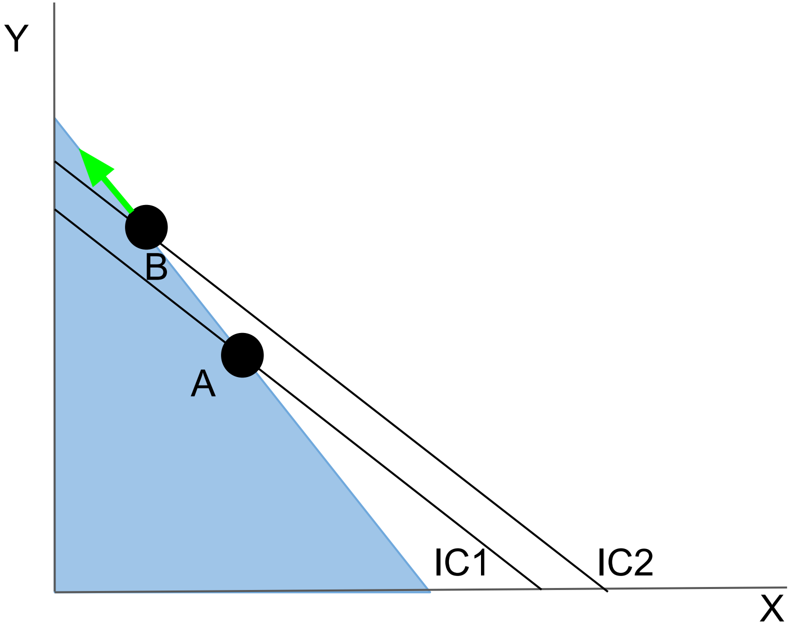


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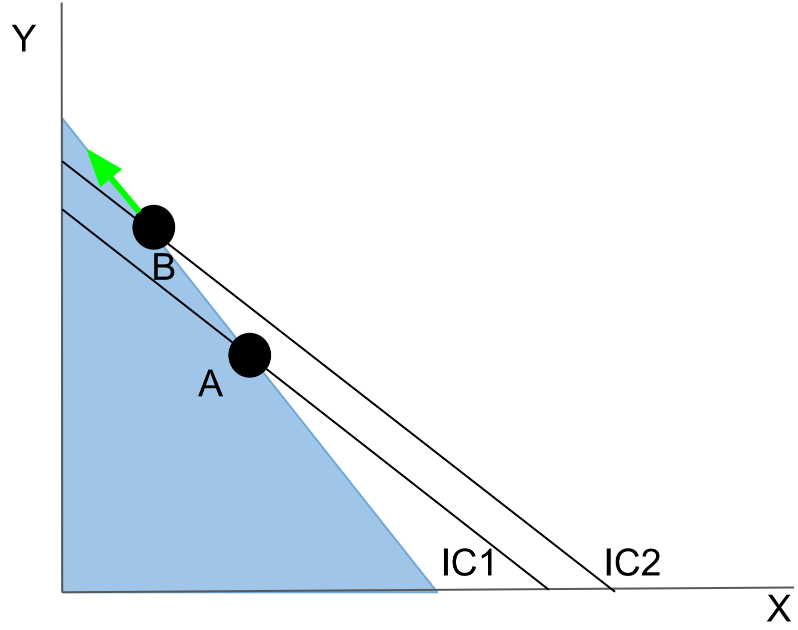


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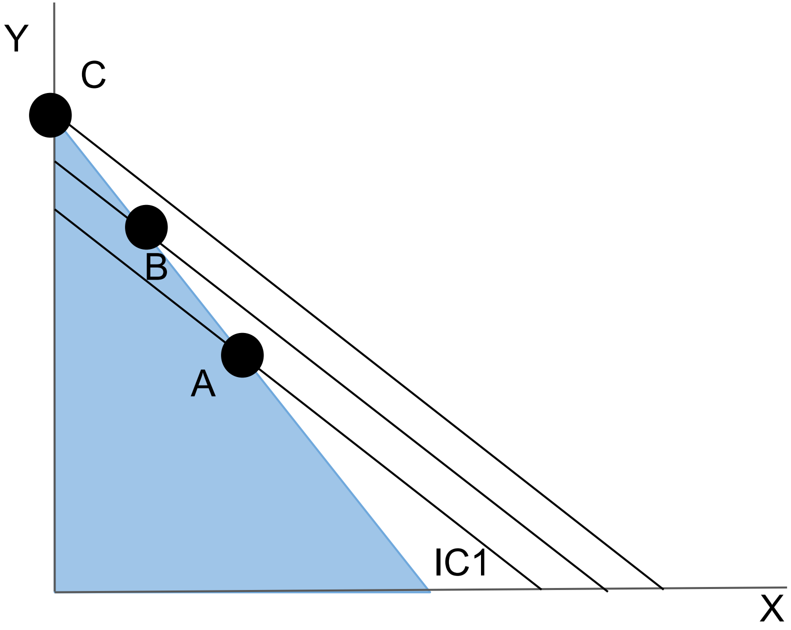


- Is bundle A optimal?
  - No
  - B is affordable and clearly better
- Is B optimal?
  - No
  - Can continue climbing up the budget line
- Where will it end?
- C is optimal
  - Consumer chooses X = 0
  - “Corner solution”

# Perfect complements

- Consider 2 goods, where each is useless without the other
  - E.g. Left shoes and right shoes
- This can be represented by a utility function like $U(X,Y) = \min\{X, Y\}$
  - If I have 1 right shoe and 6 left shoes, that's as good as having 1 right shoe and 1 left shoe
- What do ICs look like in this case?

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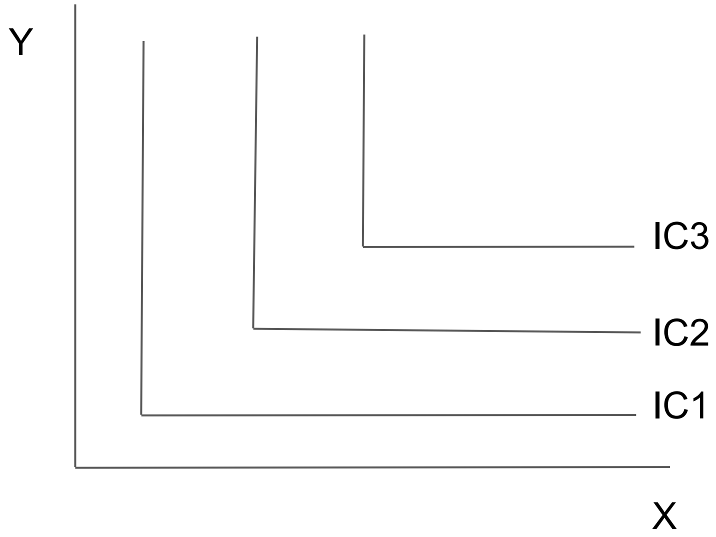

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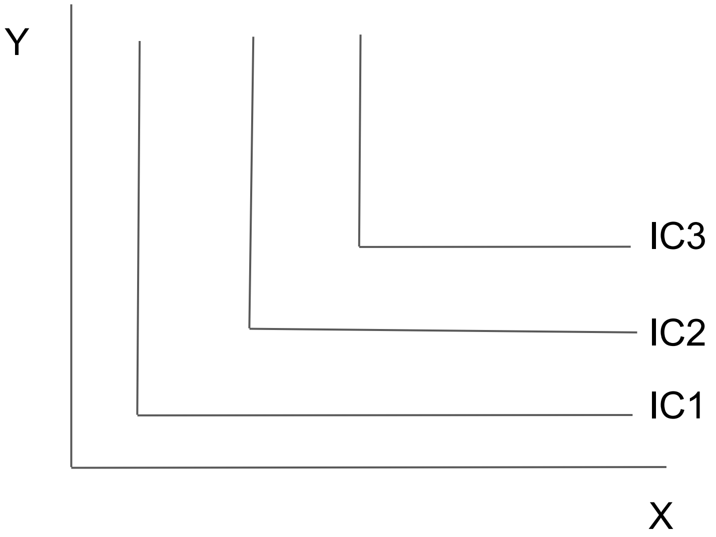
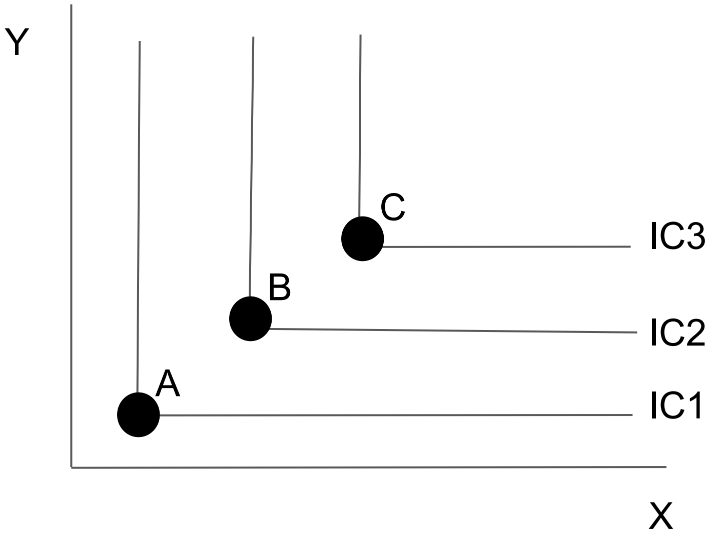


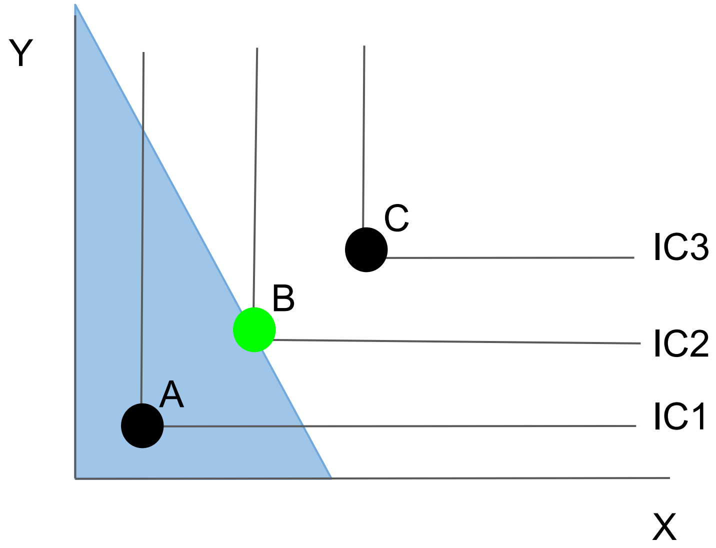
# Solution with perfect complements

- From visual inspection, clear we will choose a vertex
  - Being off-vertex involves pure waste



# Solution with perfect complements

- From visual inspection, clear we will choose a vertex
  - Being off-vertex involves pure waste
- Solution: the only vertex lying on the budget line



# Solution with perfect complements (2)

- With perfect complements, the consumer buys goods in fixed proportion
  - E.g. 1:1 for left shoes and right shoes
- In essence, it's really 1 (combined) good, with price equal to $p_X + p_Y$
- So the solution is to buy the following quantity of the combined good: $\frac{I}{p_X + p_Y}$
- From there, X and Y are immediate

# Mathematical Solution

# Introduction

- Graphical approach provides intuition for how optimizing consumers choose bundles
- With this intuition, we can now have a deep understanding of a mathematical approach to consumer optimization
- The following is a "cookbook" approach to solving the following problem:

$$Maximize\ U(X,Y)\ such\ that\ I = p_X X + p_Y Y$$

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# Step 1: Perfect complements?

Check whether the utility function describes perfect complements

If so, use "combined good" to solve the problem.

E.g. U(X,Y) = min{2X, Y}, $I = 100,\ p_X = 10,\ p_Y = 5$

Need 2X = Y, so 2Y for every X.

Combined good costs 20, so can buy 5 of them.

Solution is X = 5, Y = 10.

If not, proceed to step 2.

$$Maximize\ U(X,Y)\ such\ that\ I = p_X X + p_Y Y$$

# Step 2: Compute Marginal Rate of Substitution

Compute the MRS, the ratio of marginal utilities

Example 1: $U(X,Y) = X^{0.8}Y^{0.2}$

$$MU_X = 0.8X^{-0.2}Y^{0.2},\ MU_Y = 0.2X^{0.8}Y^{-0.8},\ MRS = \frac{0.8X^{-0.2}Y^{0.2}}{0.2X^{0.8}Y^{-0.8}} = 4\frac{Y}{X}$$

Example 2: $U(X,Y) = 8X + 2Y$

$$MU_X = 8,\ MU_Y = 2,\ MRS = 4$$

$$Maximize\ U(X,Y)\ such\ that\ I = p_X X + p_Y Y$$

# Step 3: Decreasing Marginal Rate of Substitution?

Check whether MRS is a decreasing function of X and/or an increasing function of Y

If not, it will be a corner solution. Find solution by comparing “all X” to “all Y”

E.g. Last slide showed that $U(X,Y) = 8X + 2Y$ $MRS = 4$

Suppose $I = 100,\ p_X = 10,\ p_Y = 5$

Spending on only X yields utility of 80, spending on only Y gives 40.

Solution is only X: X=10, Y=0.

If so, proceed to step 4

$$Maximize\ U(X,Y)\ such\ that\ I = p_X X + p_Y Y$$

# Step 4: Set MRS equal to price ratio and simplify

With decreasing MRS, solution has MRS equal to price ratio. Solving that equation will give us a relationship between optimal X and optimal Y.

E.g. Earlier saw that $U(X,Y) = X^{0.8}Y^{0.2}$ $\quad MRS = 4\frac{Y}{X}$

Suppose $I = 100,\ p_X = 10,\ p_Y = 5$ $\quad \frac{p_X}{p_Y} = 2$

Setting MRS equal to price ratio and solving yields:

$$4\frac{Y}{X} = 2$$

$$2Y = X$$

$$Maximize\ U(X,Y)\ such\ that\ I = p_X X + p_Y Y$$

# Step 5: Plug in to budget constraint and solve

Step 4 gave us a relationship between X and Y with no concern for budget. Process concludes by plugging into budget constraint.

E.g. With budget constraint **100 = 10X + 5Y** and optimal ratio **2Y=X**, we get:

$$100 = 20Y + 5Y$$

$$Y = 4$$

$$X = 8$$

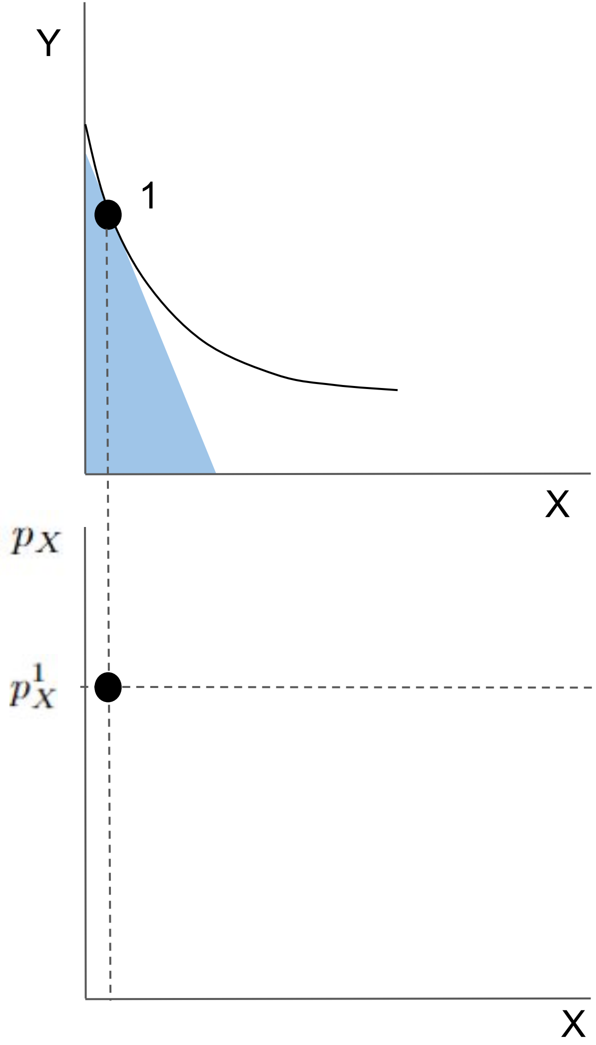
And we're done!

# Building a Demand Curve

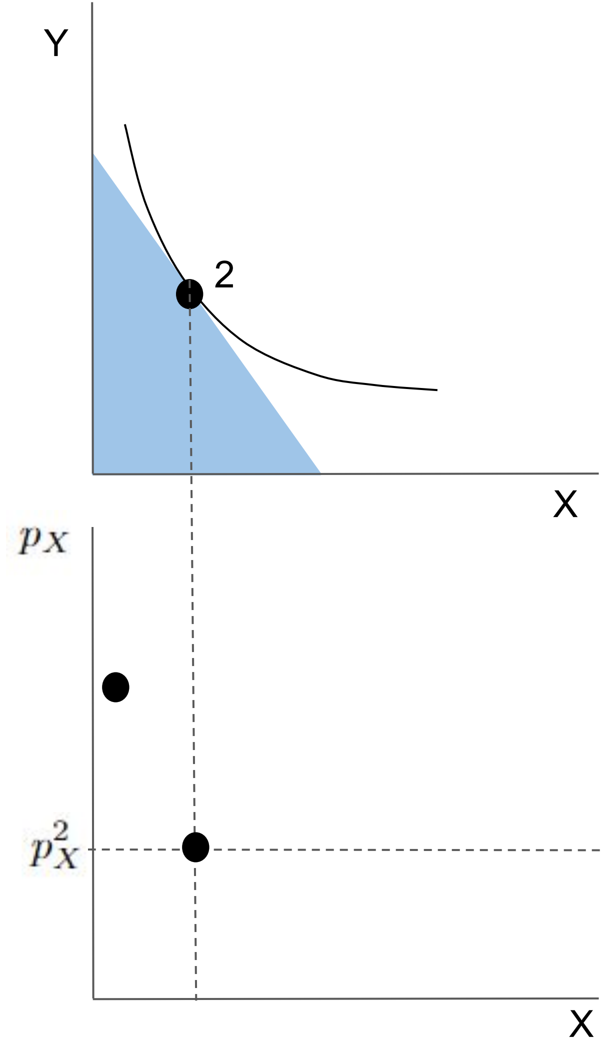
# Introduction

- We motivated consumer choice theory as the underpinning for a demand curve
- Can now look at how changes in price and income affect quantity demanded
- We now derive the demand curve, which shows how much of a good an individual will choose for each proposed price.

- Let's start with $I^1, p_X^1, p_Y^1$ and say the consumer chooses point 1 in "2-good space"
    - We can then plot that point in "price-quantity space," where the demand curve will live



- Let's start with $I^1, p_X^1, p_Y^1$ and say the consumer chooses point 1 in "2-good space"
  - We can then plot that point in "price-quantity space," where the demand curve will live
- If we lower the price to $p_X^2$, we can find the new tangency point and then plot it along the demand curve



- Let's start with $I^1,\ p_X^1,\ p_Y^1$ and say the consumer chooses point 1 in "2-good space"
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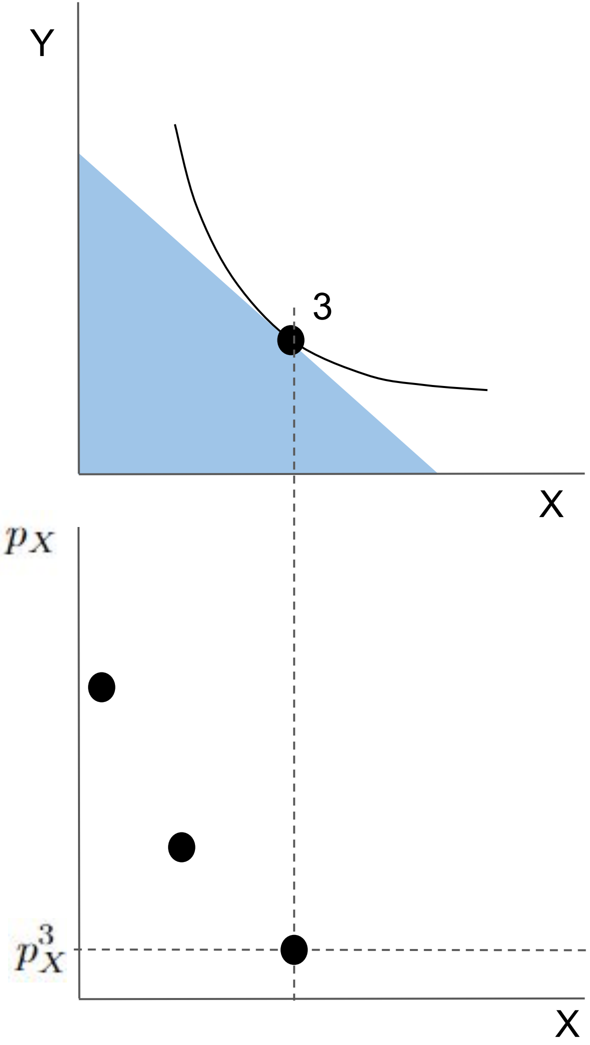

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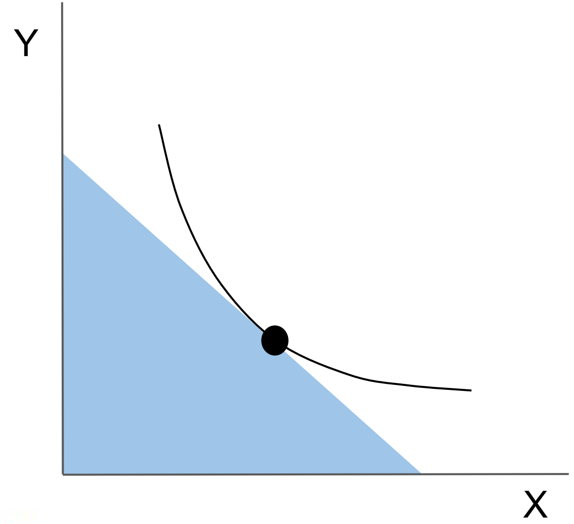


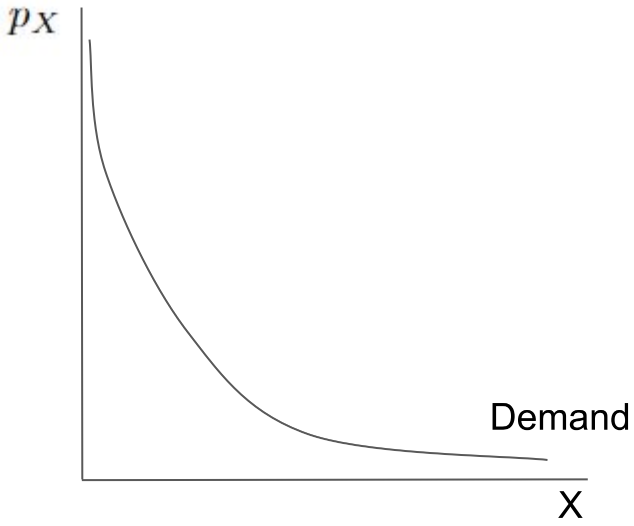

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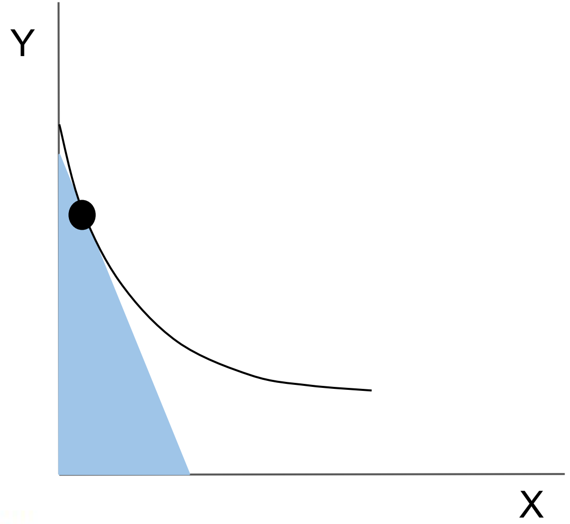


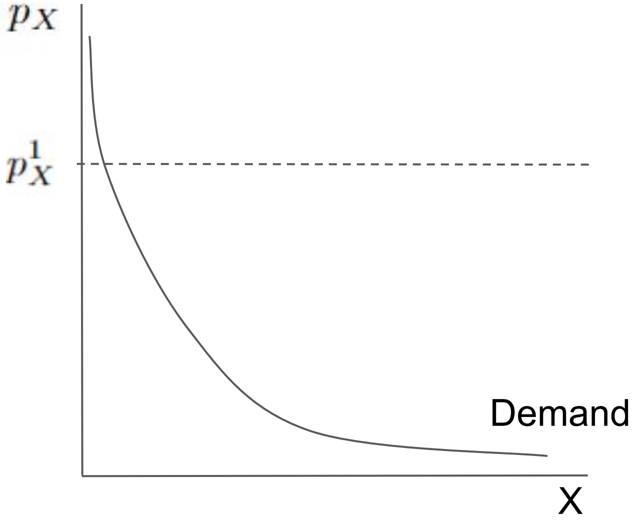

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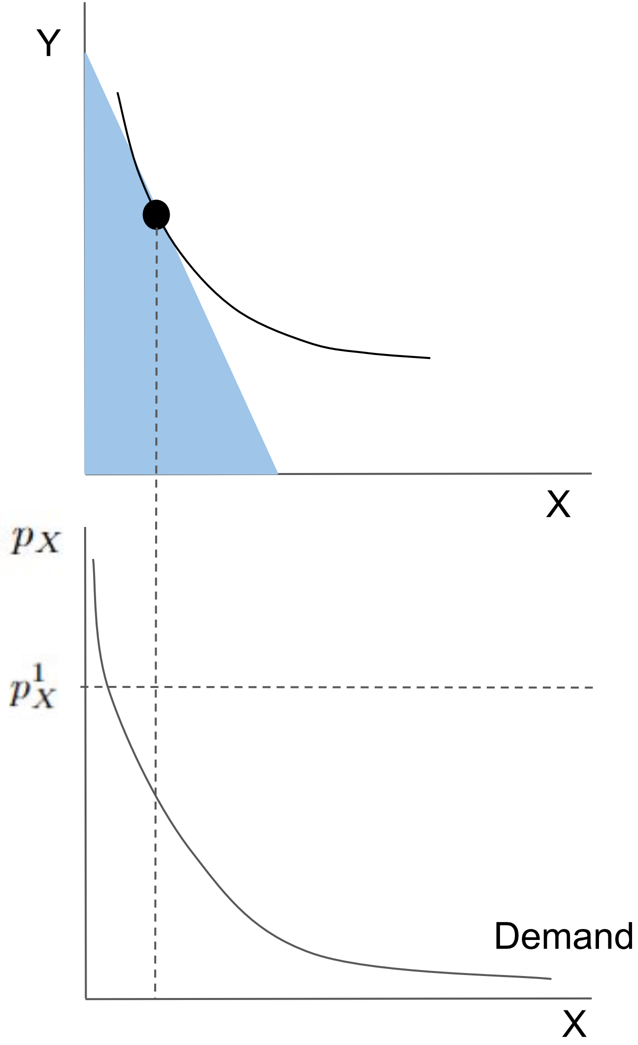

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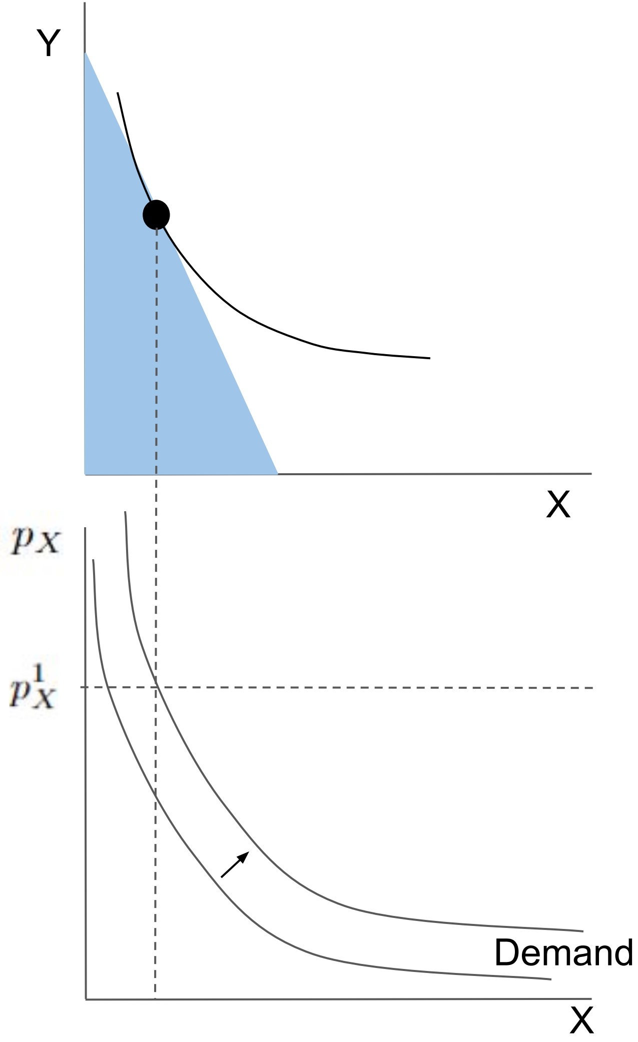

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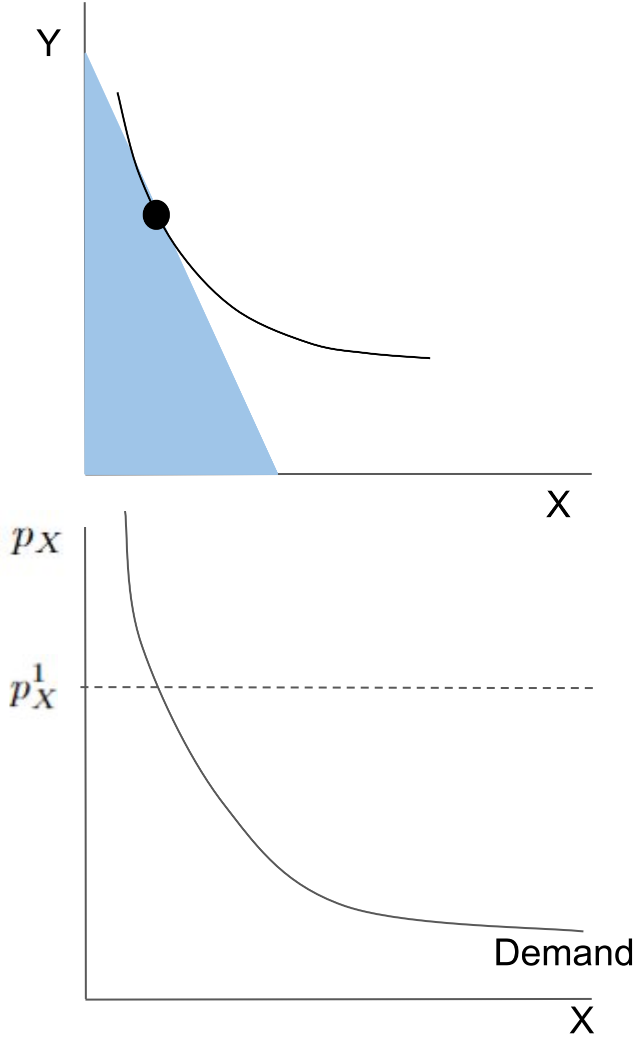

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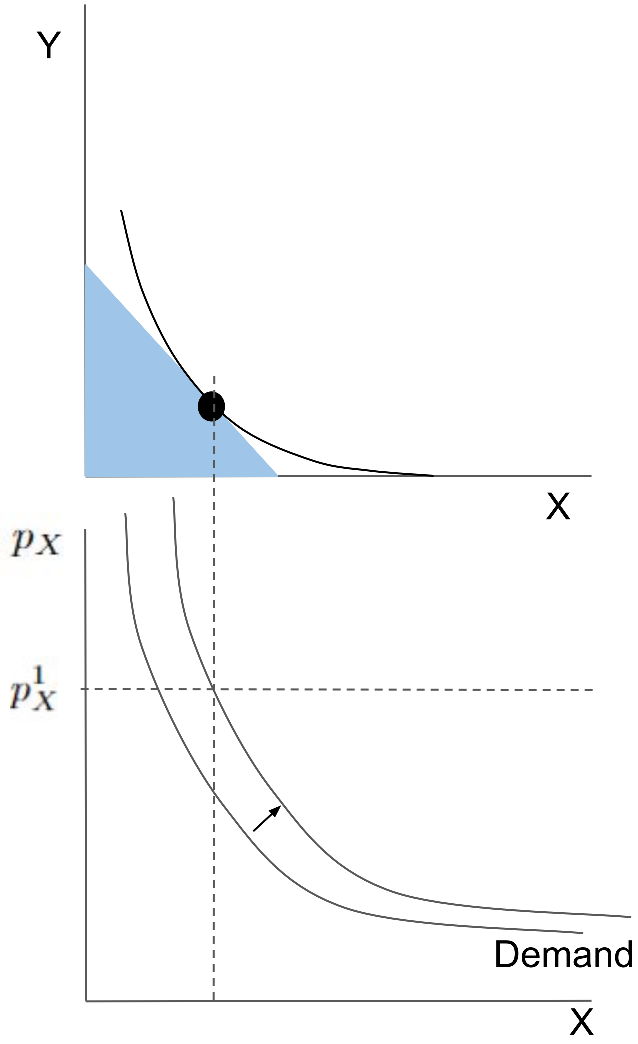
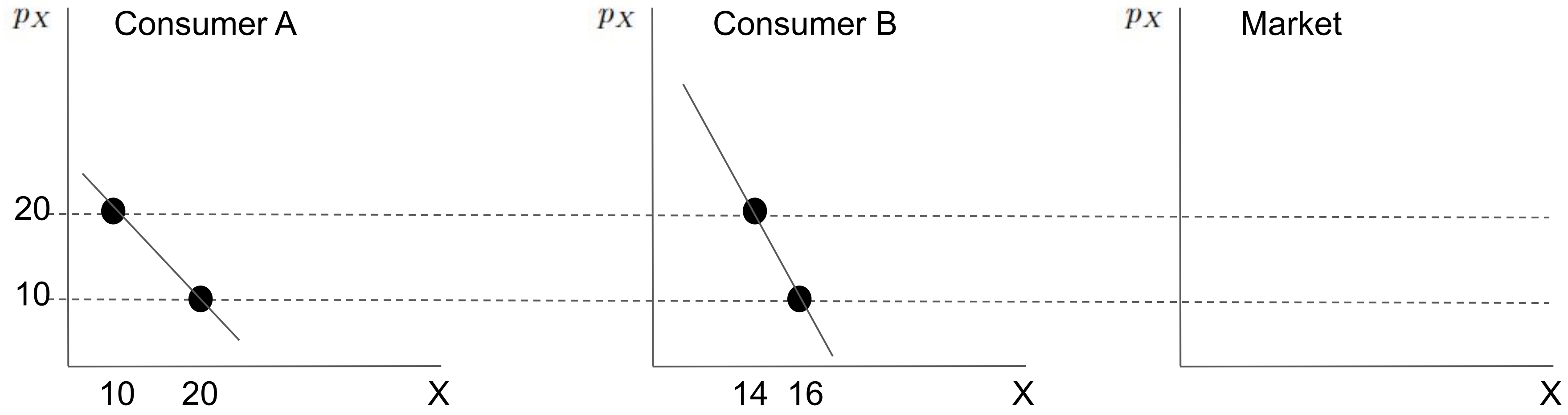


# From individual to market demand

- We just saw how consumer theory produces an individual's demand curve from the individual preferences.
- Market demand is simply the "horizontal sum" of the individual demand curves.

# Numerical example

- Consider a market with two consumers
  a. At price 10, consumer A would demand 20 units of the good. At price 20, he would demand 10
  b. At price 10, consumer B would demand 16 units of the good. At price 20, he would demand 14
- What is the market demand?



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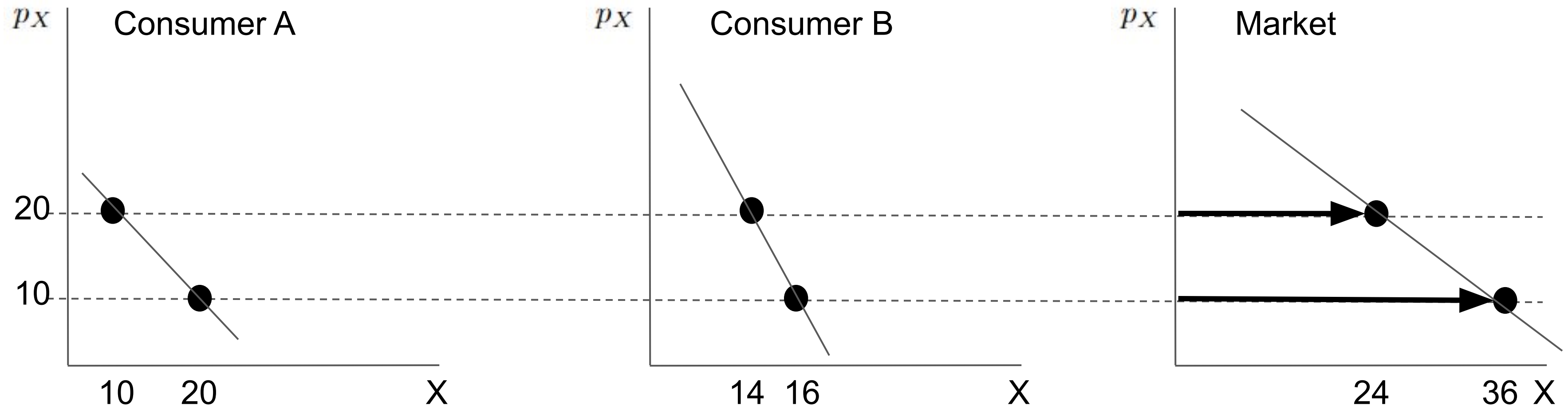


# Endowments

# Introduction

- So far, we've assumed that the consumer is endowed with some income that can then be used to go buy the goods.
- We can also imagine that the consumer is endowed with goods. She can then go to the market and change her quantities of the goods, based on market prices.
- The budget constraint then changes from

$$I \geq p_X X + p_Y Y$$

to

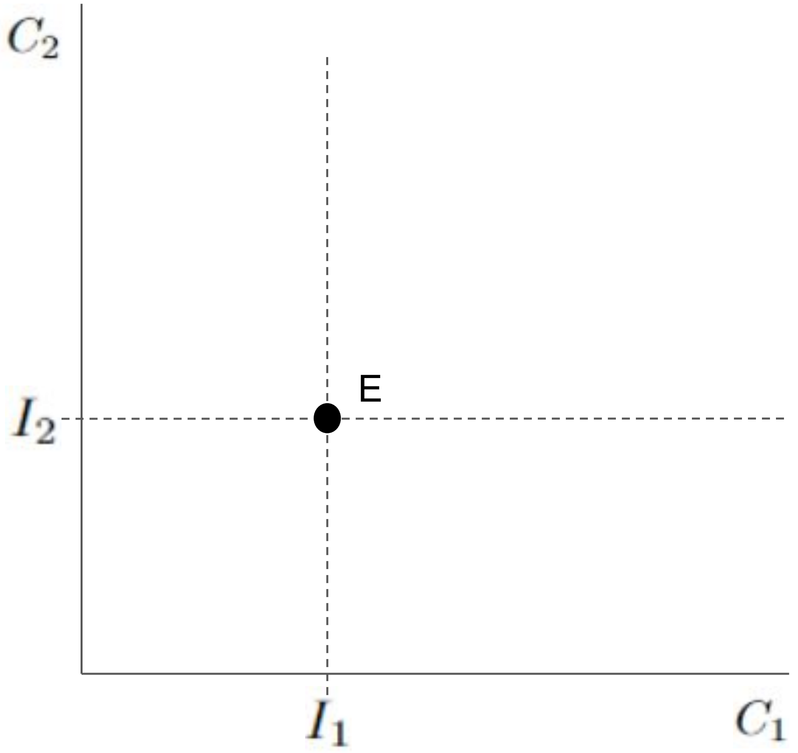
$$p_X X^e + p_Y Y^e \geq p_X X + p_Y Y$$

# "Intertemporal choice"

- A very important application of this approach is the decision to save.
- Consider a consumer who will consume in 2 different periods: 1, 2
- In each period, she will earn income: $I_1,\ I_2$
- In each period, she must choose consumption: $C_1,\ C_2$
- The consumer could just consumer her income each period: $C_1 = I_1,\ C_2 = I_2$
- But let's model the option to borrow or save
  - Assume an "interest rate" of r% (e.g. 5%)
  - If she gives up 1 unit of consumption in period 1, she can have 1.05 units of consumption in period 2
  - What will the budget set look like?

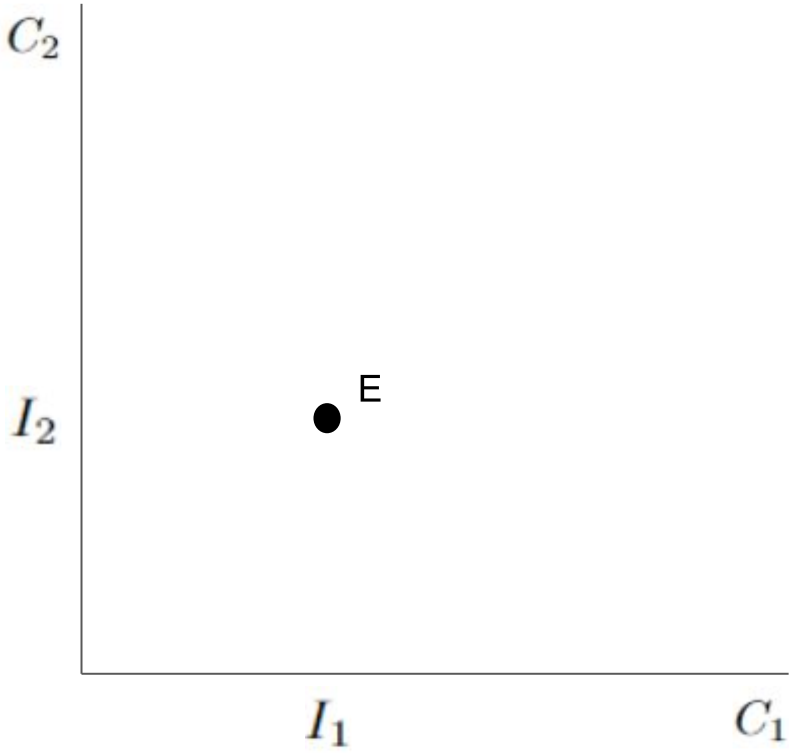
# Visual representation of intertemporal budget set

- Endowment is always affordable



# Visual representation of intertemporal budget set

- Endowment is always affordable
- So the budget line goes through E, but what is its slope?



# Visual representation of intertemporal budget set

- Endowment is always affordable
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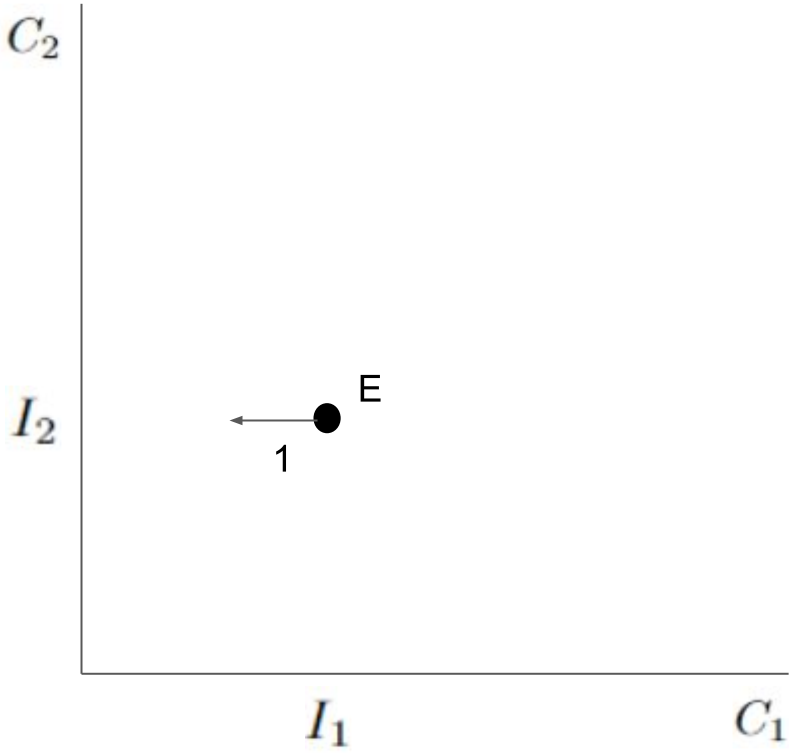

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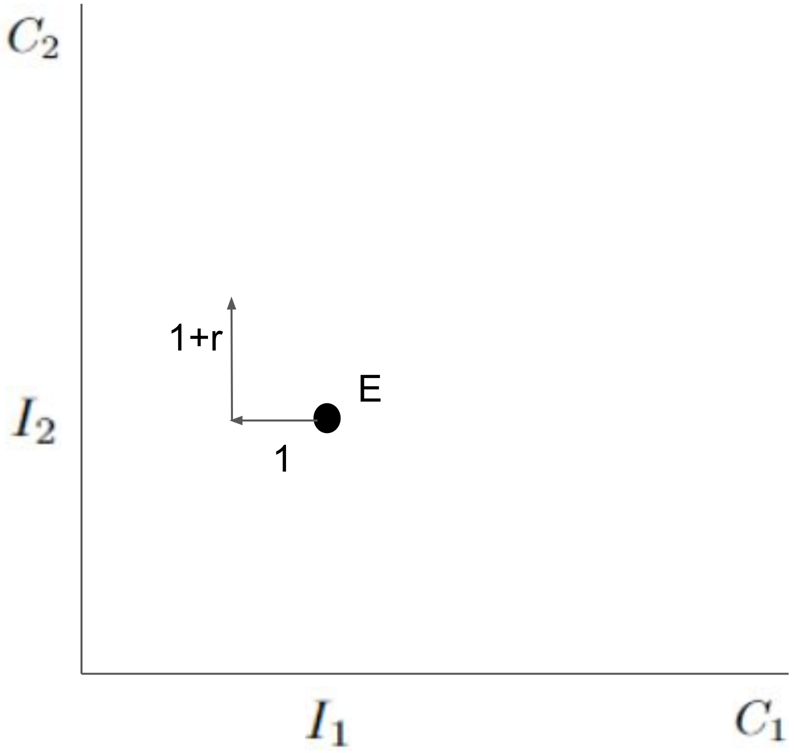
  - 1+r



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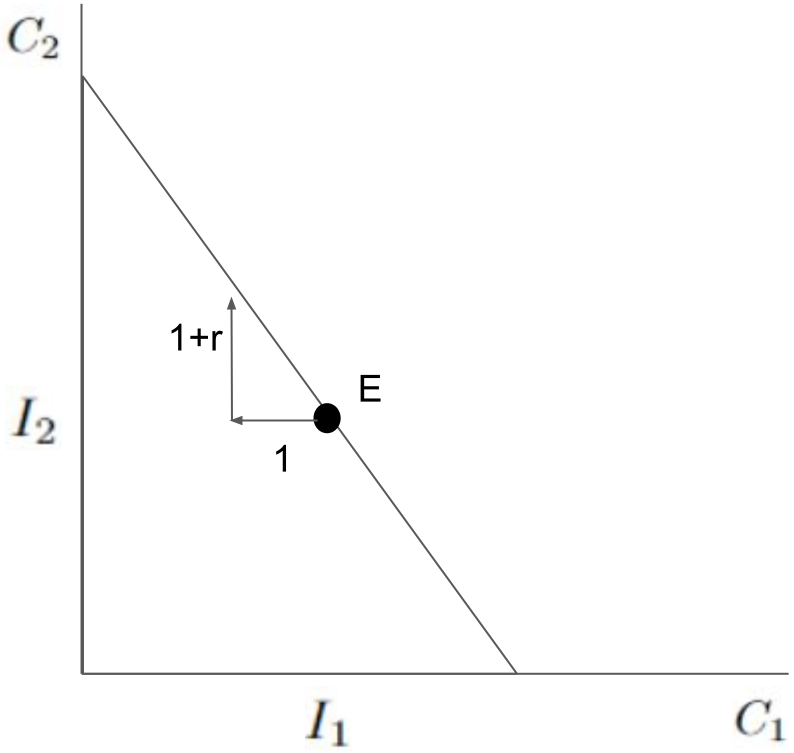

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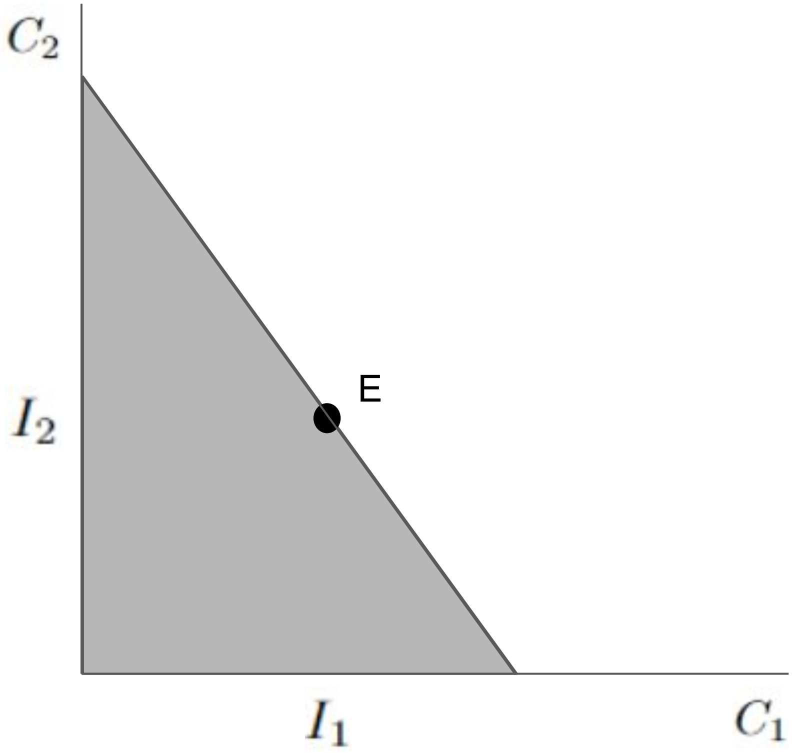
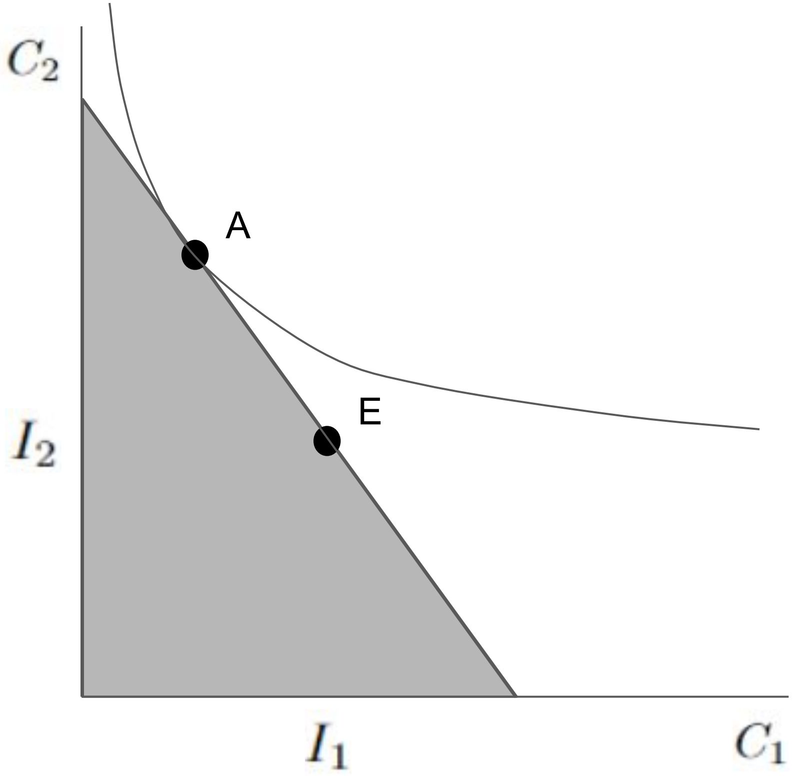


# Borrowing and saving

- Suppose she chooses point A. Is she a borrower or saver?



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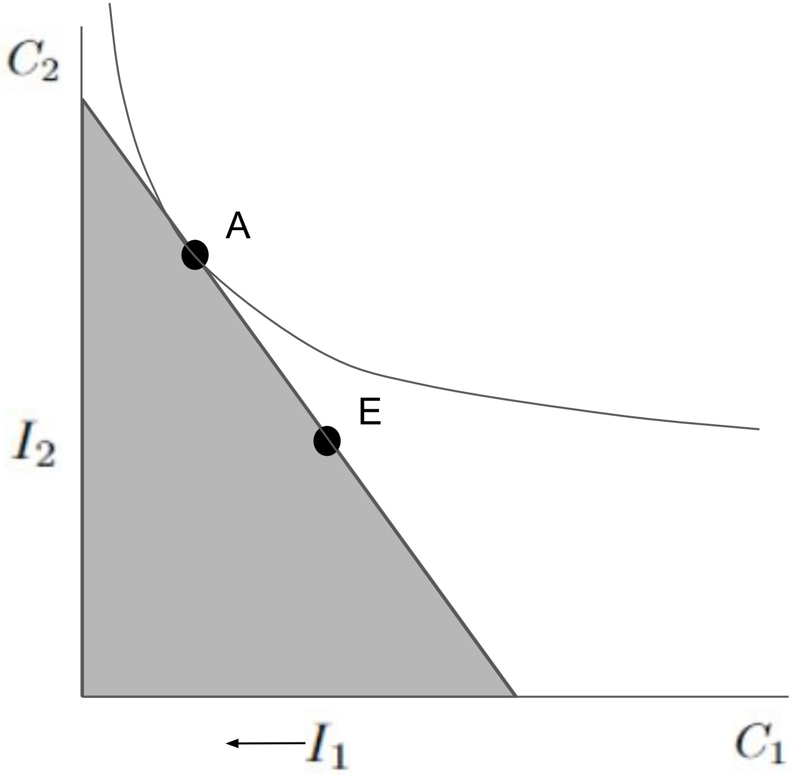

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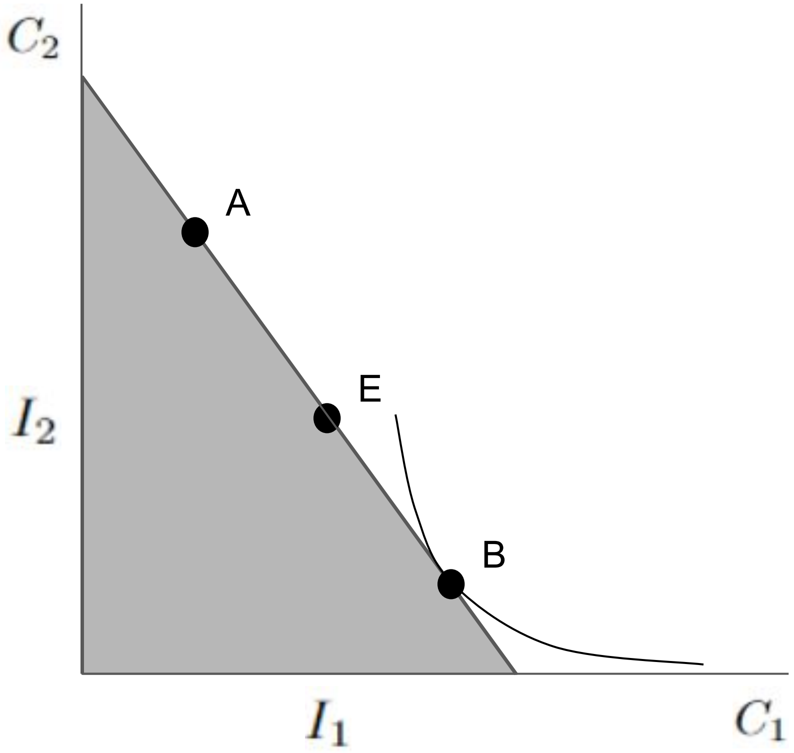

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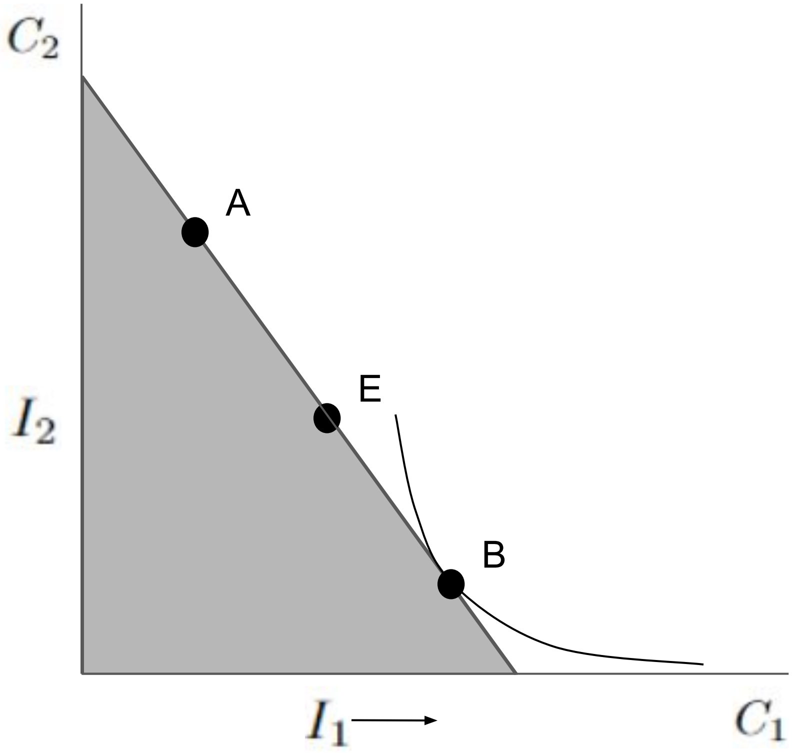
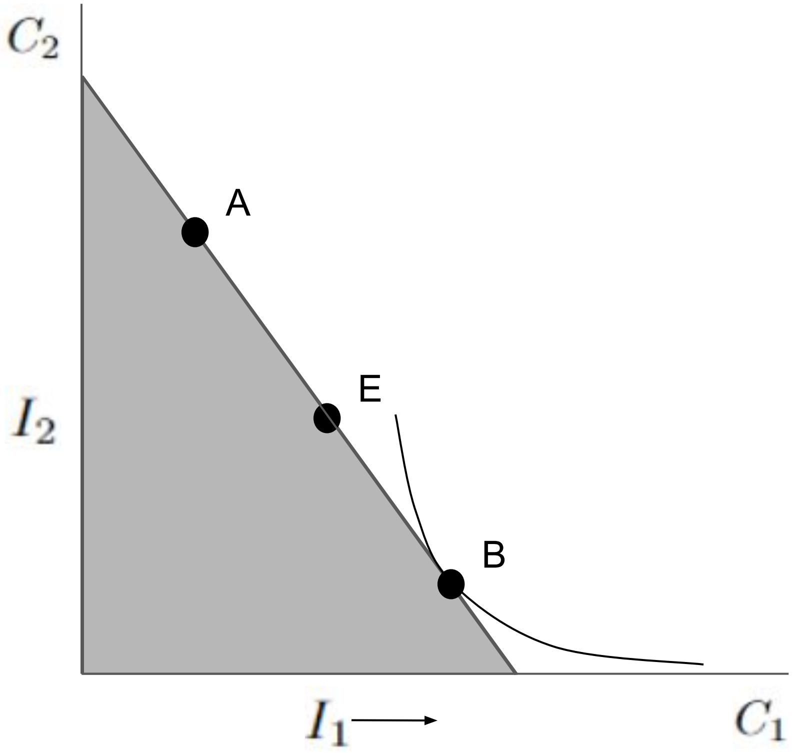


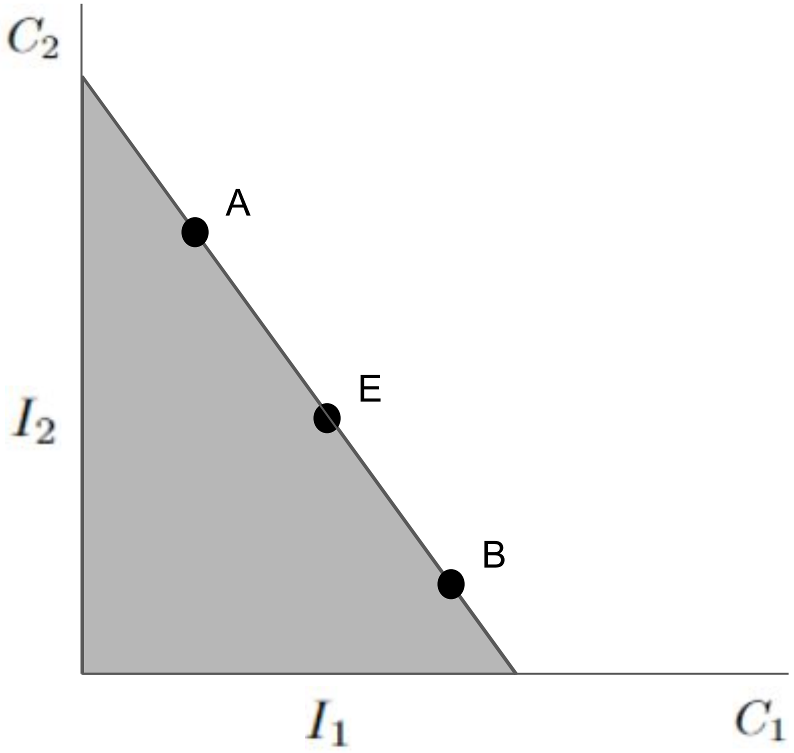
# Borrowing and saving

- Suppose she chooses point A. Is she a borrower or saver?
  - Saver
- What if she chooses B?
  - Borrower
- In either case, set MRS = 1+r



# Change in interest rate

- Now suppose the interest rate decreases. How will the budget set change?



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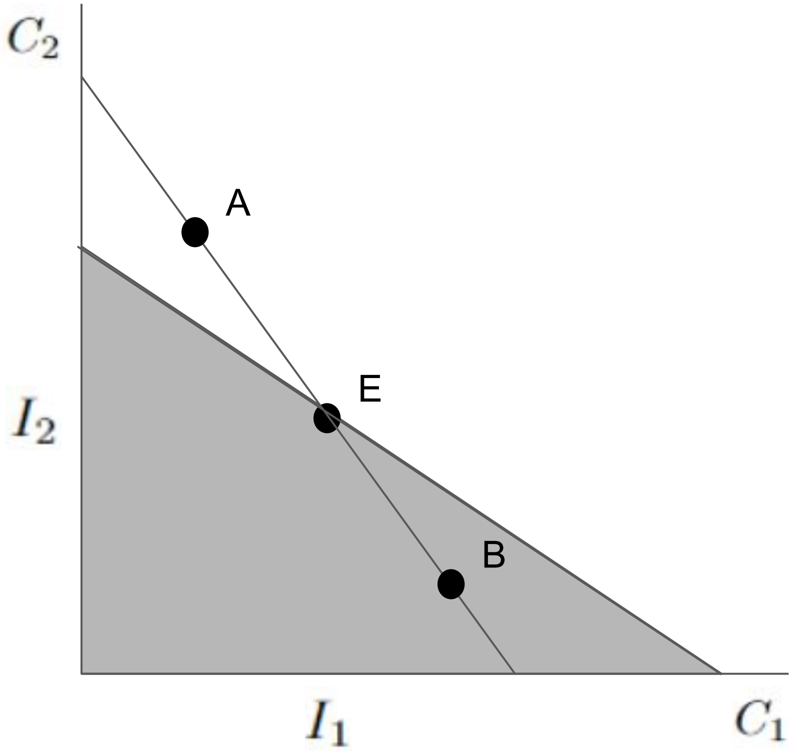

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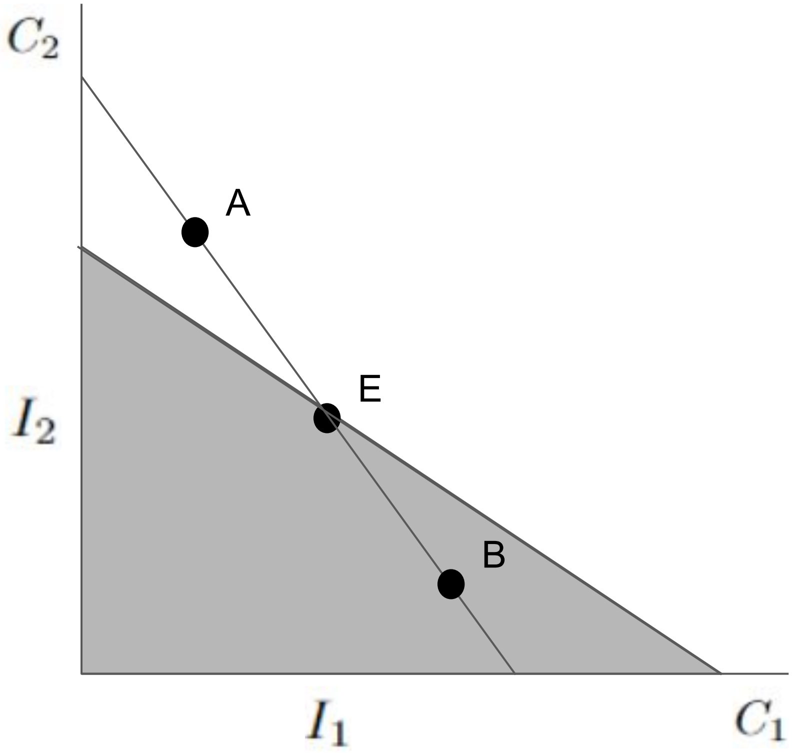

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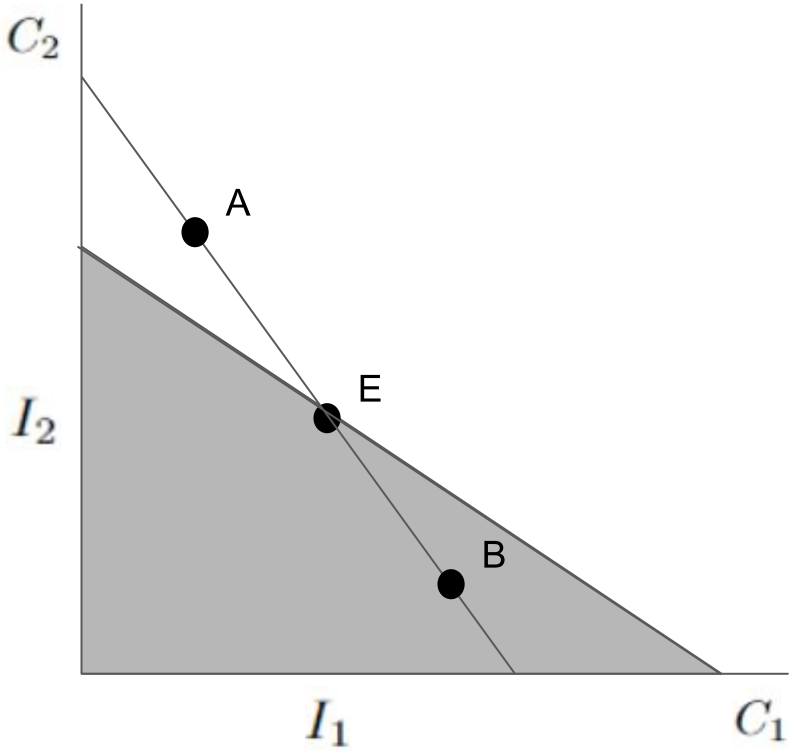

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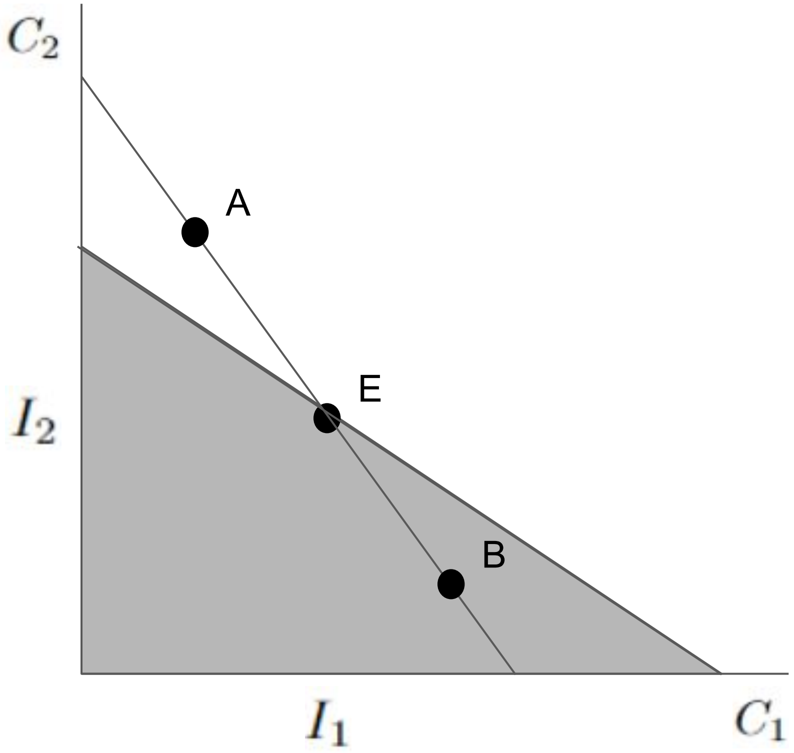

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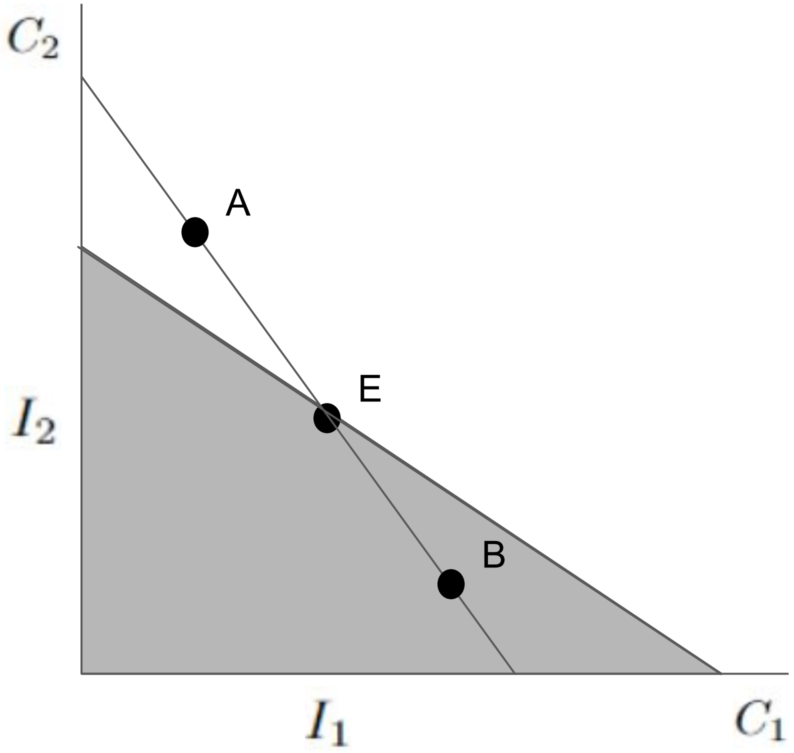
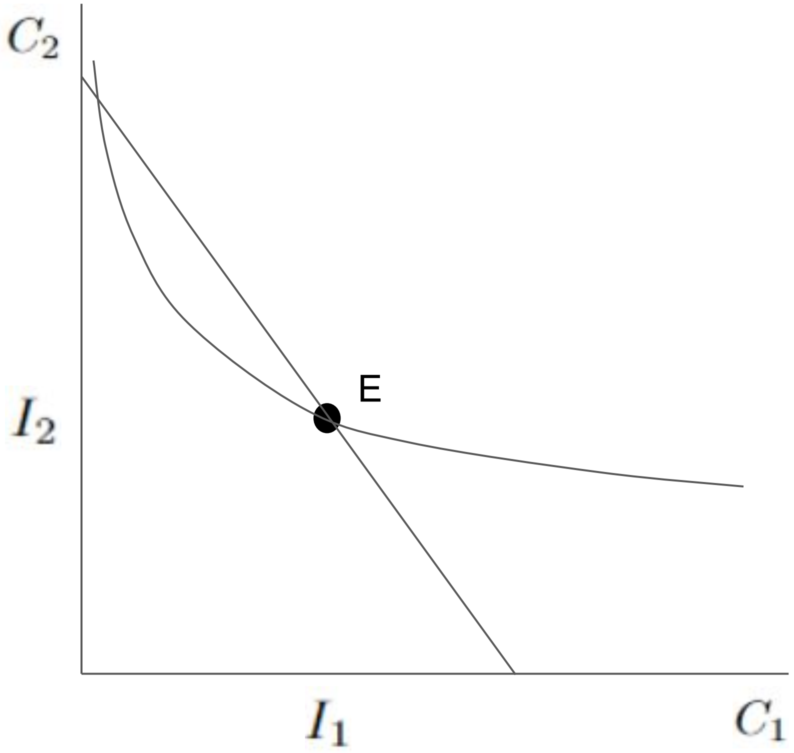


# Borrowing and saving

- This person has MRS < 1+r at E
- Should she borrow or save?



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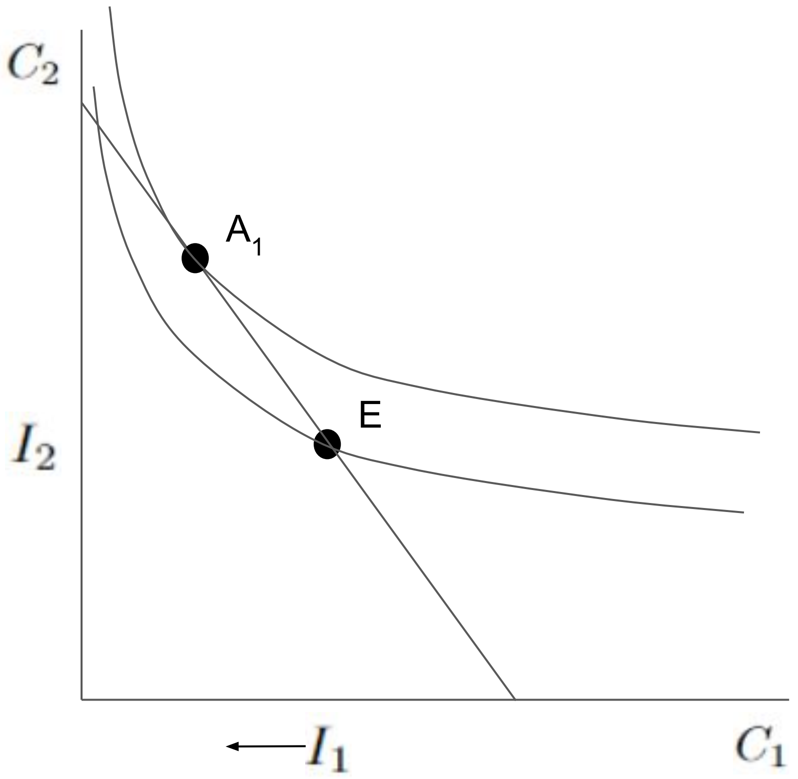

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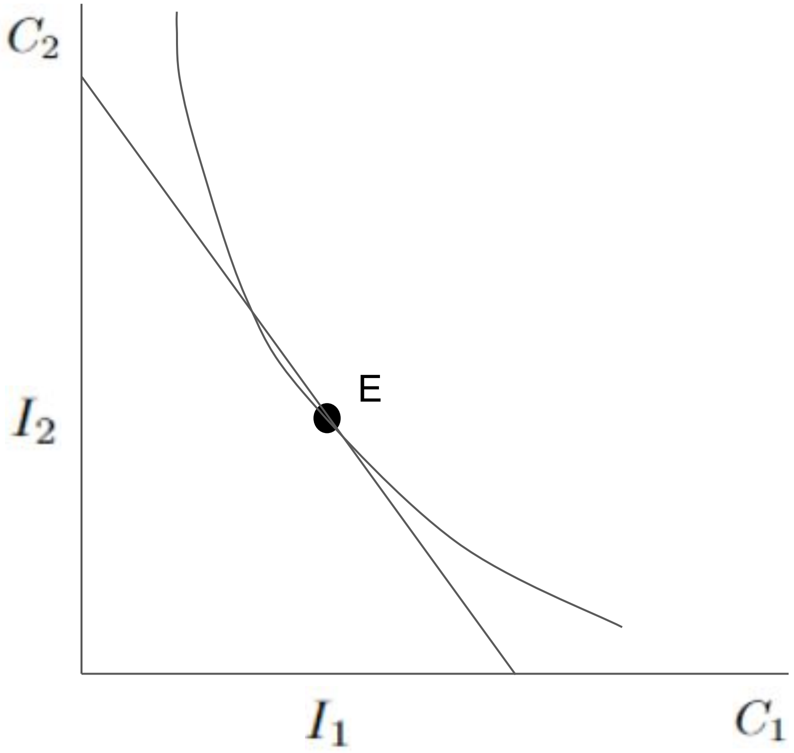

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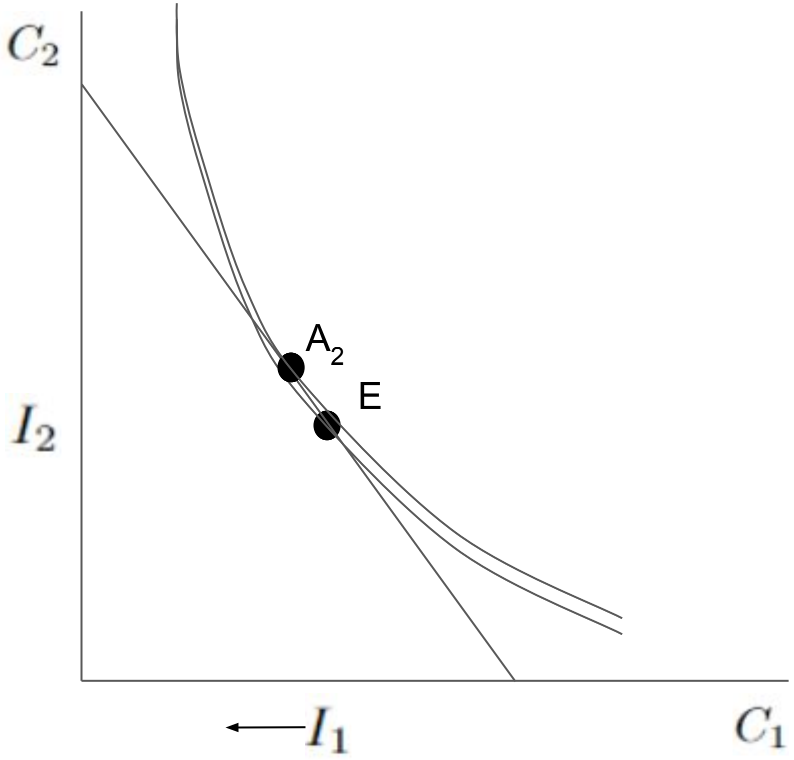

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- This person has MRS > 1+r at E
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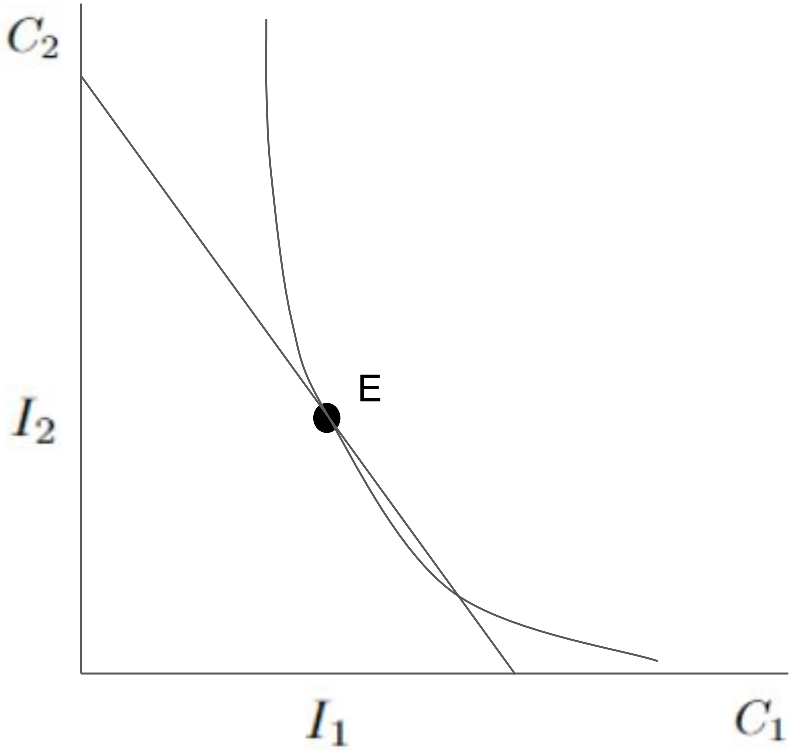
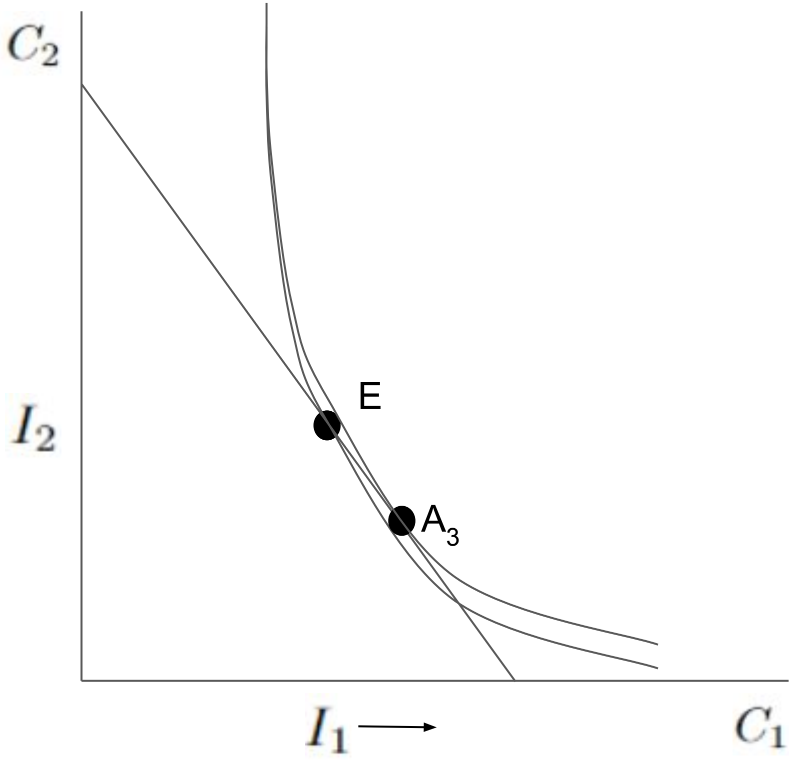


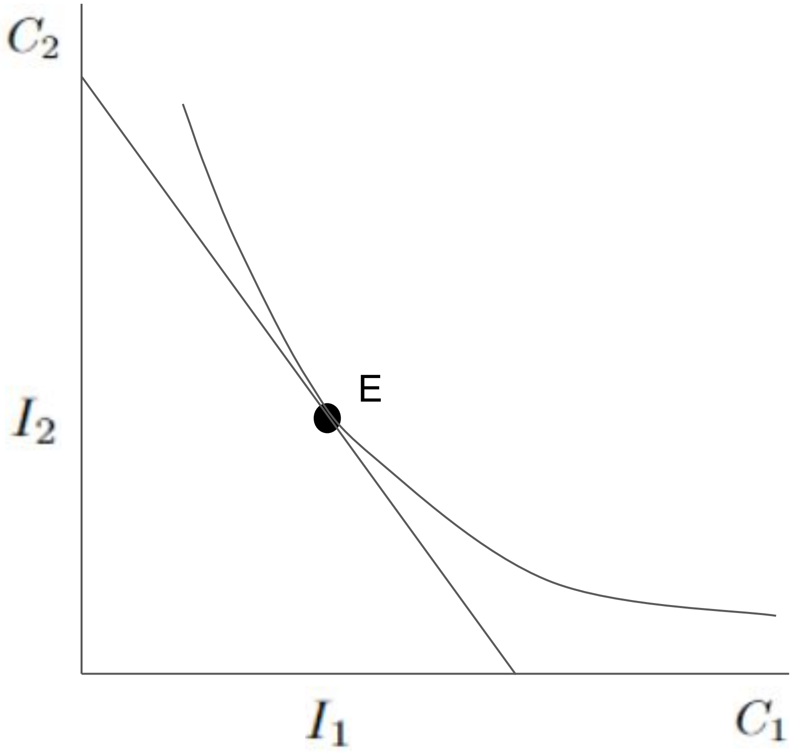
# Borrowing and saving

- This person has MRS > 1+r at E
- Should she borrow or save?
    - Borrow: values future consumption relatively little



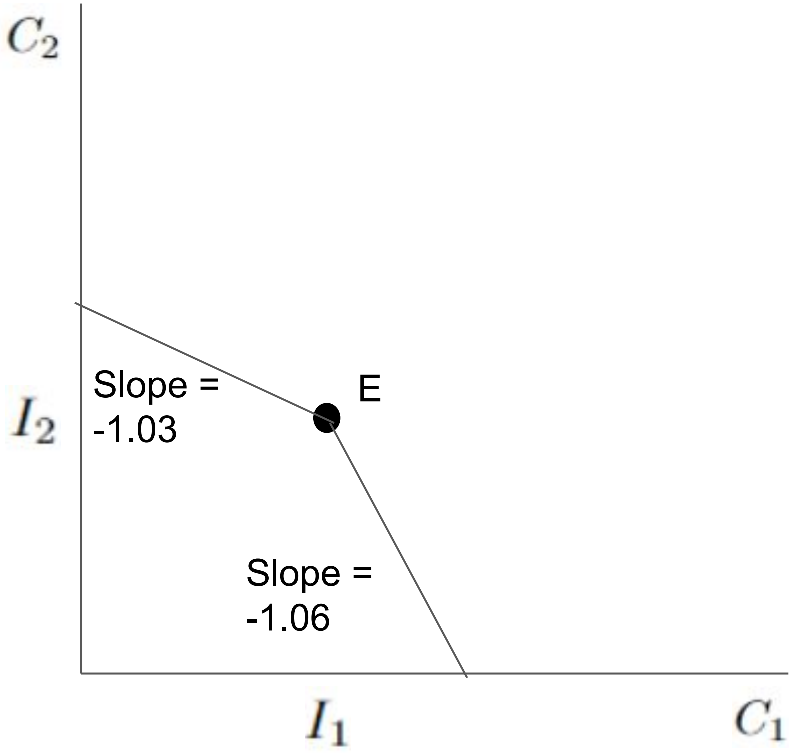
# Borrowing and saving

- Only someone who just somehow happens to have MRS = 1+r at E will neither borrow nor save.
  - I.e. We would expect almost everyone to either have meaningful savings or debt
- But many people do not…



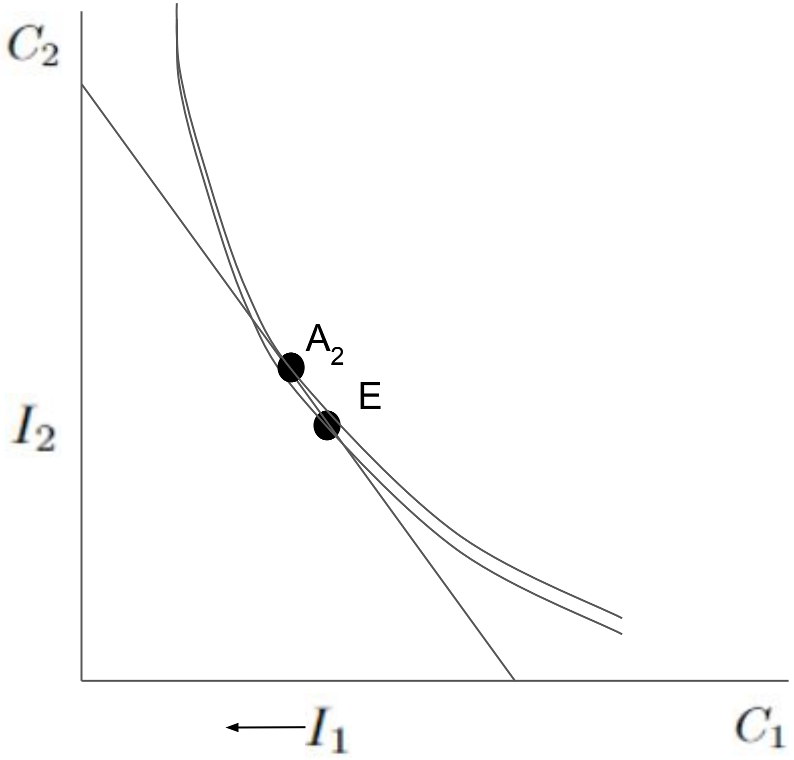
# Kinked budget set

- If interest rate is higher for borrowing than saving, budget line has a concave kink
  - E.g. pay 6% on credit card, earn 3% on savings account



# Kinked budget set

- Consider the person who wanted to save a little bit with a flat budget line…



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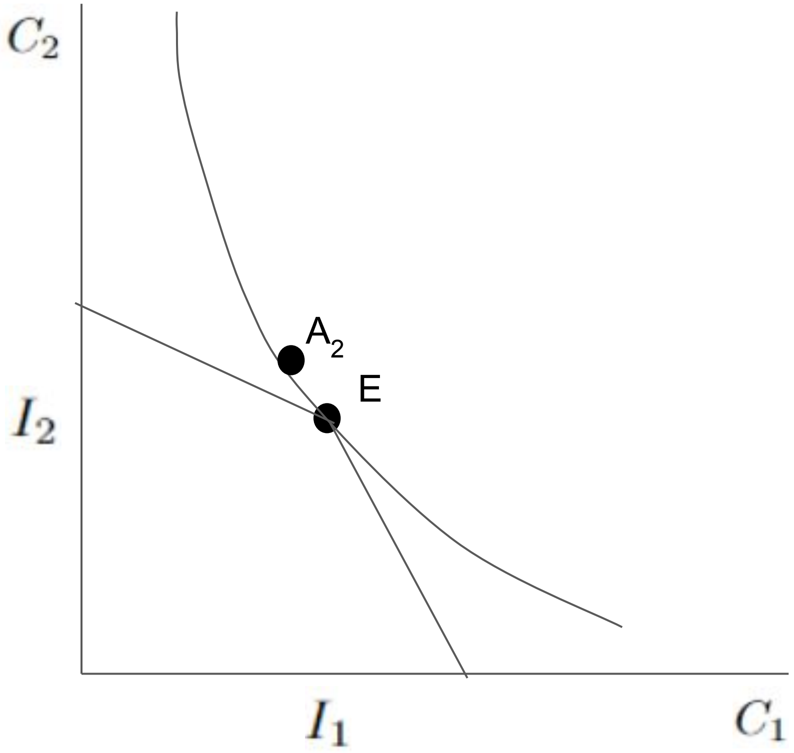
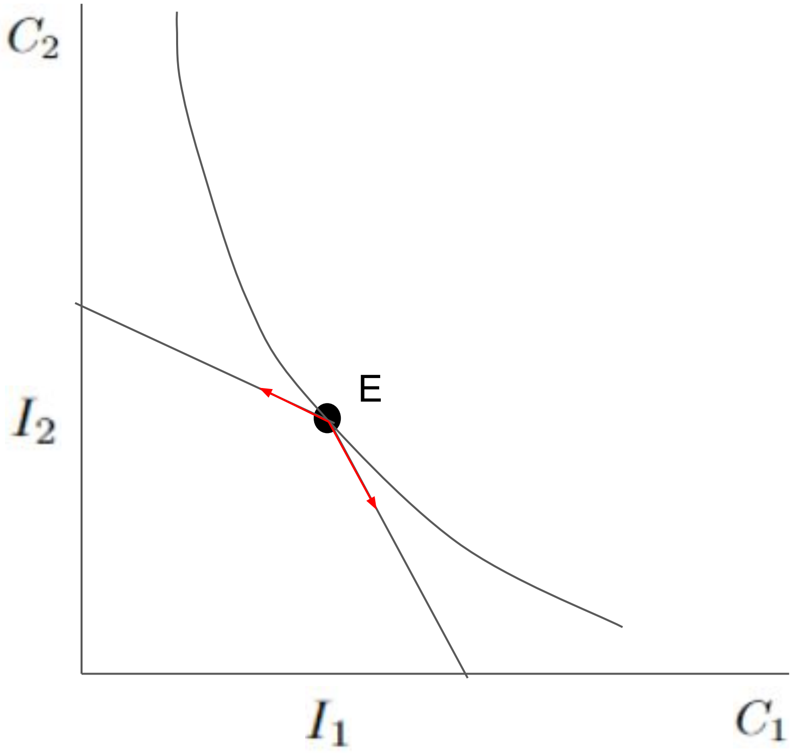
- Consider the person who wanted to save a little bit with a flat budget line…
- With a kink, $A_2$ is no longer affordable



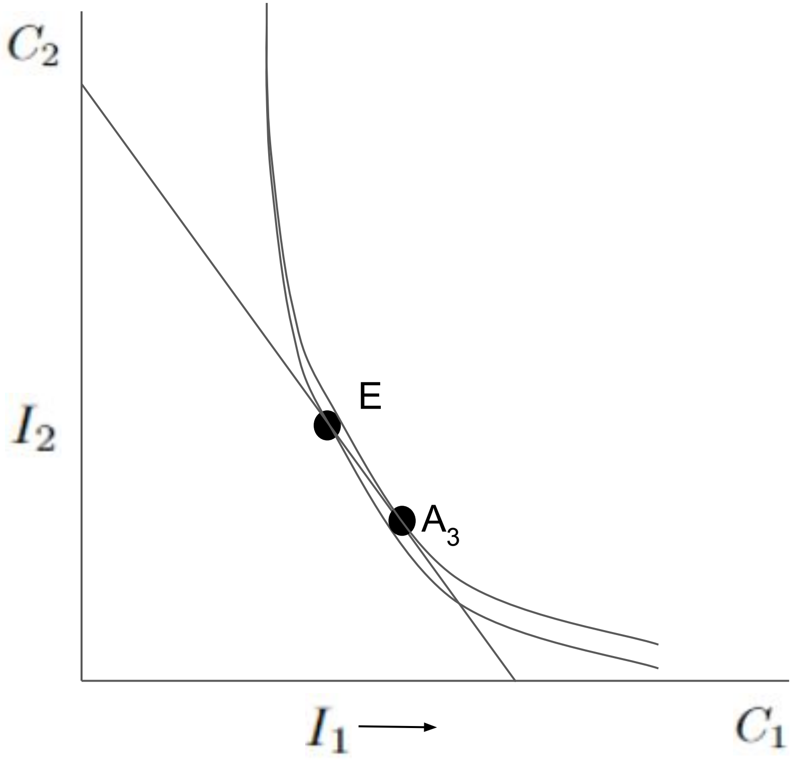
# Kinked budget set

- Consider the person who wanted to save a little bit with a flat budget line…
- With a kink, $A_2$ is no longer affordable
- And all affordable bundles will put her on a lower IC
- This person will neither borrow nor save – stick at endowment
    - “You don’t pay me enough to save”
    - “You charge too much to borrow”



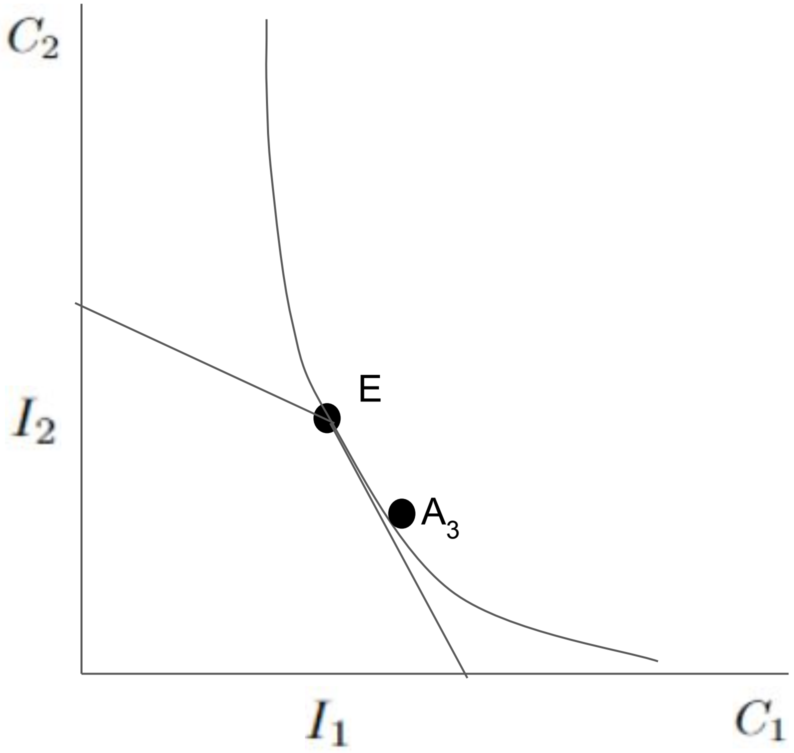
# Kinked budget set

- Consider the person who wanted to borrow a little bit with a flat budget line…



# Kinked budget set

- Consider the person who wanted to borrow a little bit with a flat budget line…
- With a kink, $A_3$ is no longer affordable



# Kinked budget set

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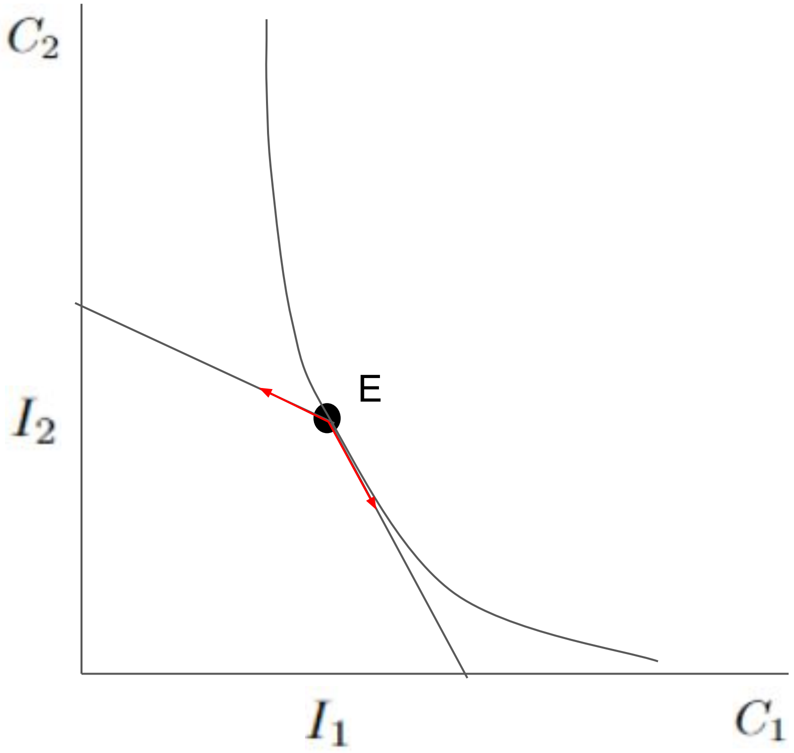

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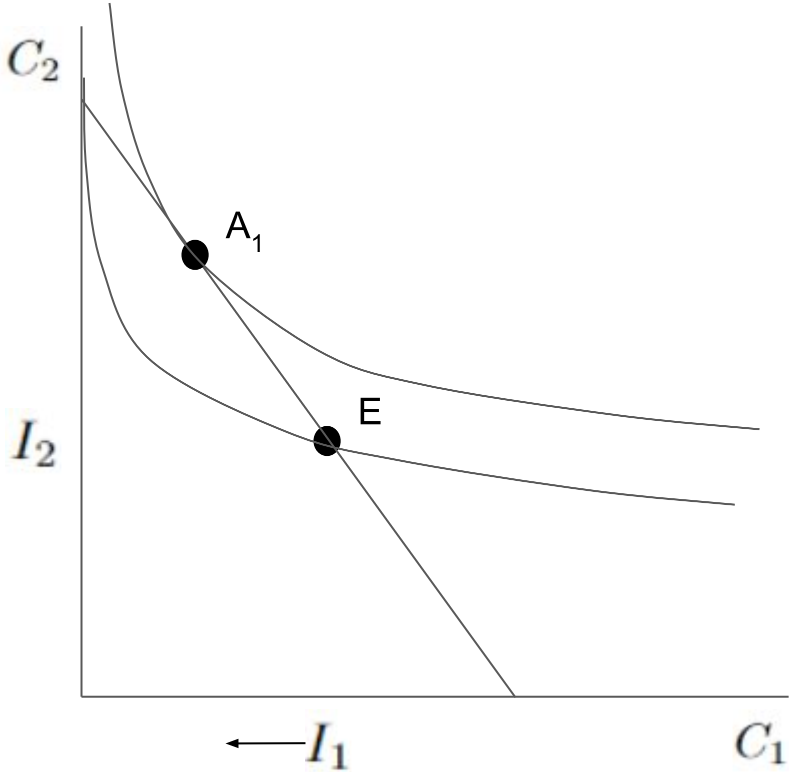
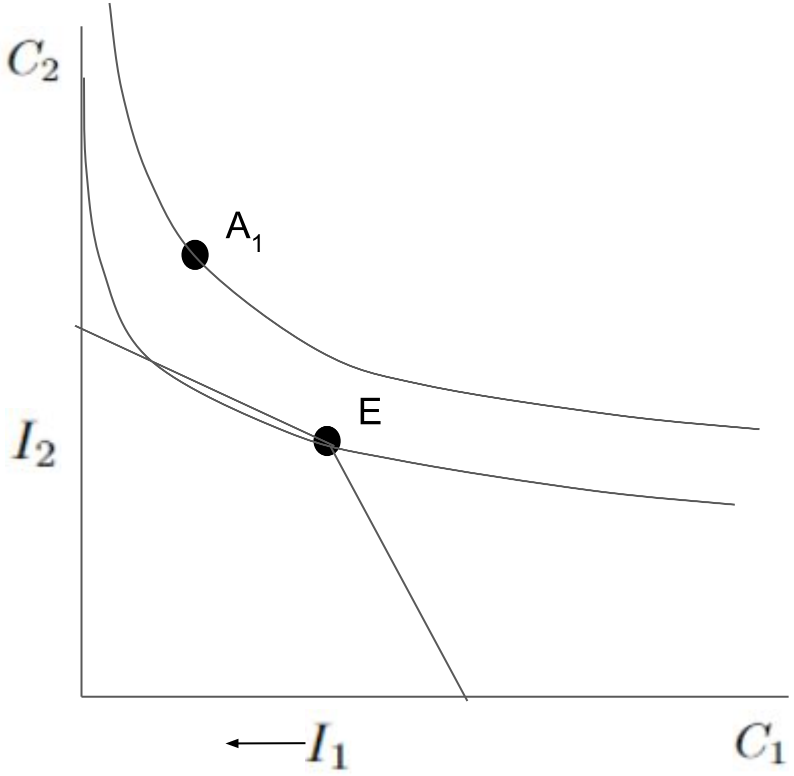


# Kinked budget set

- Consider the person who wanted really wanted to save with a flat budget line…
- What should she do with a kinked budget set?



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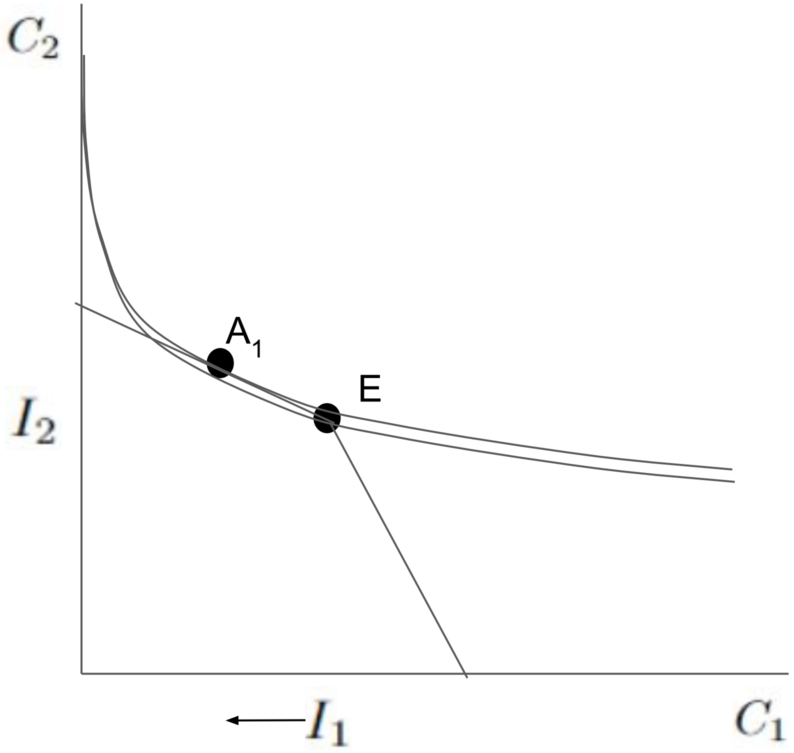
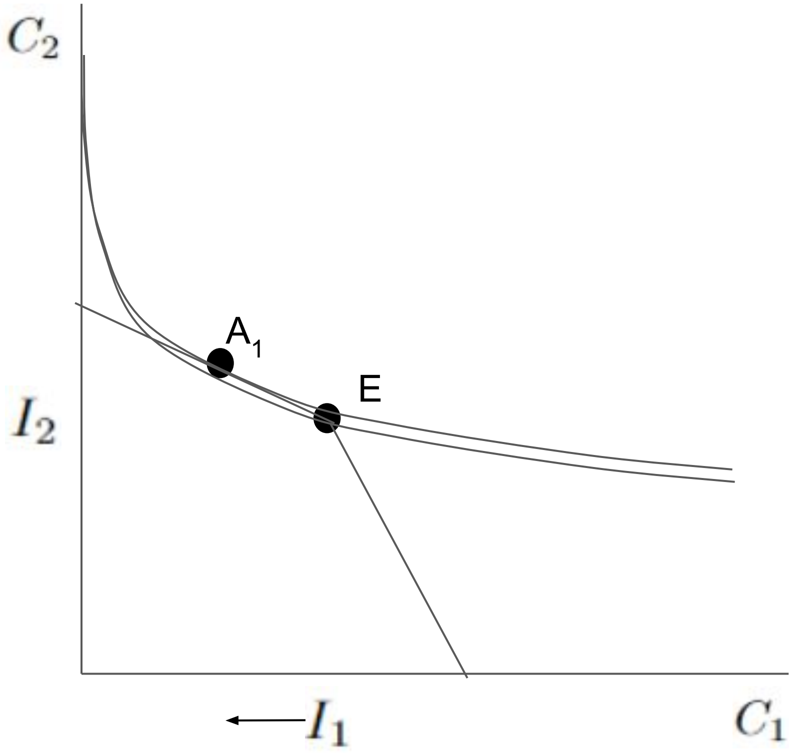
- Consider the person who wanted really wanted to save with a flat budget line…
- What should she do with a kinked budget set?
  - She'll still be a saver



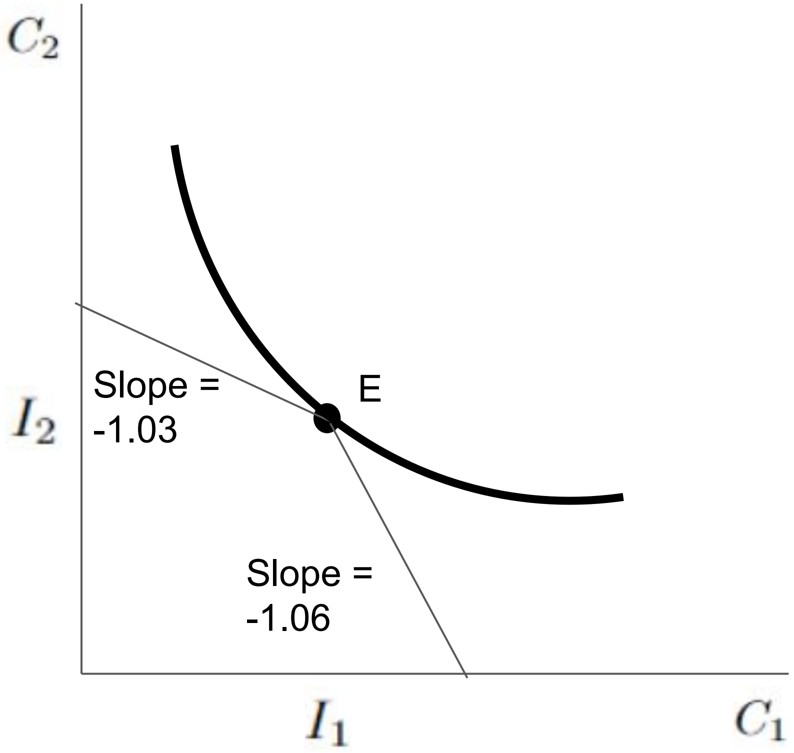
# Kinked budget set

- Consider the person who wanted really wanted to save with a flat budget line…
- What should she do with a kinked budget set?
  - She’ll still be a saver
- Takeaway: kink in the budget set makes model more successful
  - Some people will borrow or save, but many will do neither



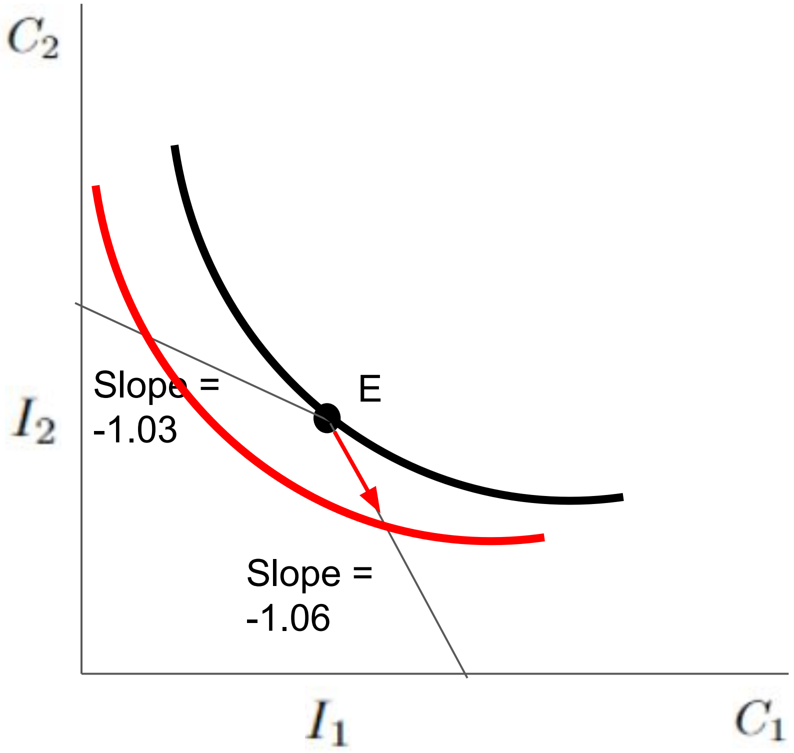
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E



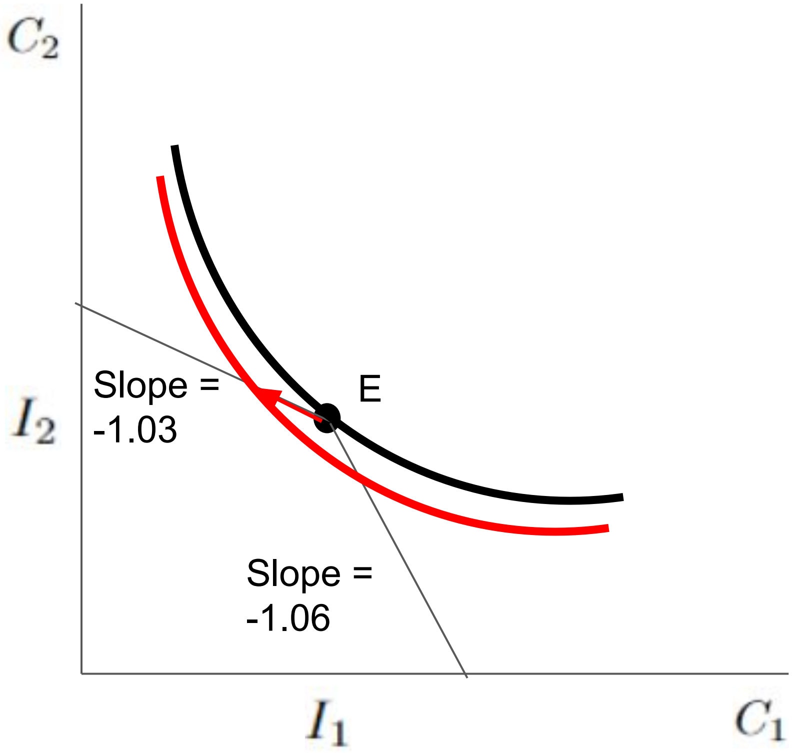
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
  - "You charge too much for me to borrow."



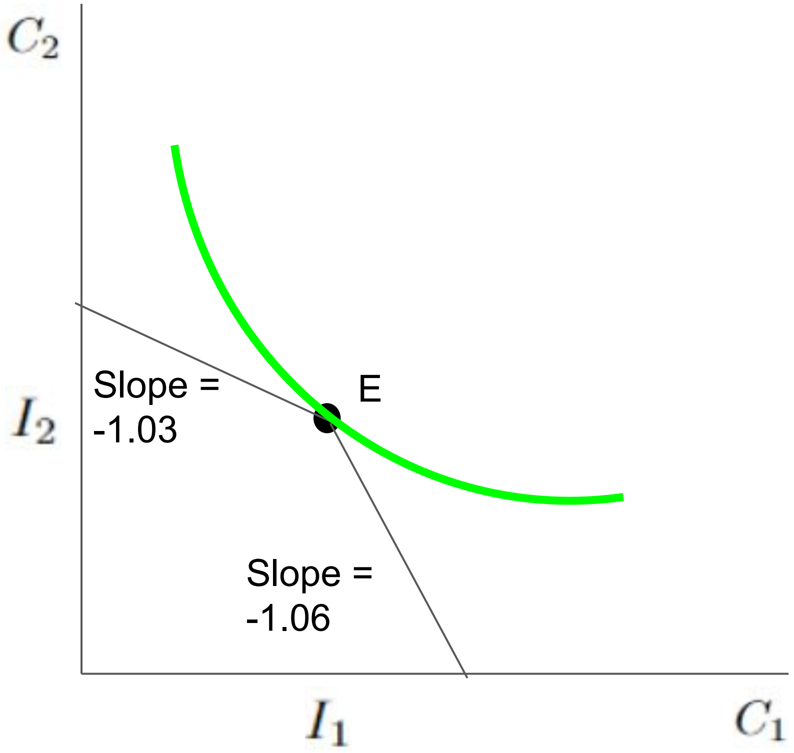
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
  - "You don't pay enough for me to save."



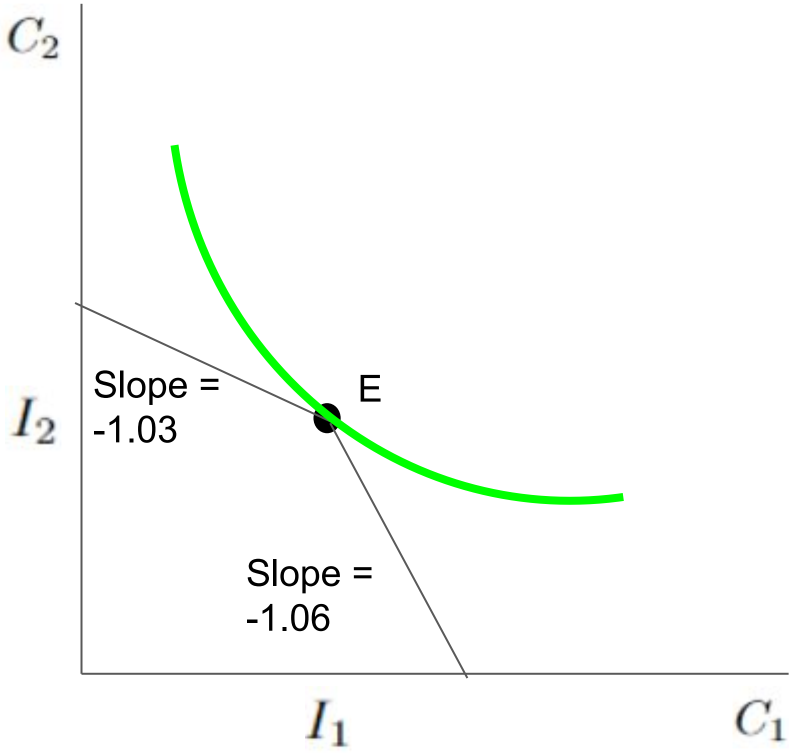
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
    - "I'll stick at my endowment"



# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
- Endowment is optimal for a range of MRSs



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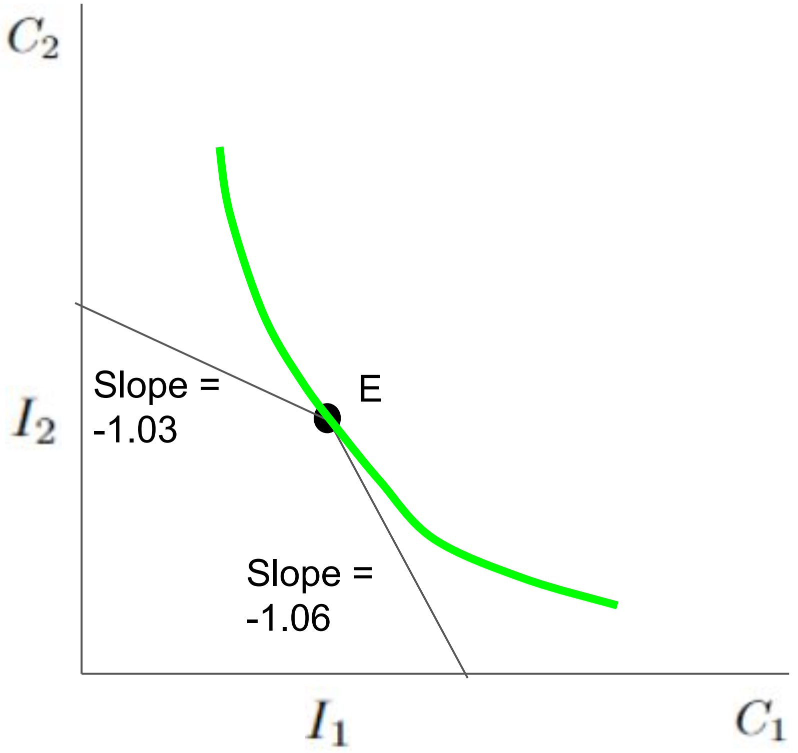

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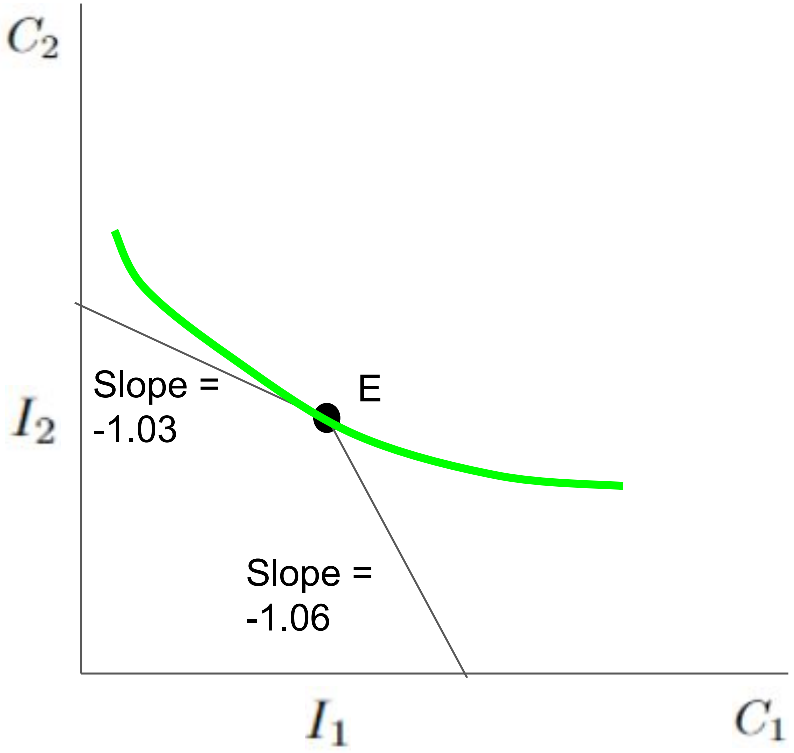
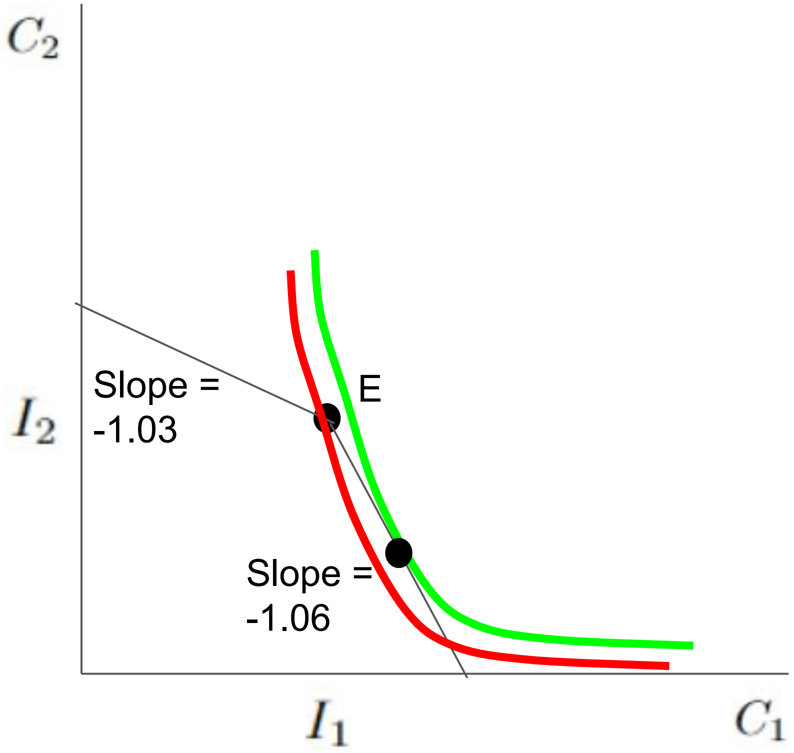


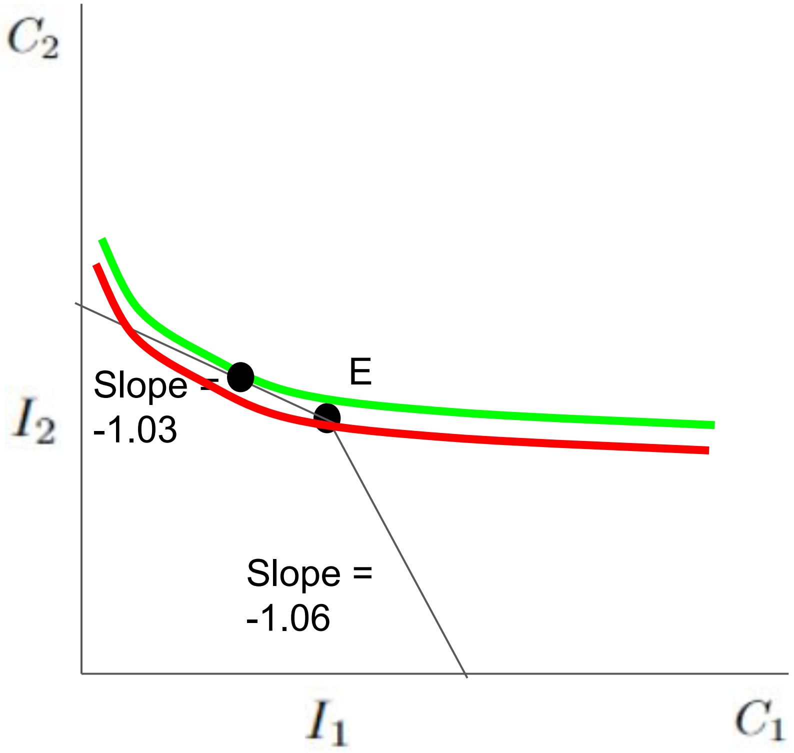
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
- Endowment is optimal for a range of MRSs
- Only if you really want consumption today will you borrow



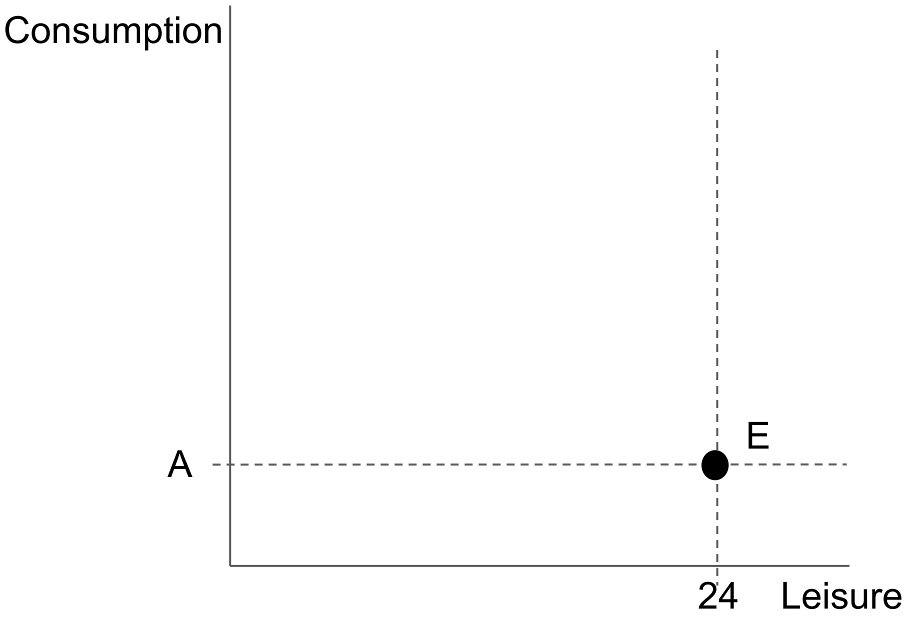
# Kinked budget set: solution

- Many will choose to neither borrow nor lend – stay at endowment E
- If your MRS at E is between 1.03 and 1.06, you won't leave E
- Endowment is optimal for a range of MRSs
- Only if you <u>really</u> want consumption today will you borrow
- And only if you <u>really</u> want it tomorrow will you save



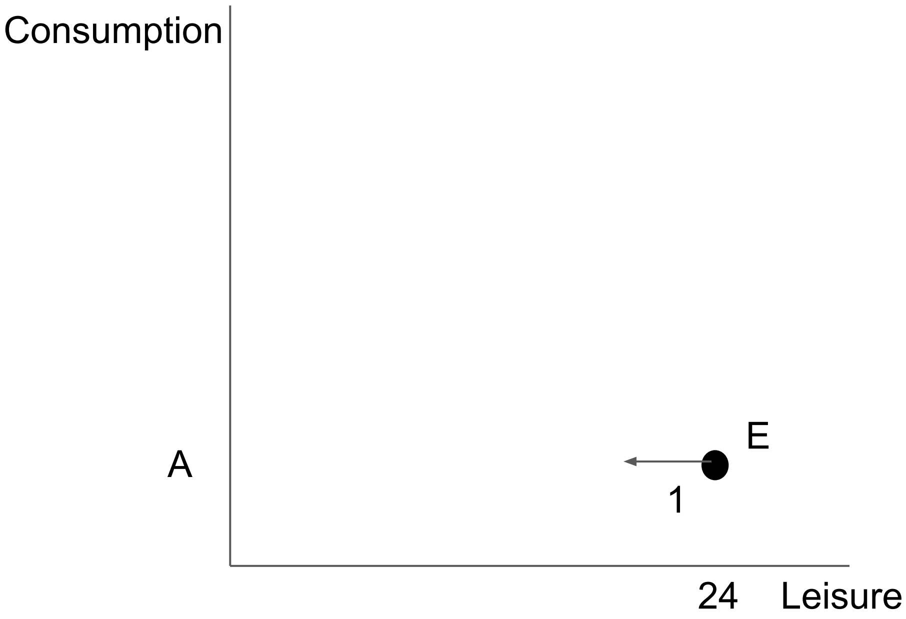
# “Labor/leisure”

- Another important setting is the labor market, where people sell their **time**
- Suppose she has wealth/asset A, and there are 24 hours in a day
- One option she has is to spend 24 hours on “leisure” and consume A



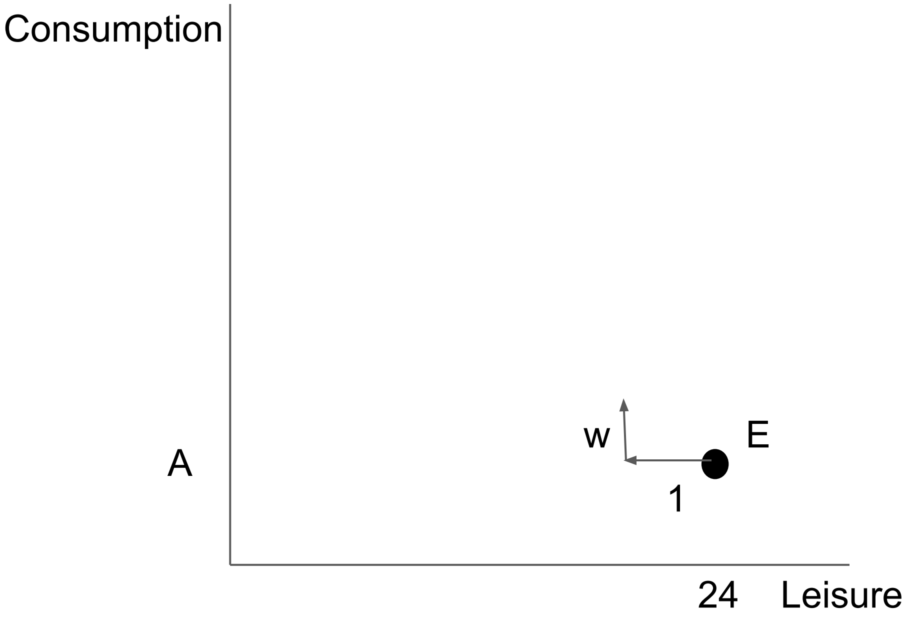
# Labor/leisure budget set

- But maybe she gets bored and feels like she needs more stuff than A can buy
- If she gives up 1 hour of leisure to work, how much more stuff can she buy?



# Labor/leisure budget set

- But maybe she gets bored and feels like she needs more stuff than A can buy
- If she gives up 1 hour of leisure to work, how much more stuff can she buy?
- The typical term for the price of leisure is the “wage” (w)



# Labor/leisure budget set

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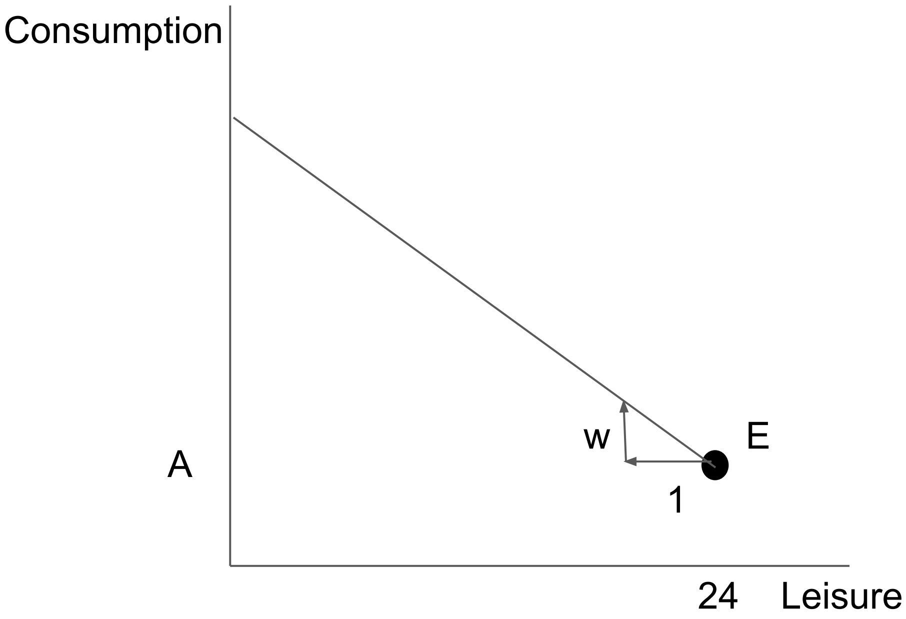
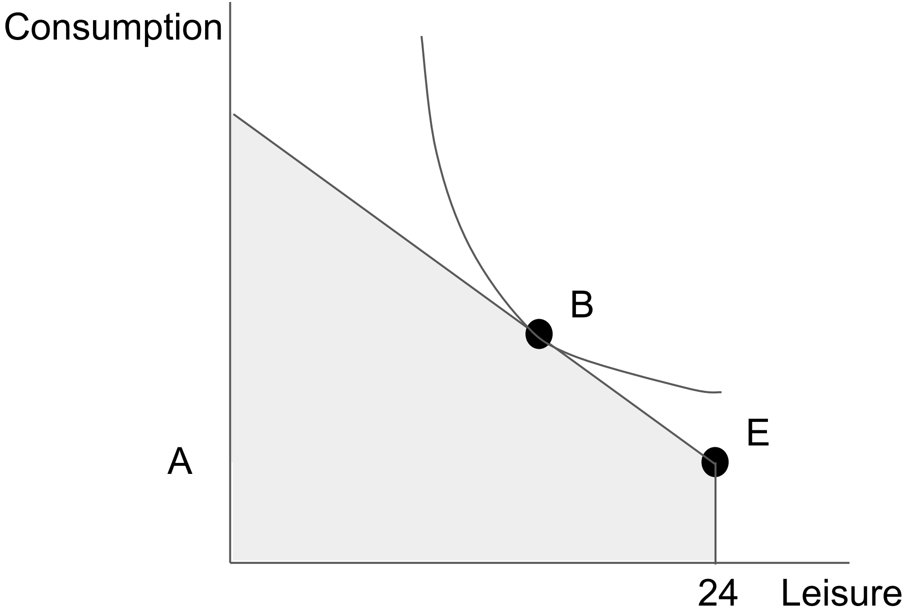


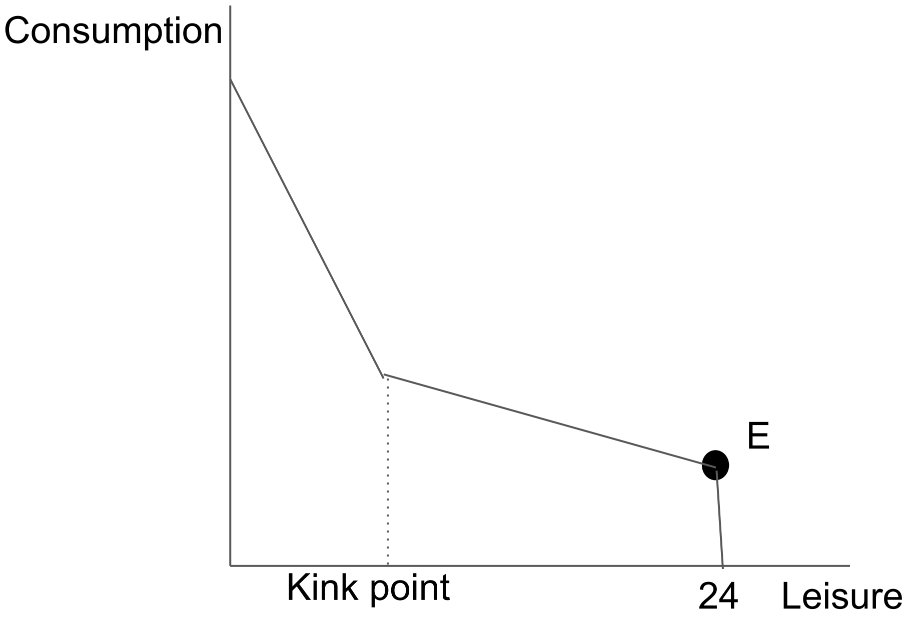
# Labor/leisure: solution

- Choose a point where the MRS is equal to the wage



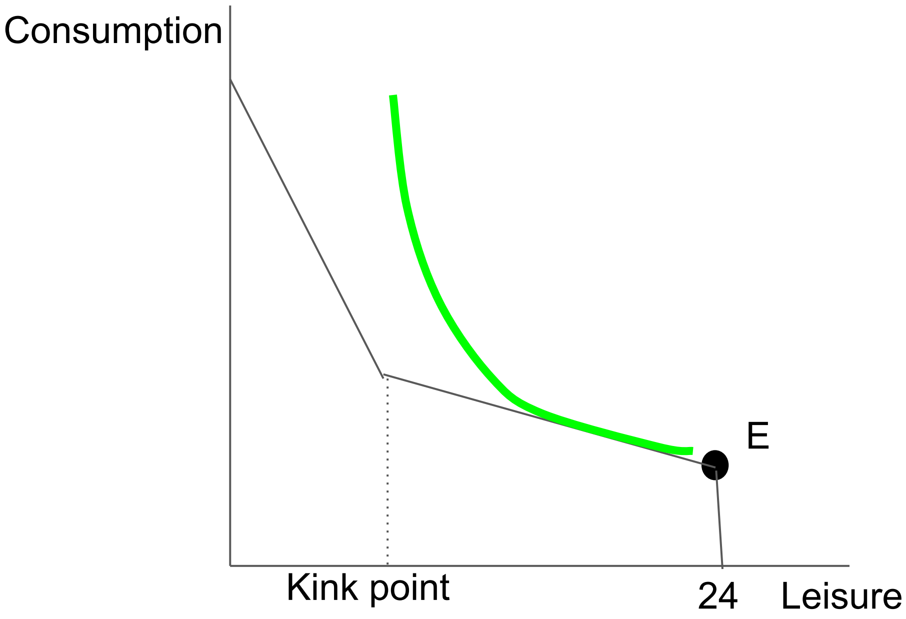
# Labor/leisure budget set with overtime pay

- Higher wage for overtime creates a kink convex kink in the budget set



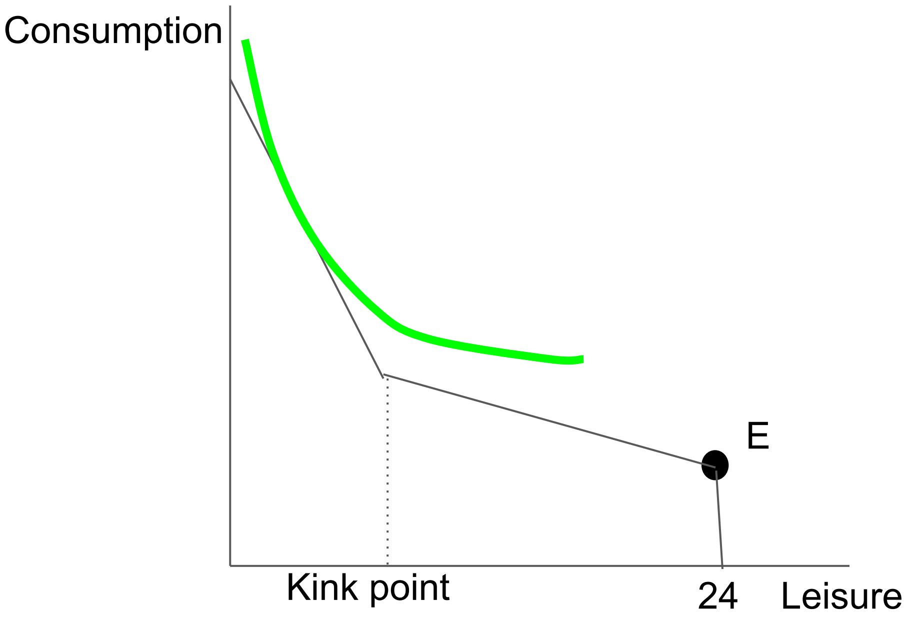
# Labor/leisure budget set with overtime pay, solution

- “I really value leisure, so I won’t work up to earn overtime.”



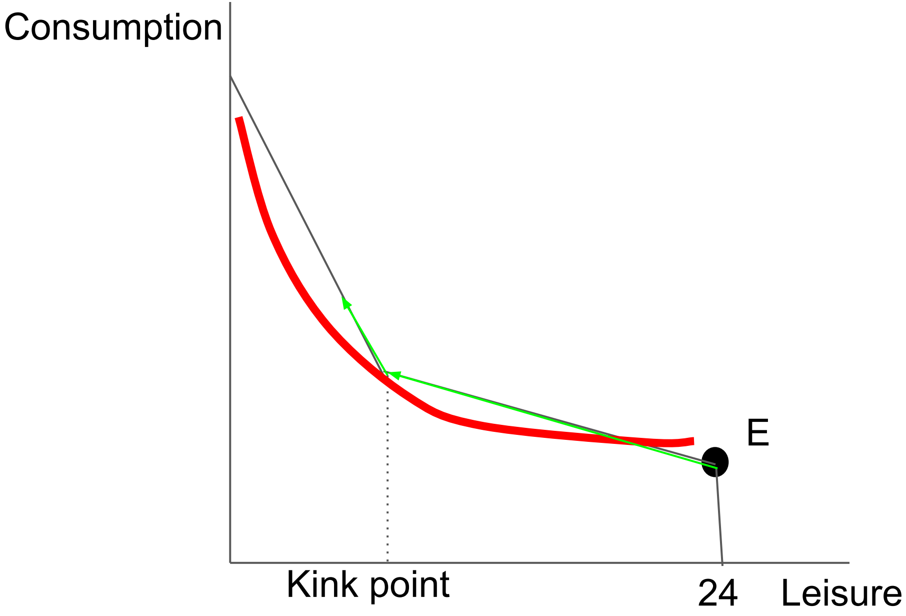
# Labor/leisure budget set with overtime pay, solution

- "I really value consumption, so I'll work extra to get overtime pay."



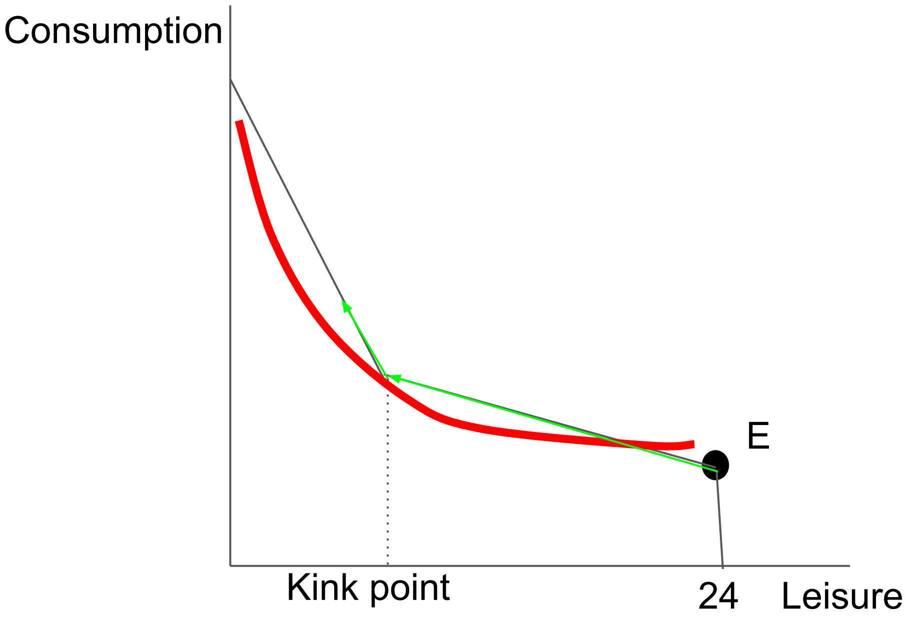
# Labor/leisure budget set with overtime pay, solution

- No one should end up at the kink
    - If your MRS is low enough to get you to the kink point, it will be low enough to pash you past it, given the discrete jump in the slope from overtime



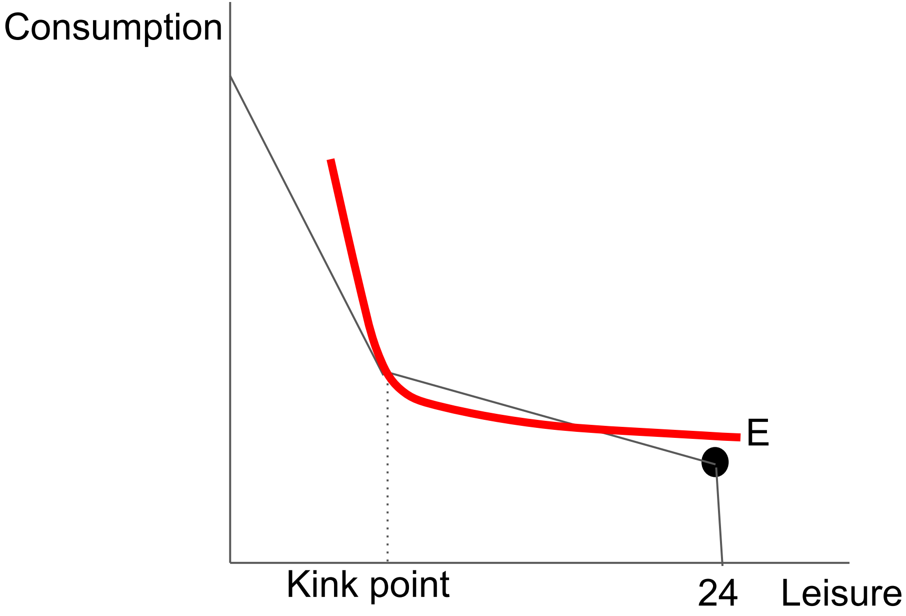
# Labor/leisure budget set with overtime pay, solution

- No one should end up at the kink
- In reality, plenty of people work full-time but not overtime. Why?



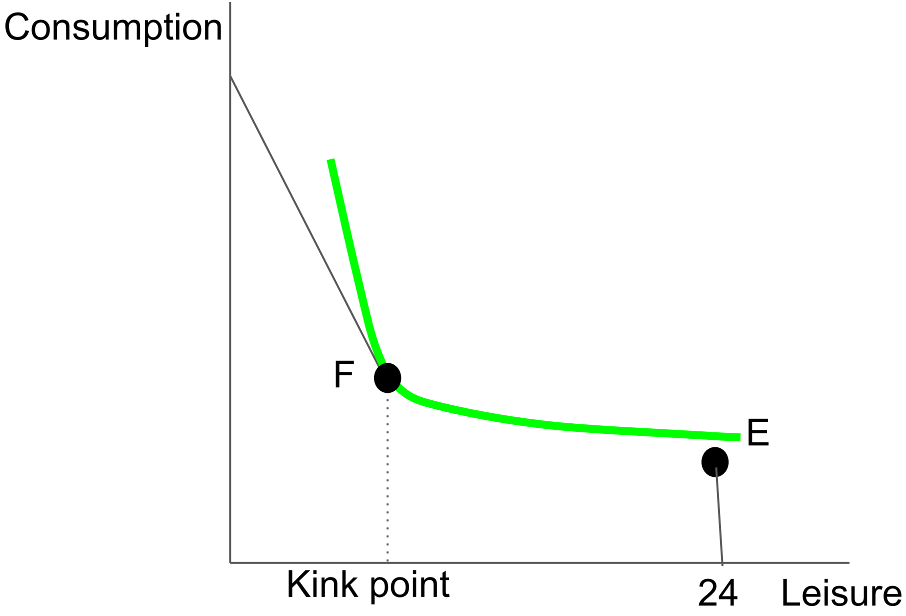
# Labor/leisure budget set with overtime pay, solution

- People don't have full flexibility in choosing their hours
- This person would like to work below the kink – i.e. less than "full-time"
  - At full-time, his MRS is very steep, so he'd give up consumption to work less



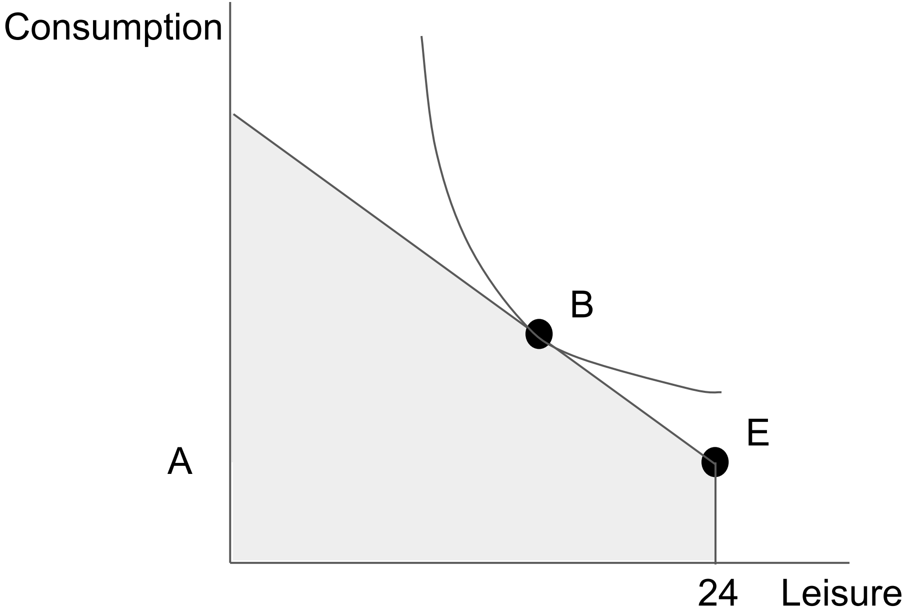
# Labor/leisure budget set with overtime pay, solution

- People don't have full flexibility in choosing their hours
- This person would like to work below the kink – i.e. less than "full-time"
  - But if the employer doesn't allow part-time work, the kink could be optimal



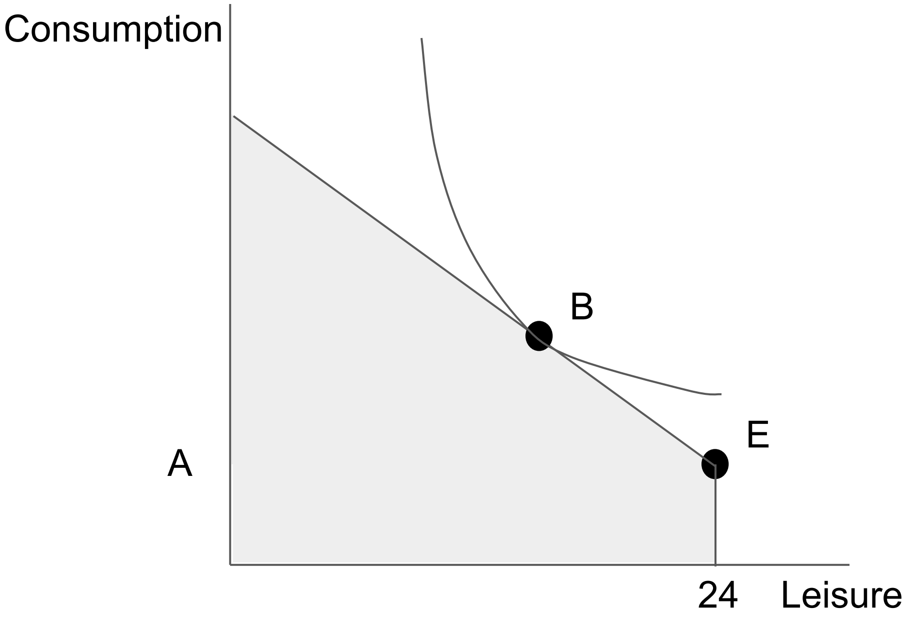
# An increase in the wage

- Suppose she chooses point B



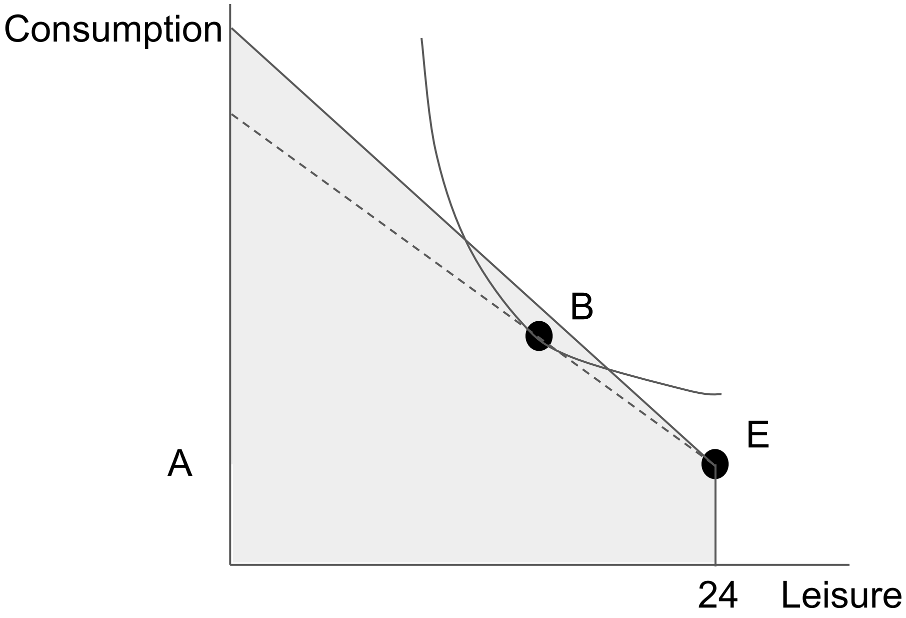
# An increase in the wage

- Suppose she chooses point B
- And then the wage goes up. What will happen to the budget set?



# An increase in the wage

- Suppose she chooses point B
- And then the wage goes up. What will happen to the budget set?
- What will happen to her optimal choice? It's complicated: see next section!



# Income and Substitution Effects

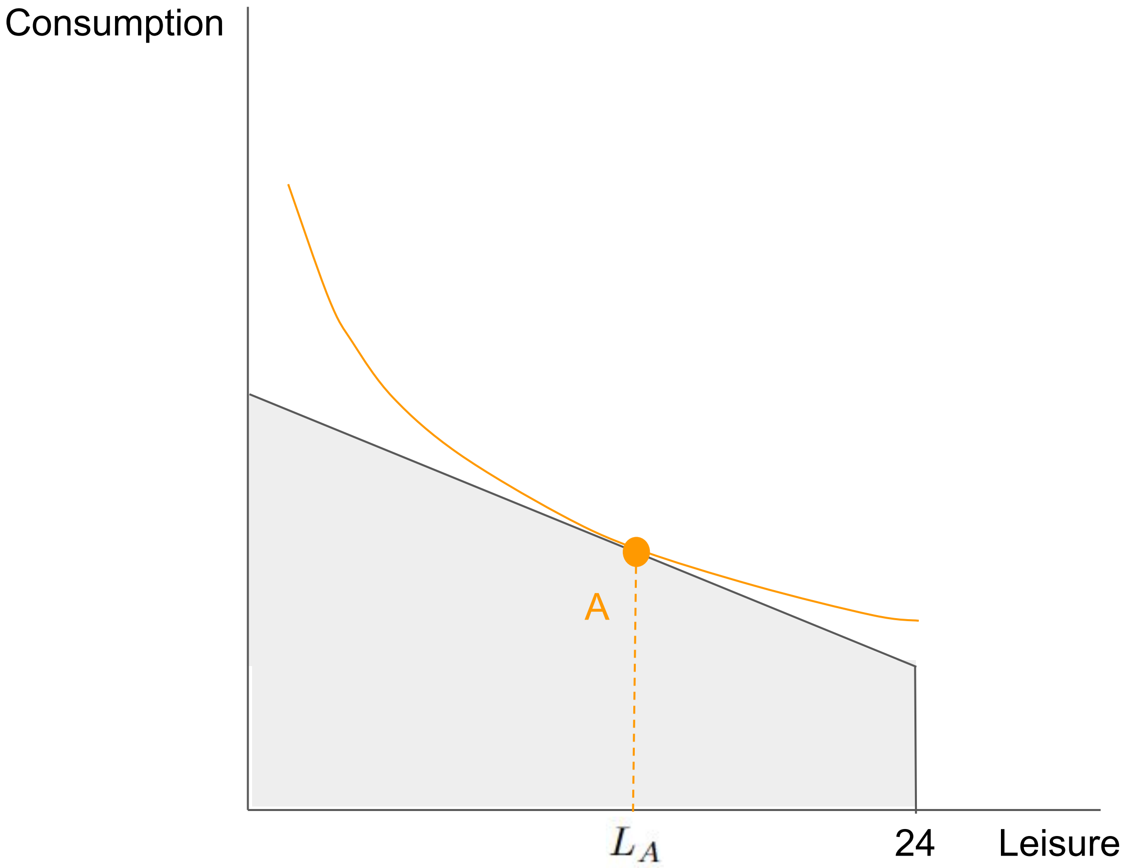
# Effect of a wage change

- If your wage increases, your incentive to work is impacted in multiple ways
- On the one hand, the returns to working are higher.
  - A different way to say this is that leisure is now more expensive
- On the other hand, because your labor is more valuable, you are richer
  - A few slides ago, we saw that the budget set <u>expanded</u>, which is a good thing!
  - This may encourage you to work less, as you can make the same amount of money with fewer hours worked
  - But it could also encourage you to work more, maybe because it brings some expensive goals within reach

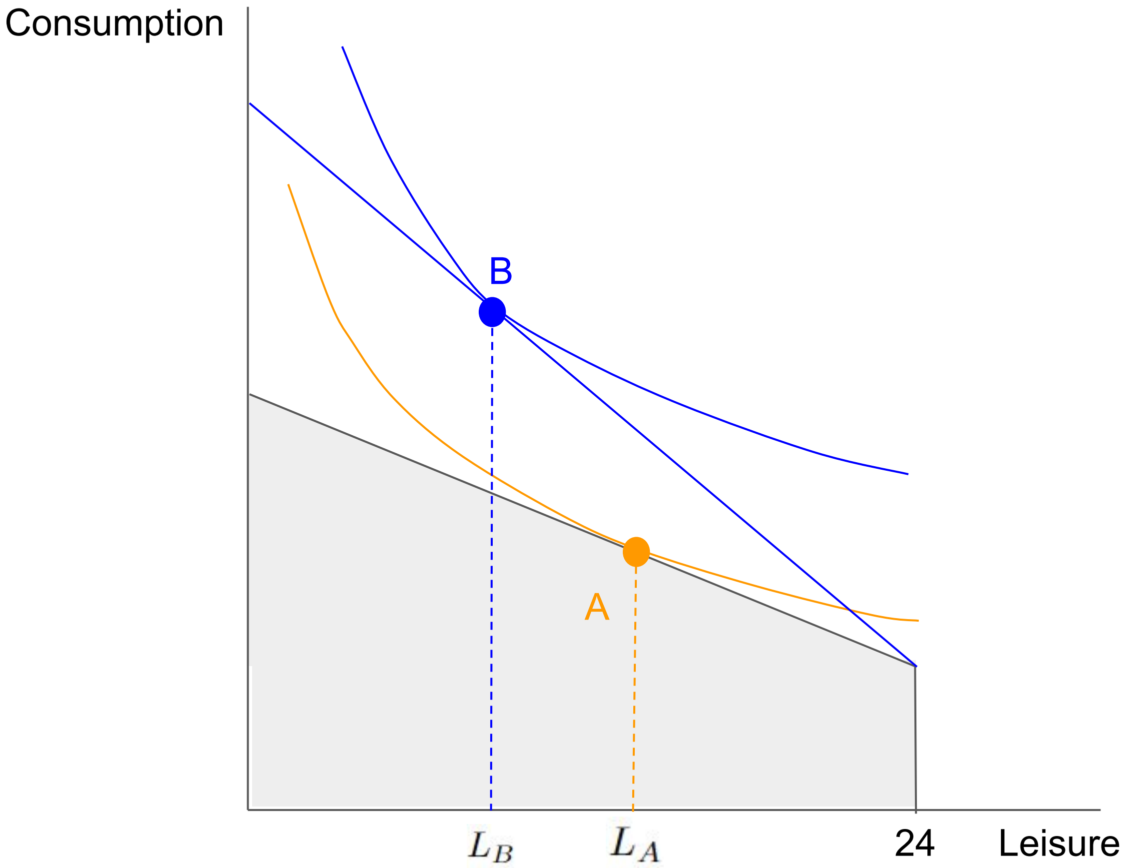
# Income and substitution effects

- Economists formally describe those two impacts as income and substitution effects
- Substitution effect
    - Isolates the effect of relative price. “Is working the **next hour** now worth it with the new wage?”
    - This effect always goes in the opposite of the price change: wage up, leisure down (labor up)
- Income effect
    - Captures the fact that you are “wealthier” when your wage goes up. “I’m getting paid more for all the **hours I already worked**: should I work more, now?”
    - This can go in either direction.
        - Positive income effect (income up, leisure up) means leisure is a “normal good”
        - Negative income effect (income up, leisure down) means leisure is an “inferior good”

# Graphical depiction of income and substitution effects

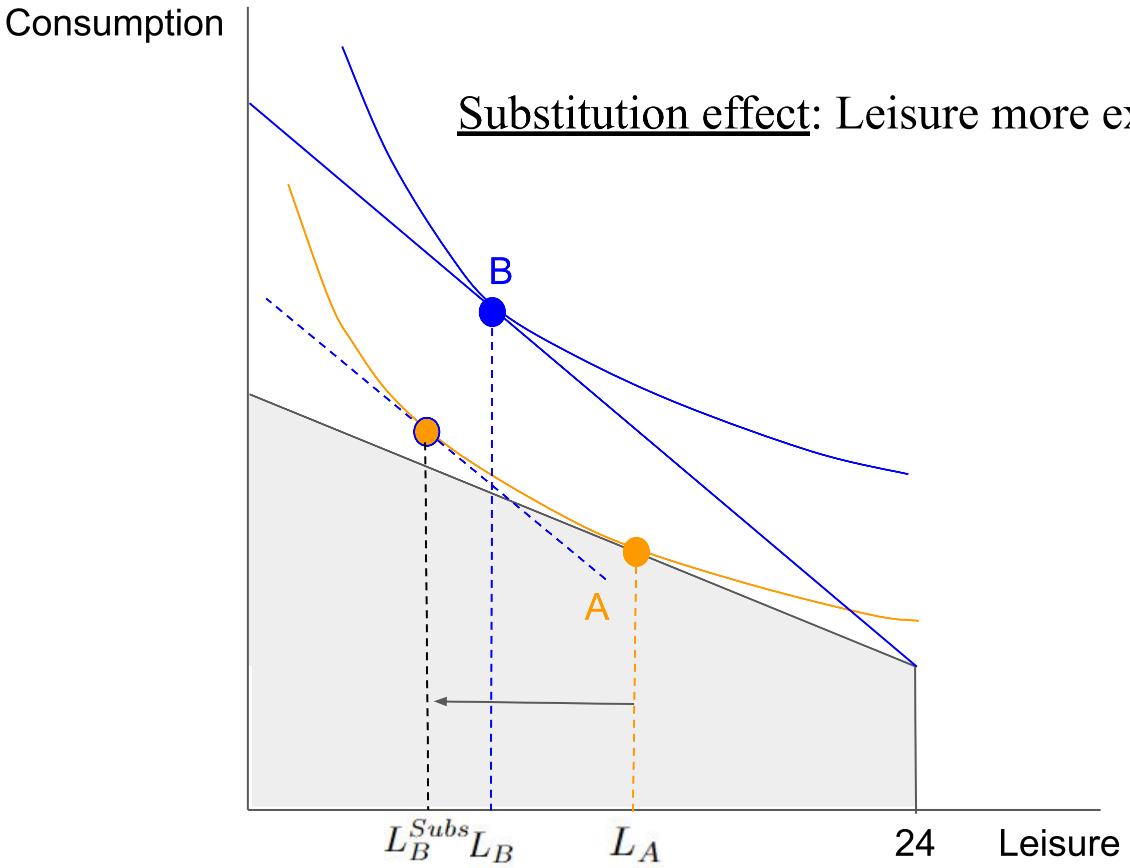


# Graphical depiction of income and substitution effects

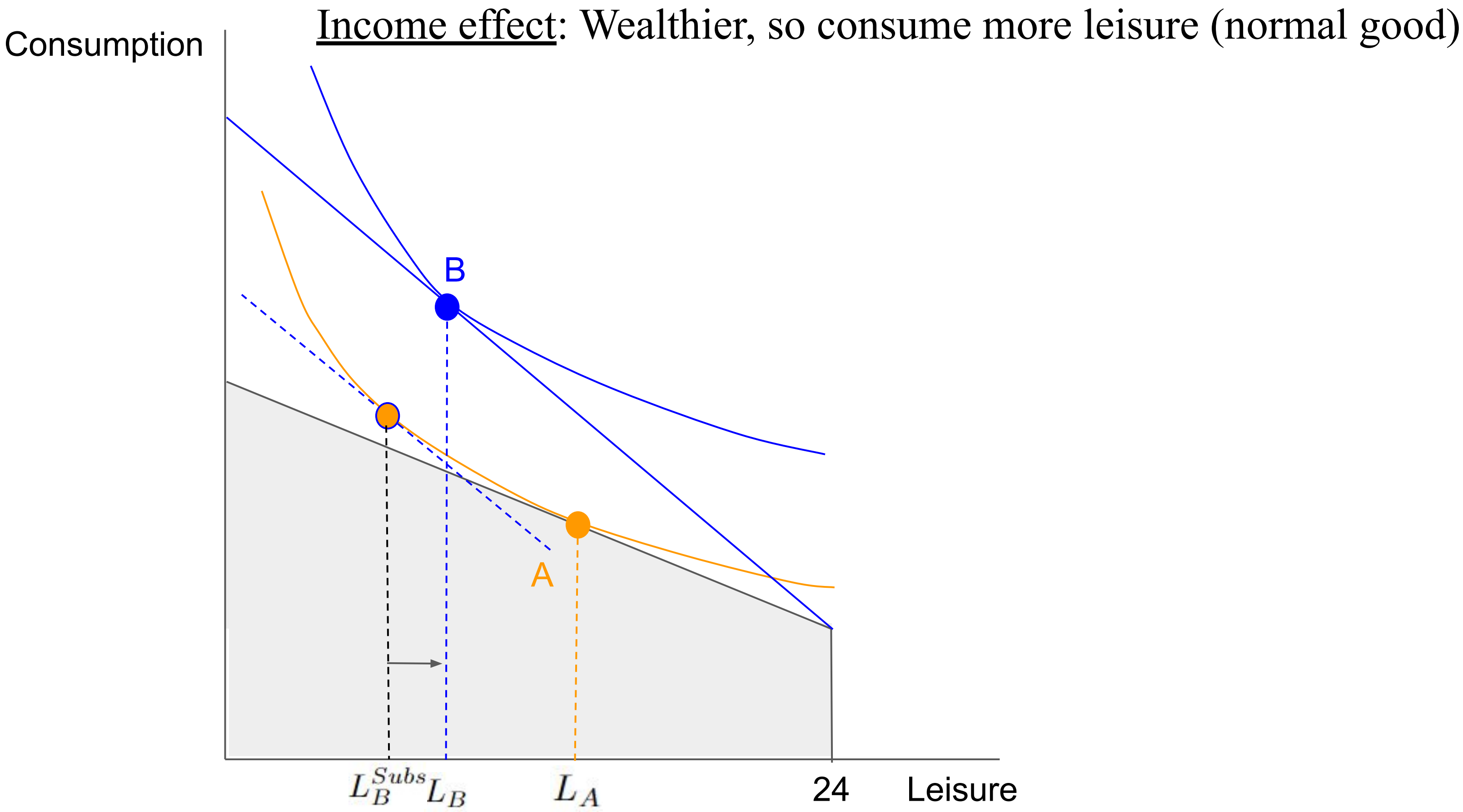


# Graphical depiction of income and substitution effects

<u>Substitution effect</u>: Leisure more expensive, so choose less

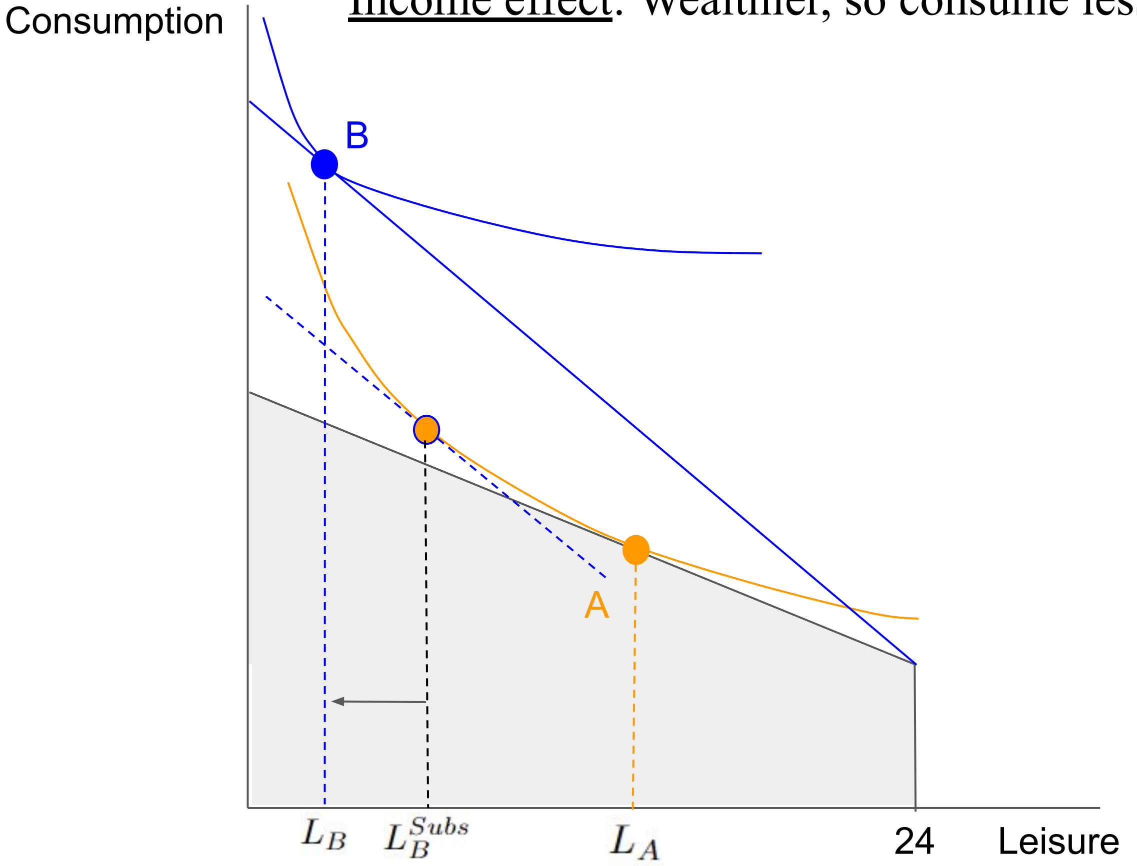


# Graphical depiction of income and substitution effects

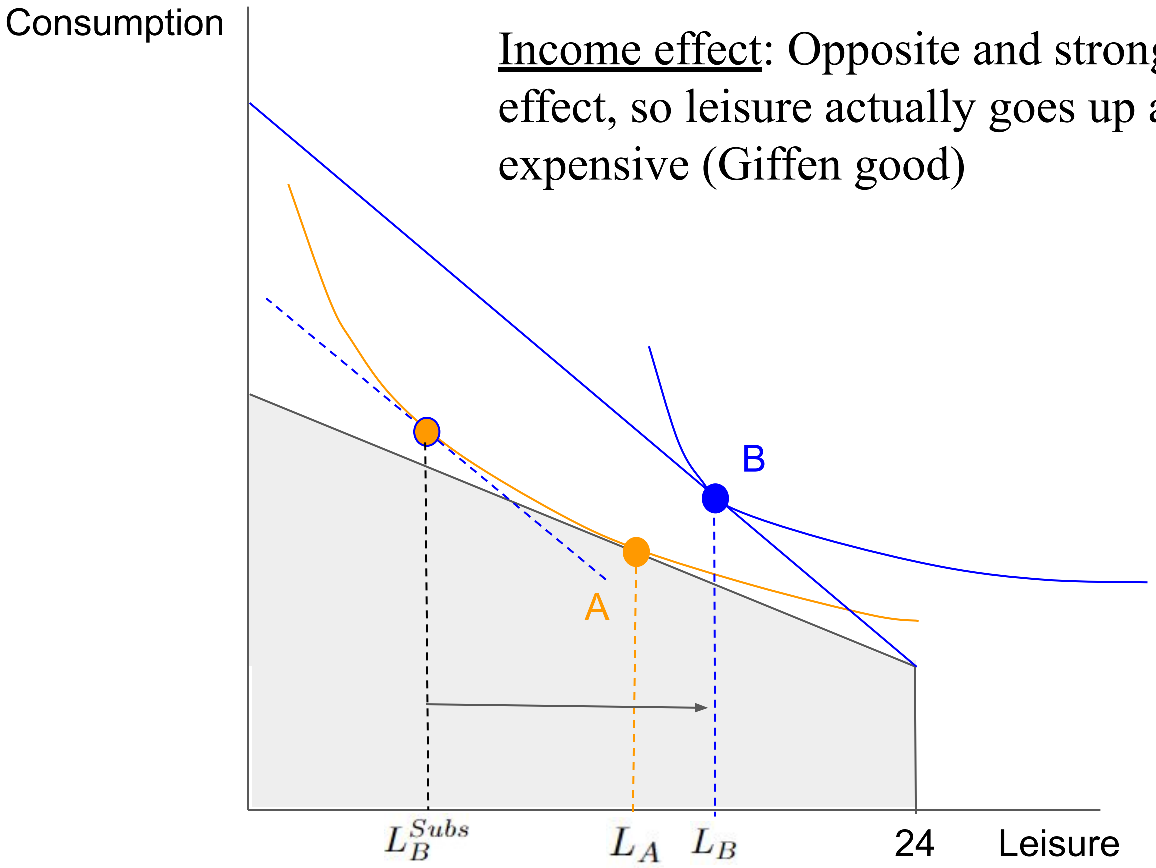


# Graphical depiction of income and substitution effects

__Income effect__: Wealthier, so consume less leisure (inferior good)

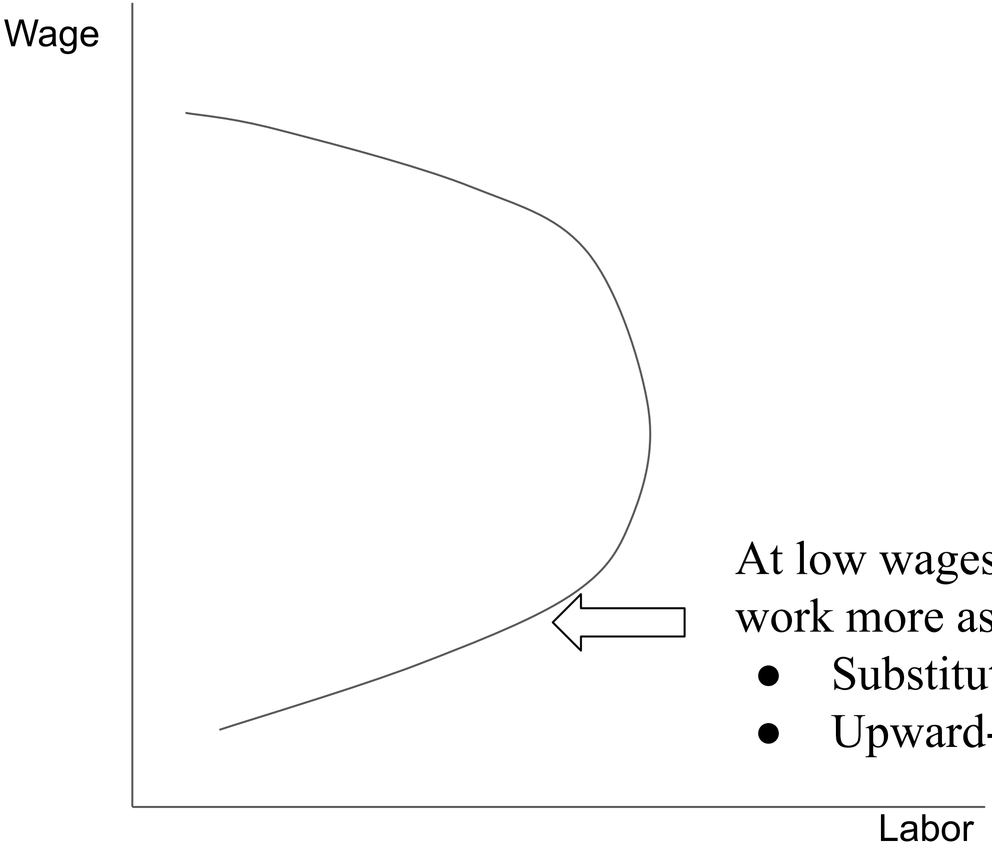


# Graphical depiction of income and substitution effects




Income effect: Opposite and stronger than substitution effect, so leisure actually goes up as it gets more expensive (Giffen good)

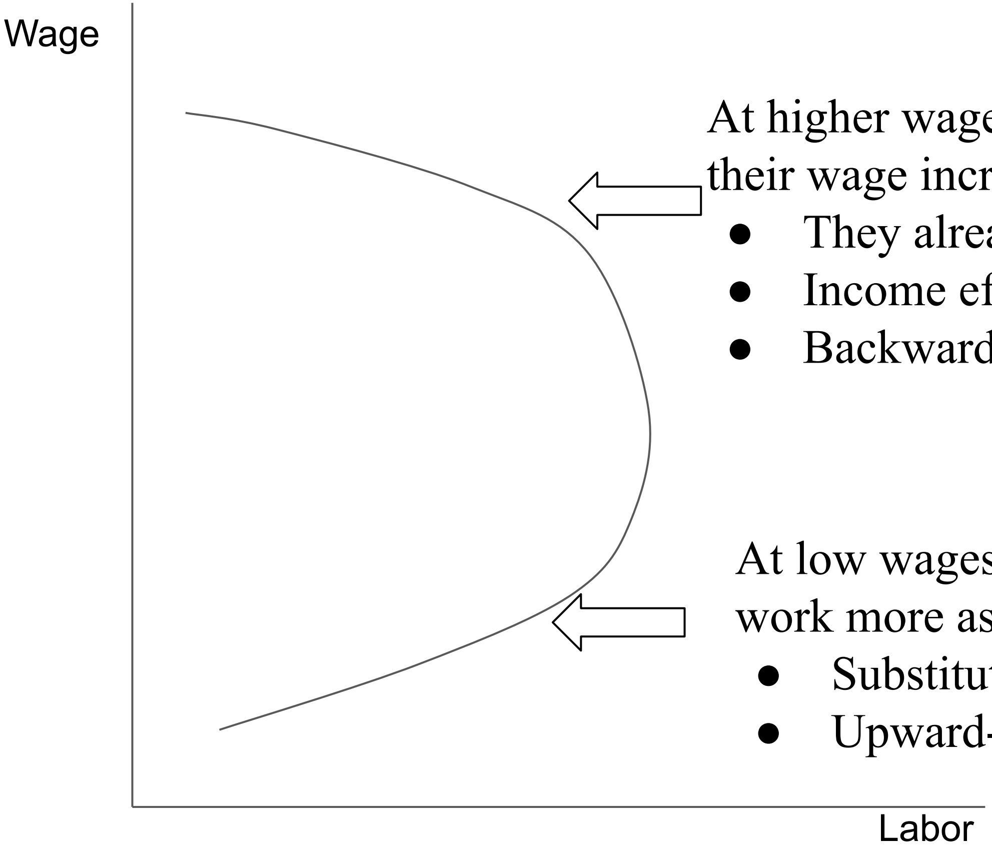
# Backward-bending labor supply




At low wages, we expect people to work more as their wage increases

- Substitution effect dominates
- Upward-sloping labor supply

# Backward-bending labor supply




At higher wages, people work *less* as their wage increases

- They already have enough money!
- Income effect dominates
- Backward-bending labor supply

At low wages, we expect people to work more as their wage increases

- Substitution effect dominates
- Upward-sloping labor supply

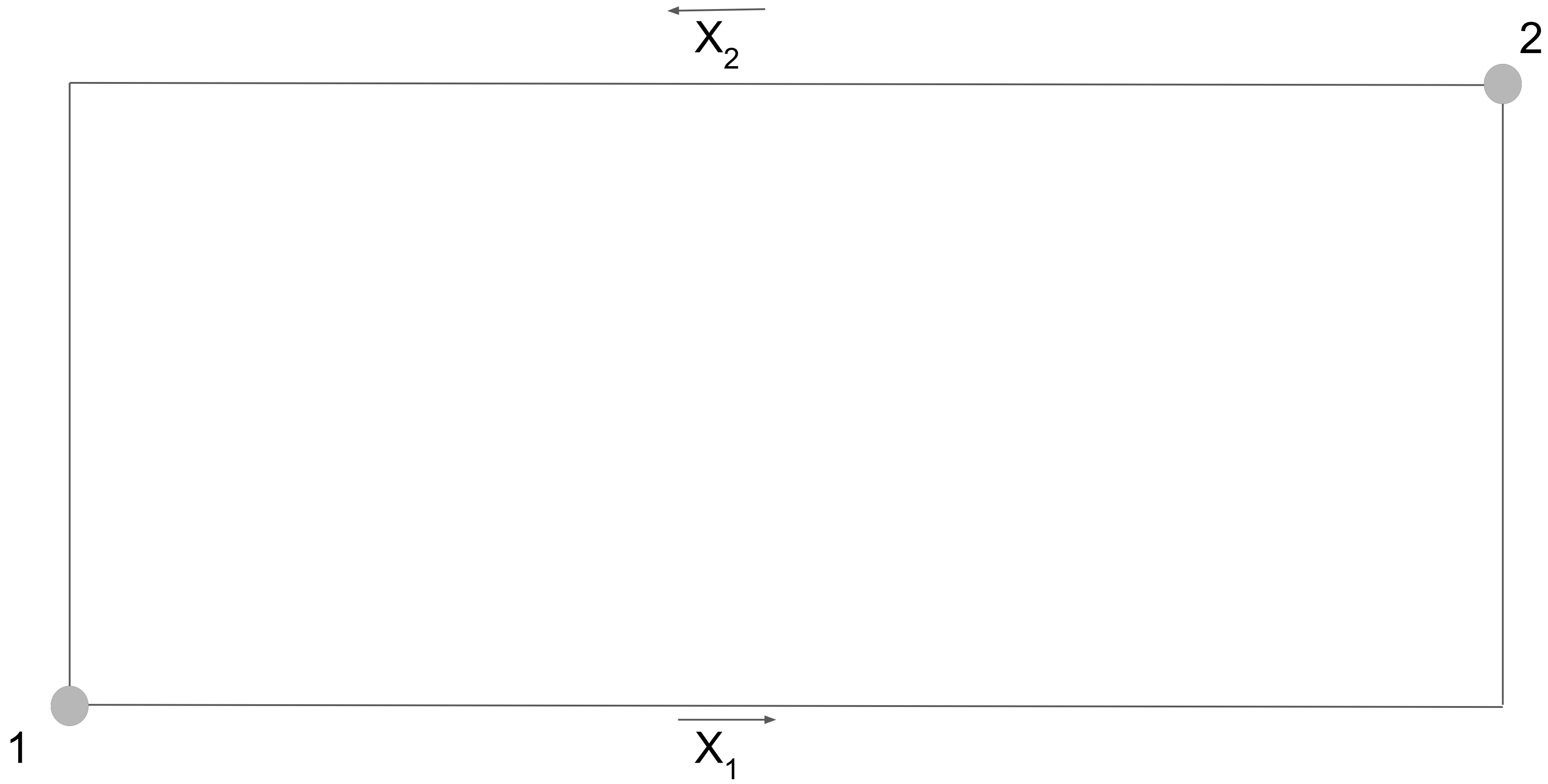
# General Equilibrium (in Exchange), Efficiency, and Welfare Economics

# Introduction

- We can now study a simple economy (i.e. full collection of markets)
  - 2 (types of) consumers
  - No production (goods are endowed to the agents)
- This will introduce general equilibrium (i.e. all markets studied together)
- Allows a more nuanced and rigorous notion of efficiency than supply-and-demand graphs
  - Supply-and-demand graphs are partial equilibrium – not all markets analyzed
- Will introduce key insights from "Welfare Economics"
- Will allow us to think about distribution/equity in a rigorous way
- Delivers rigorous, practical, and (arguably) reasonable insights about policy
  - Leaves plenty of room for debate!
  - Very powerful way to at least think through tradeoffs in policy

# Edgeworth Box

# Edgeworth Box



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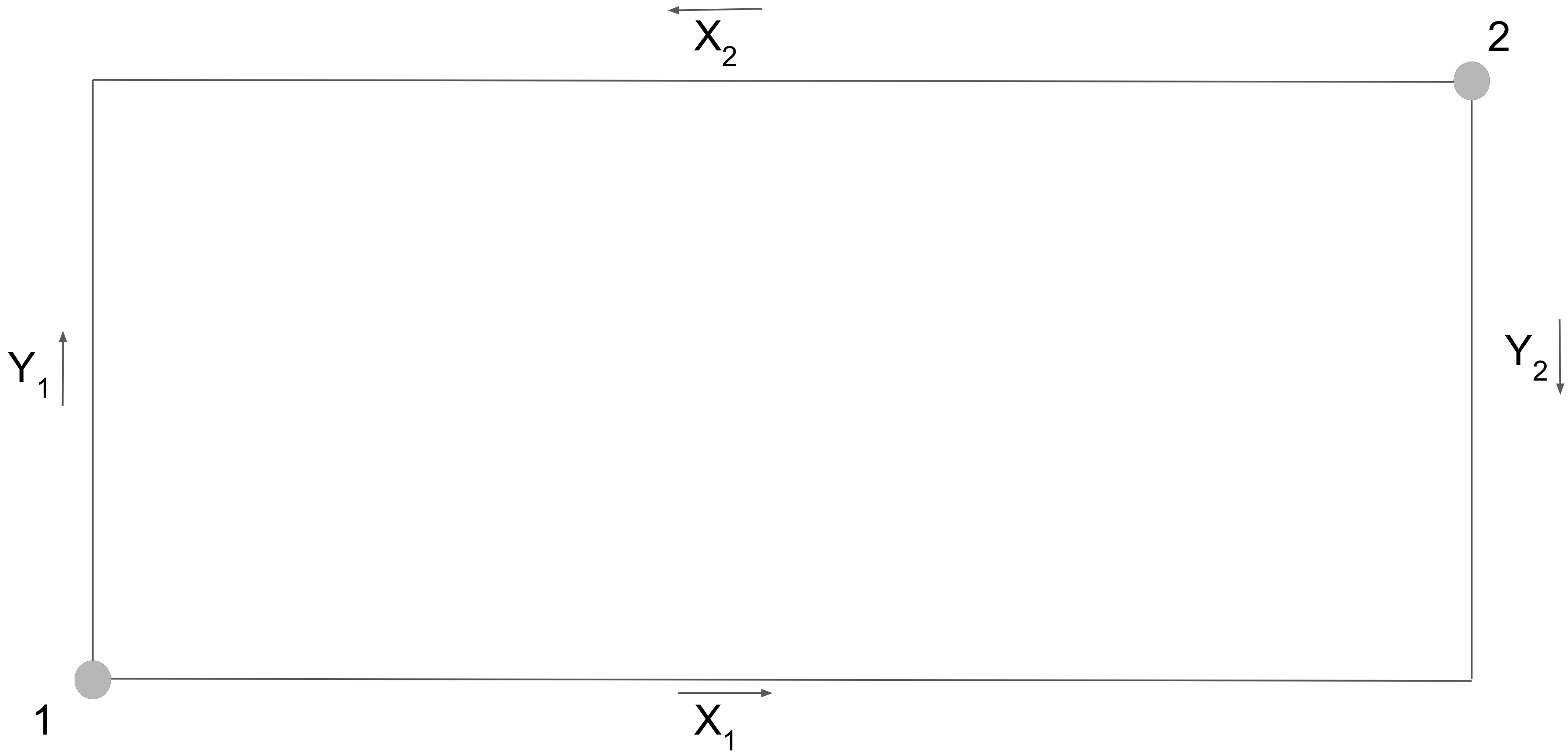

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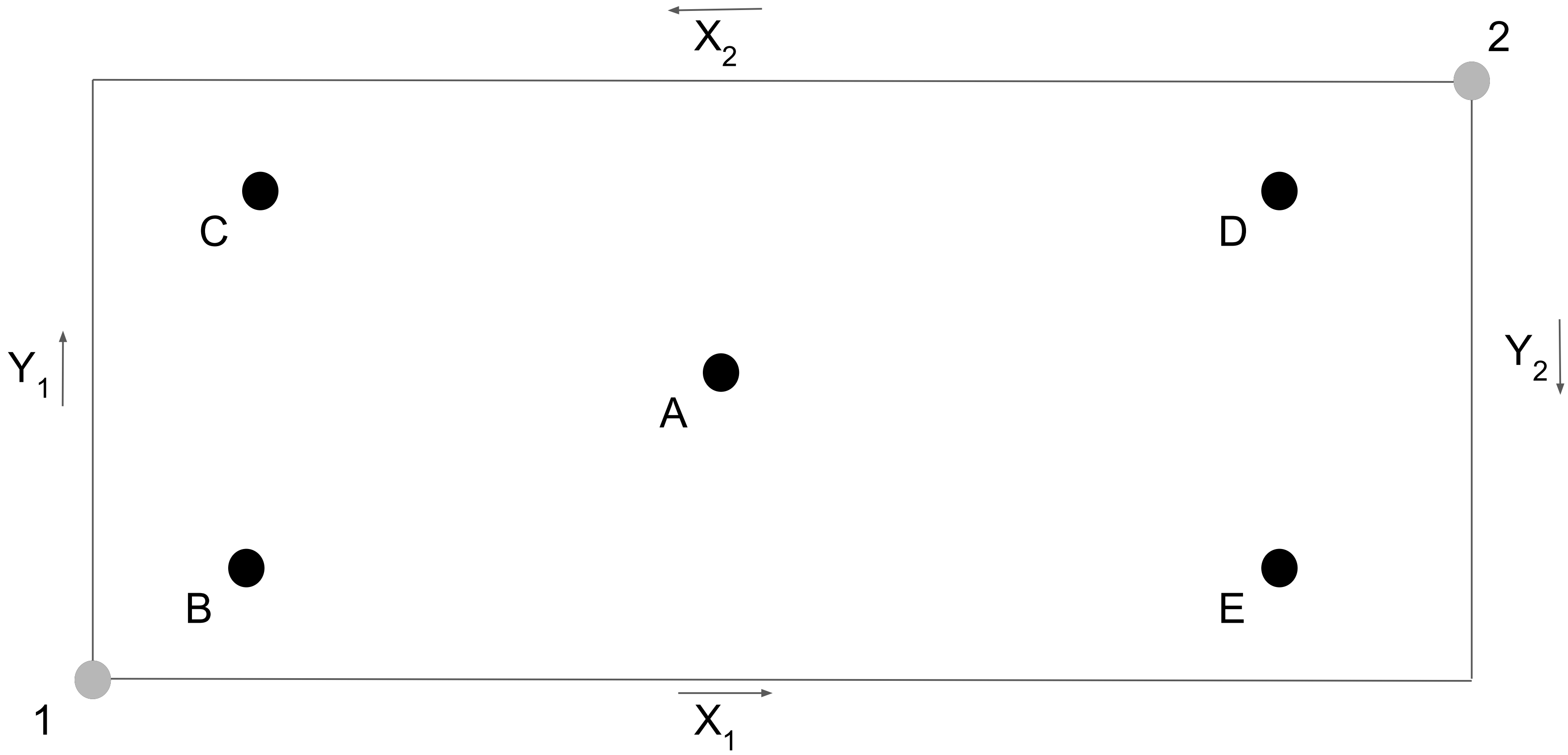

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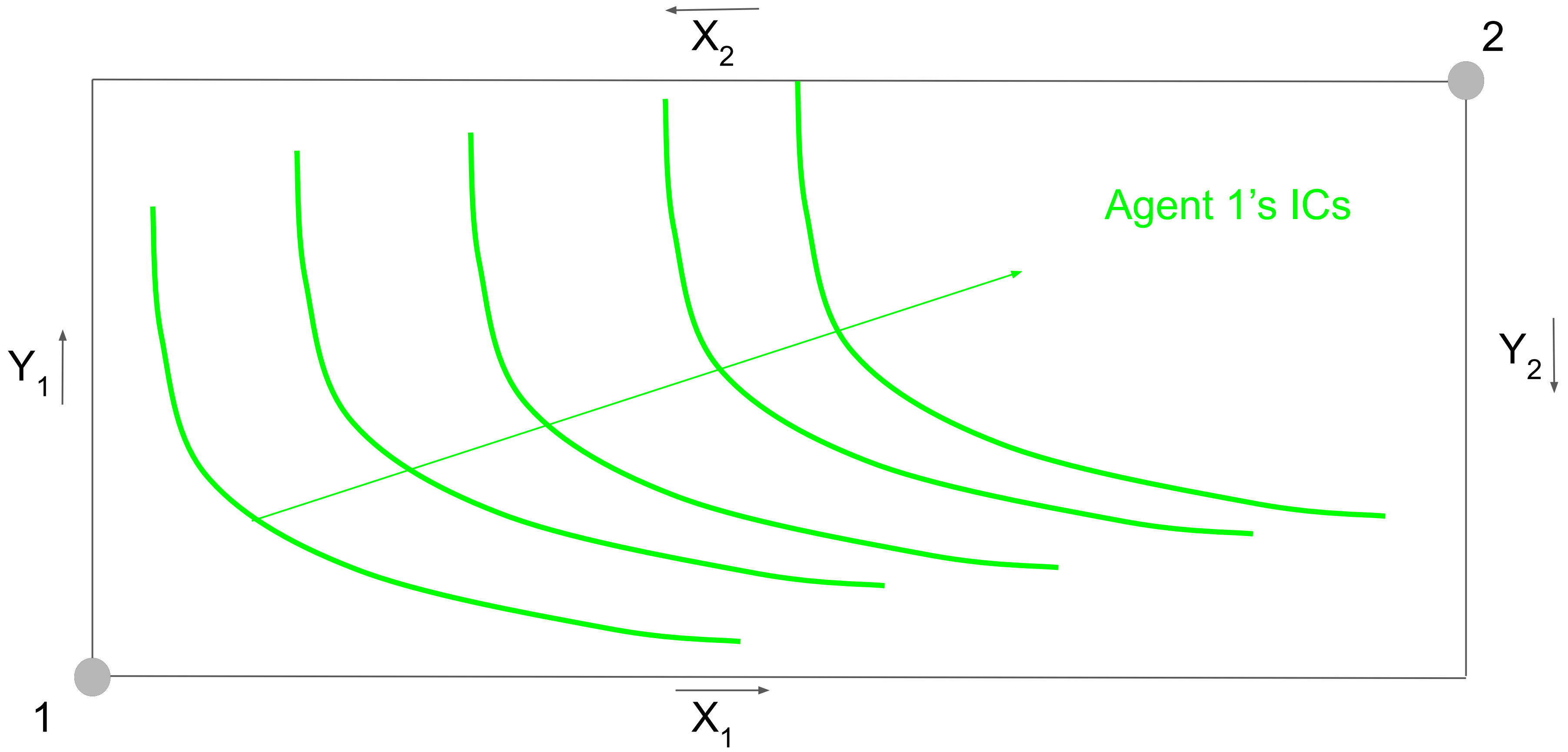

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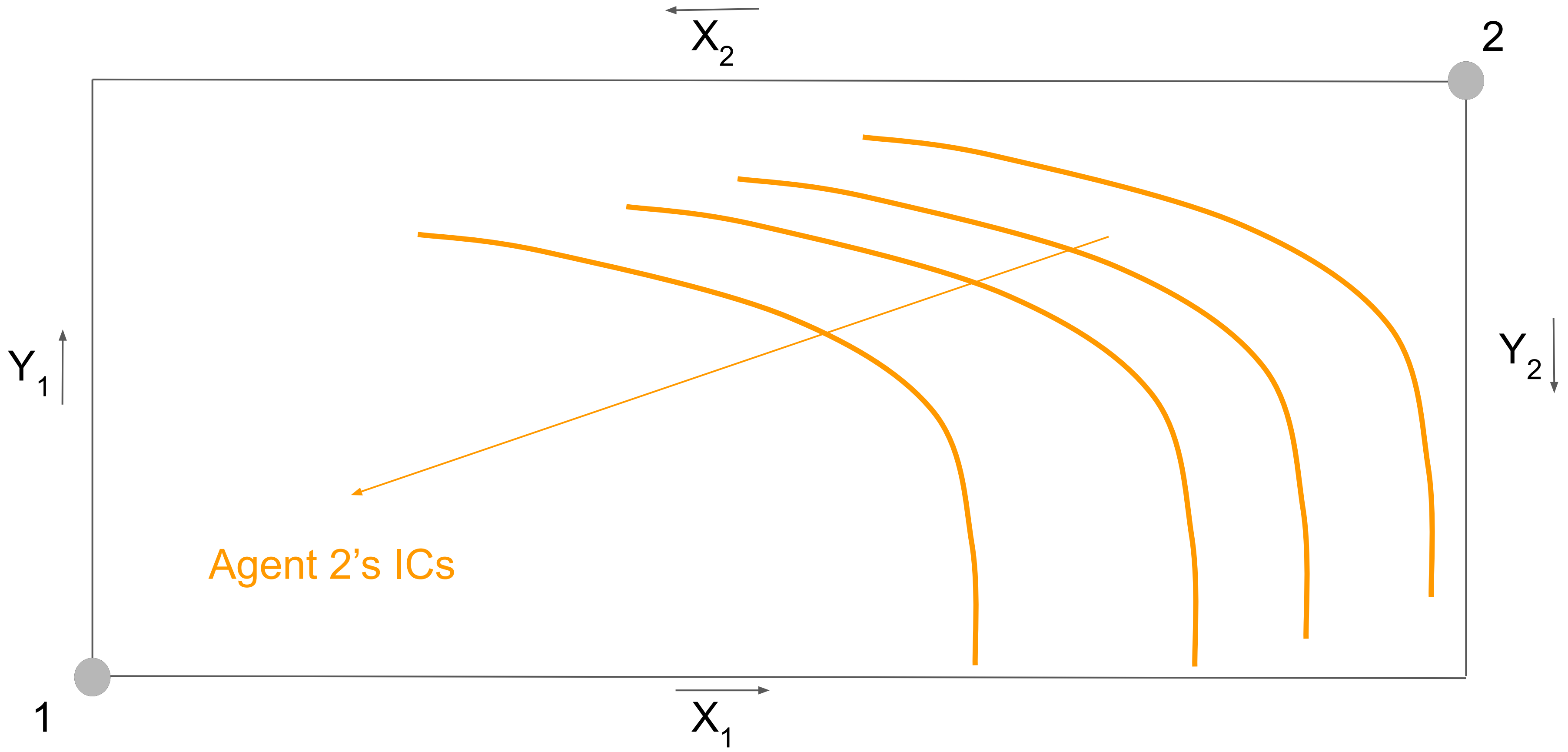

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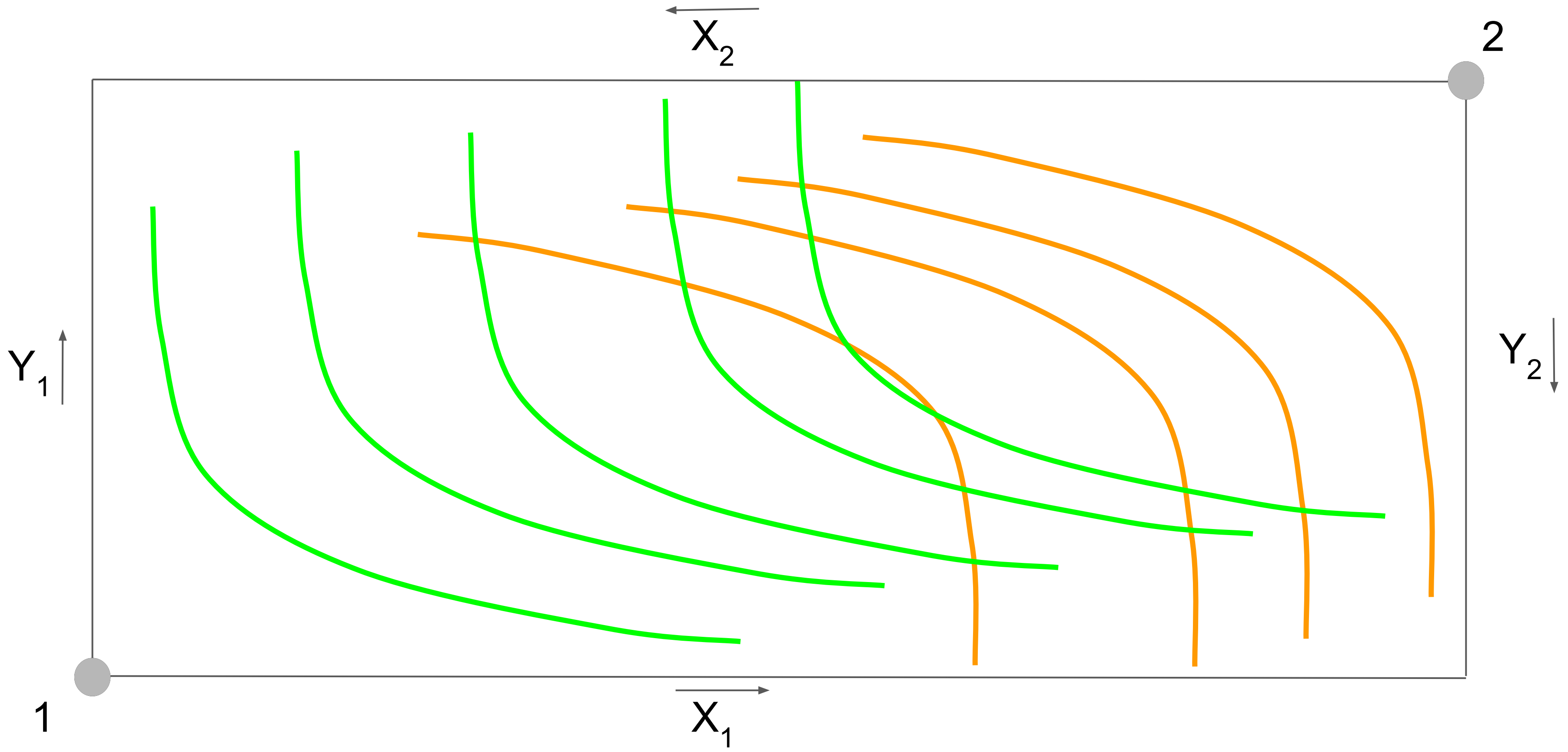

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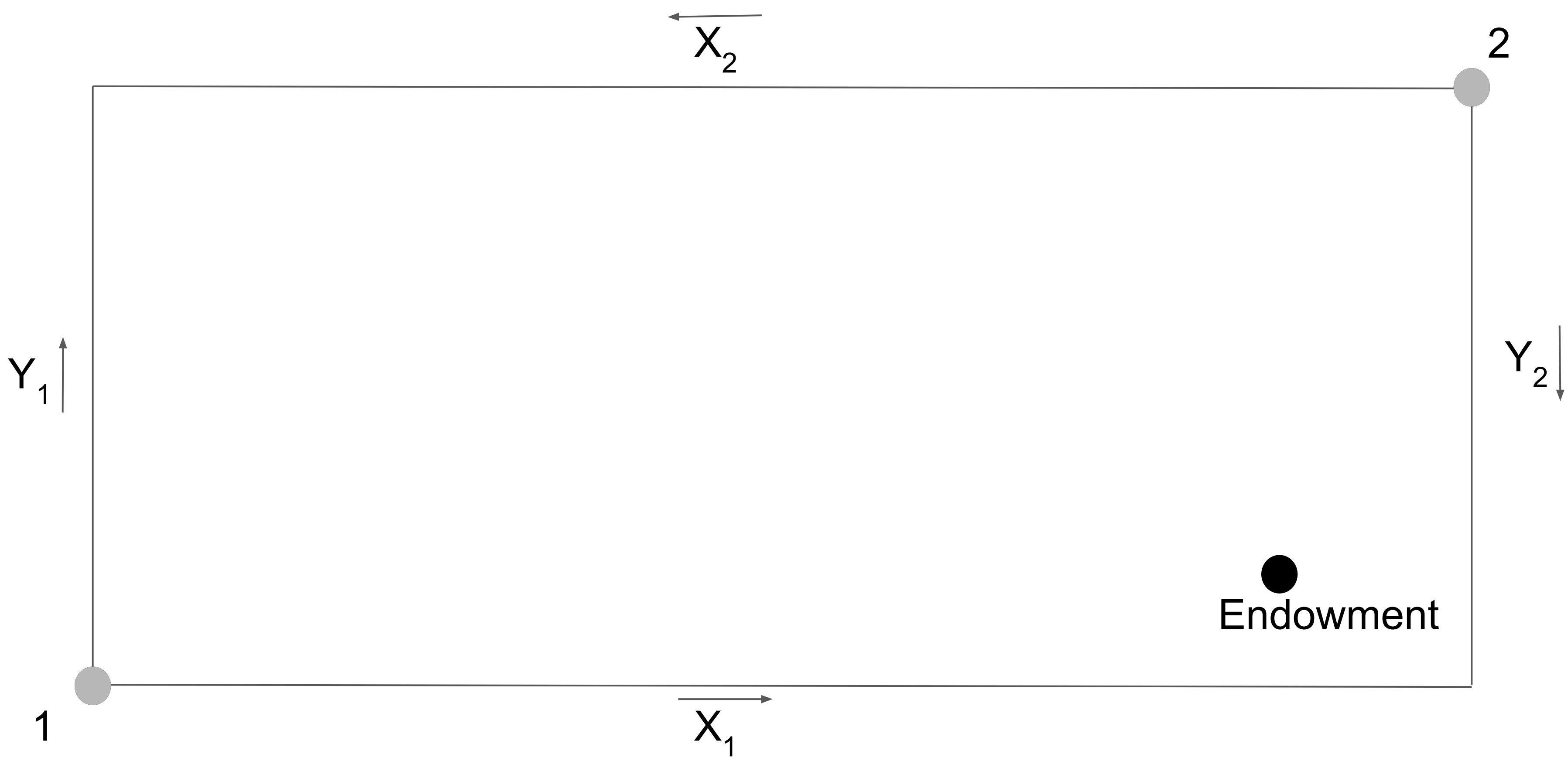

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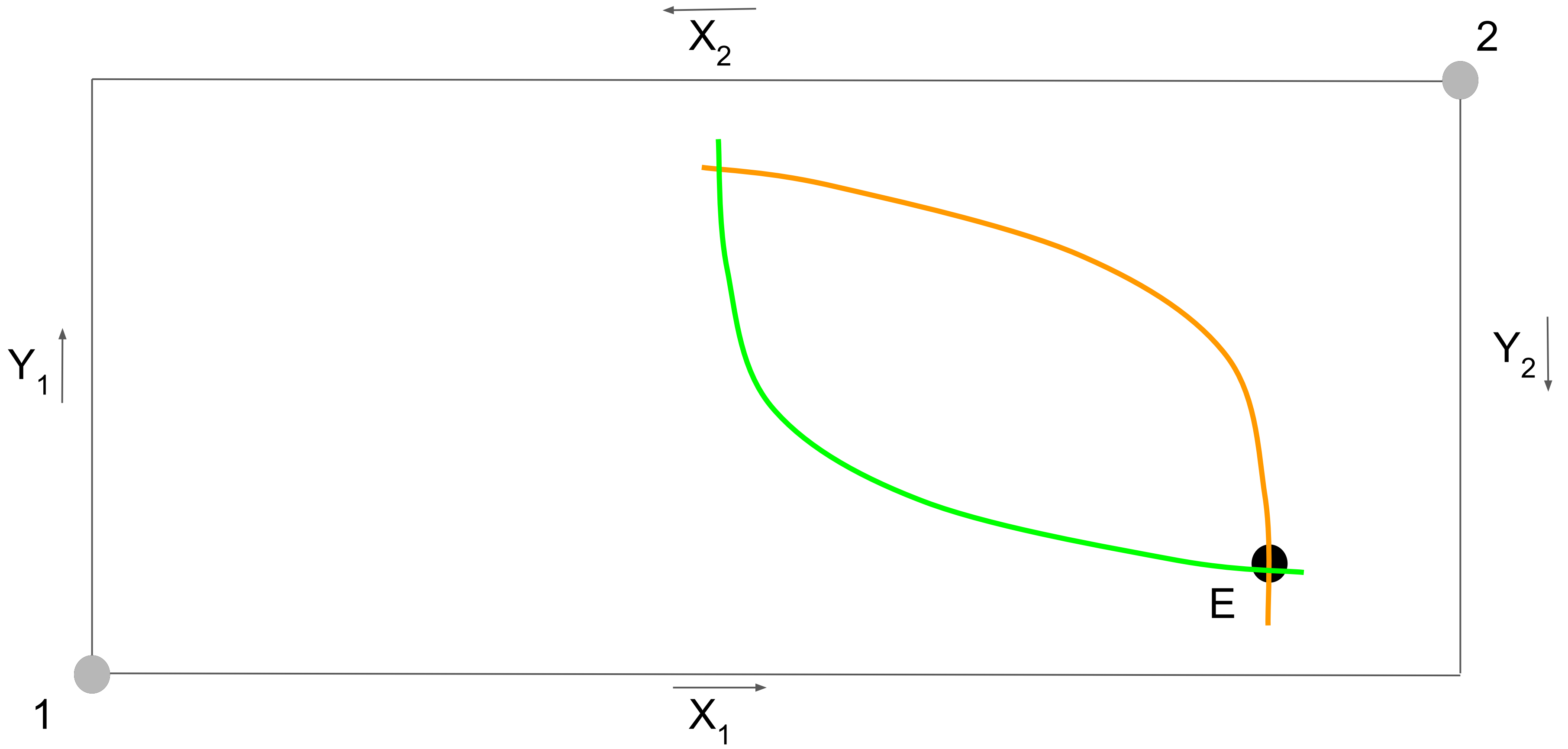

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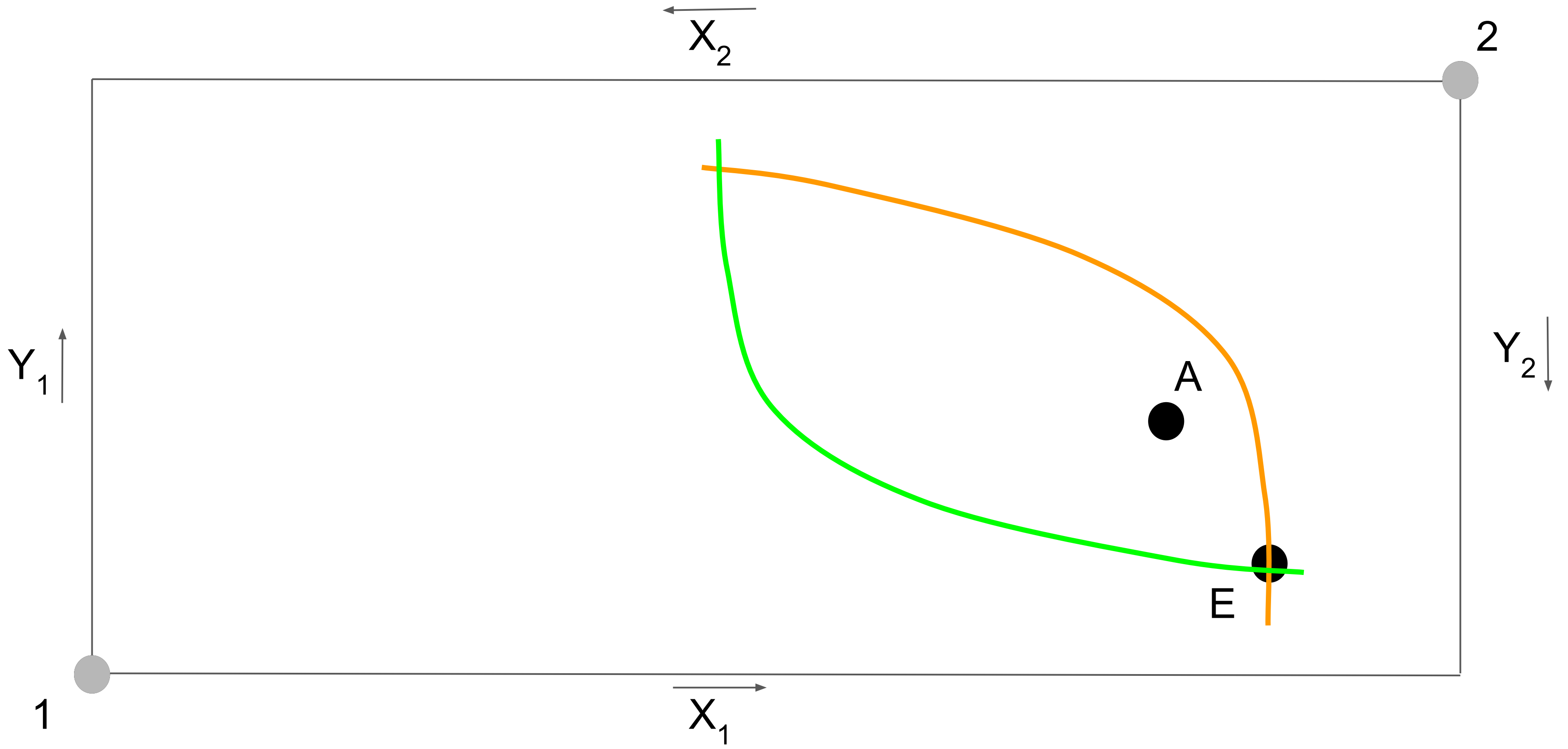

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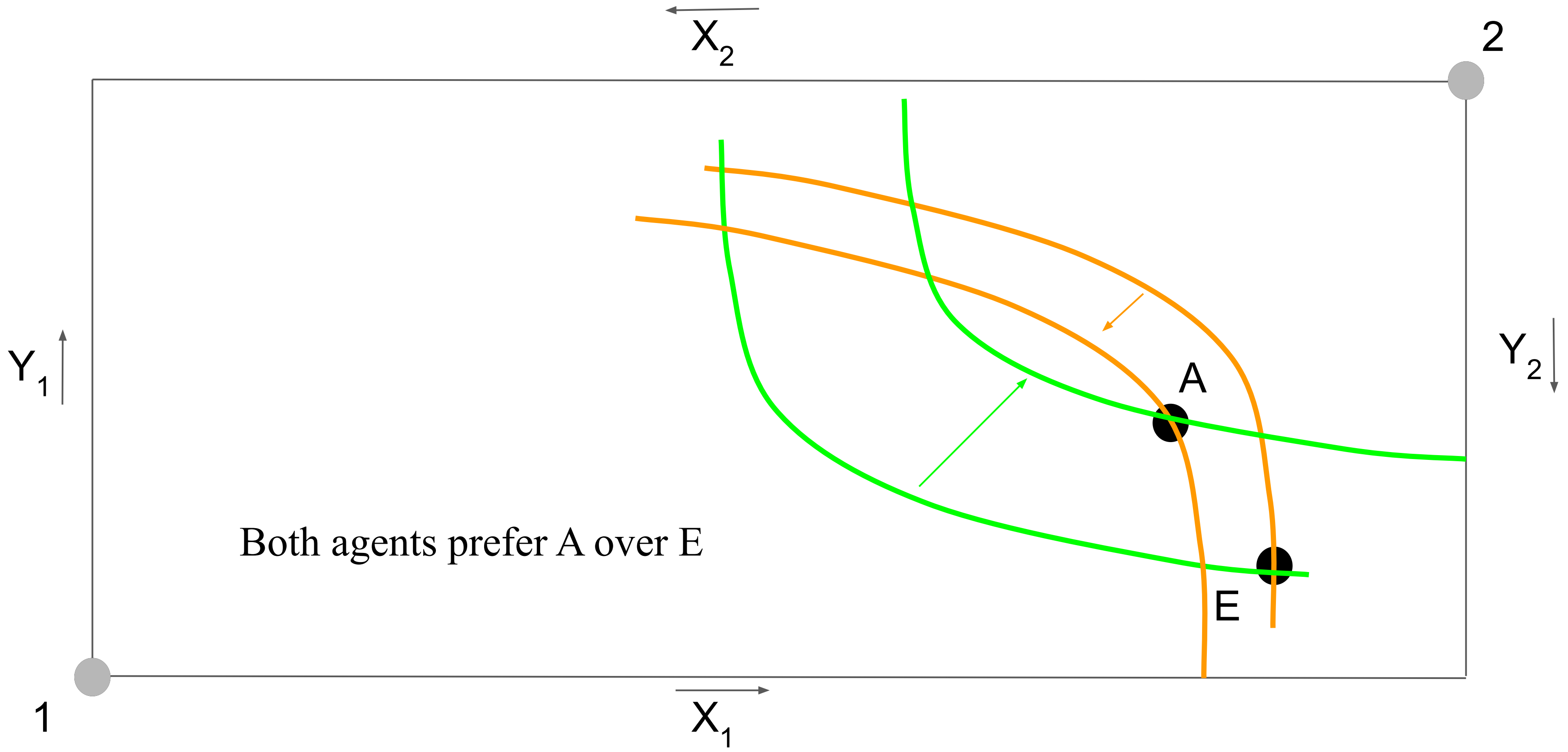
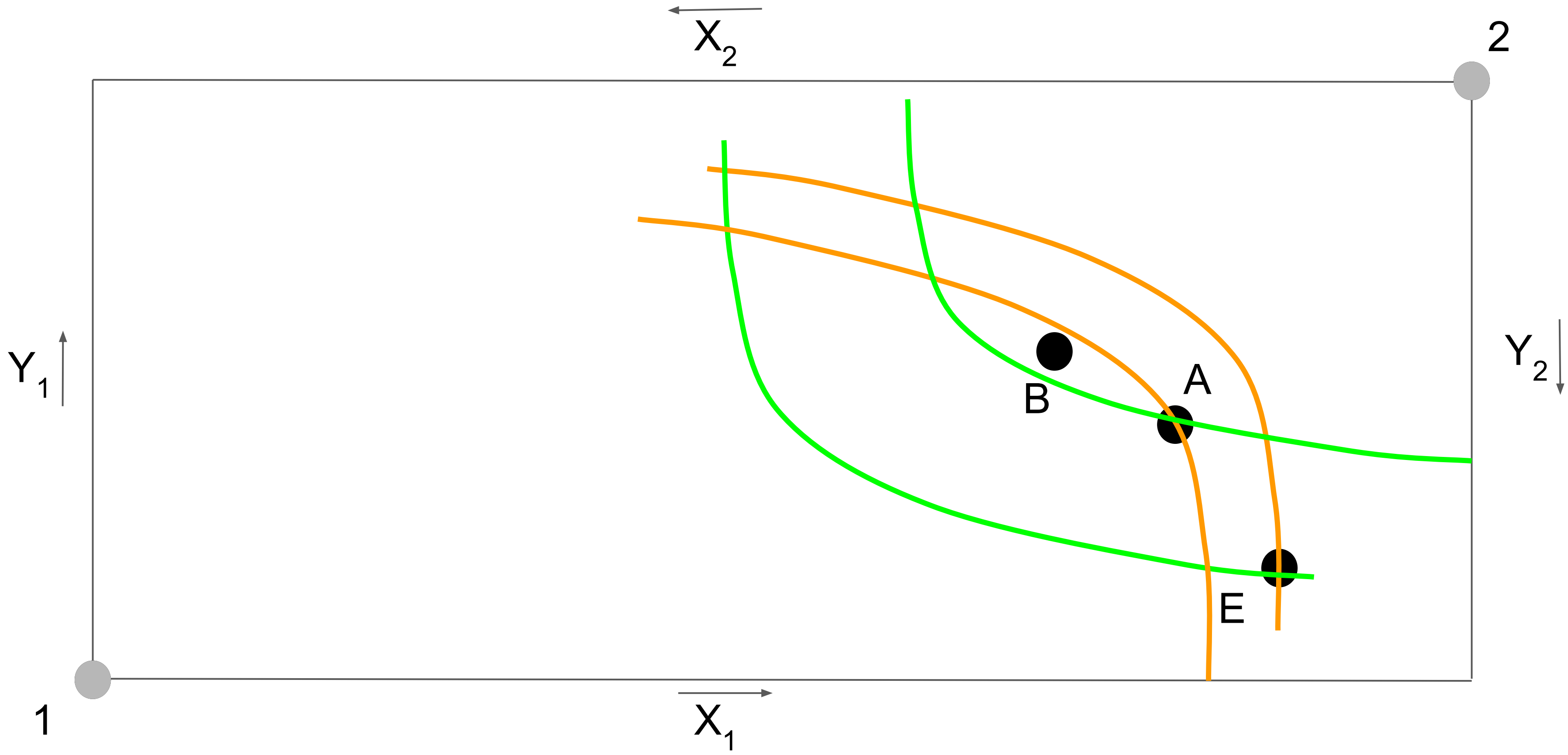



Both agents prefer A over E

# Edgeworth Box



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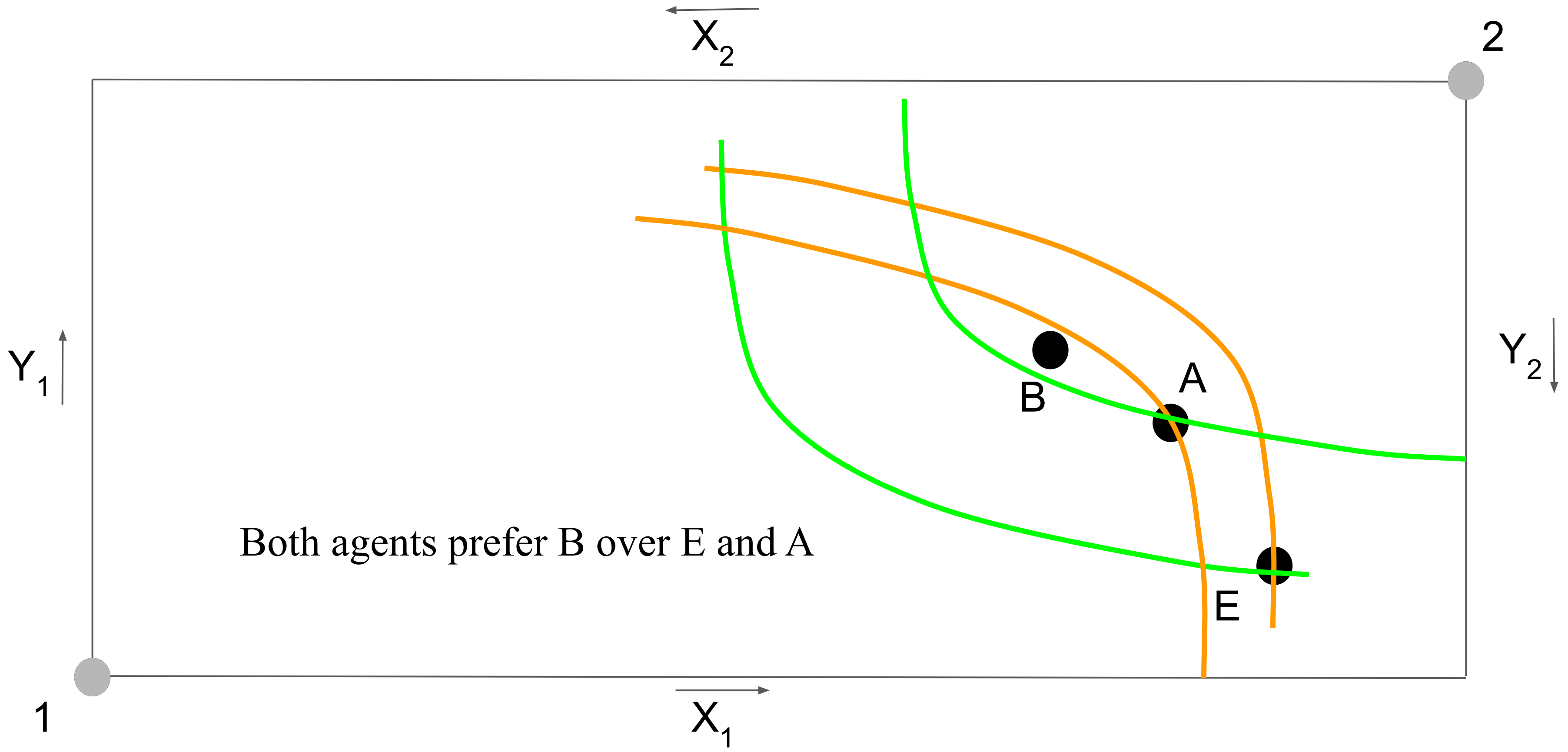
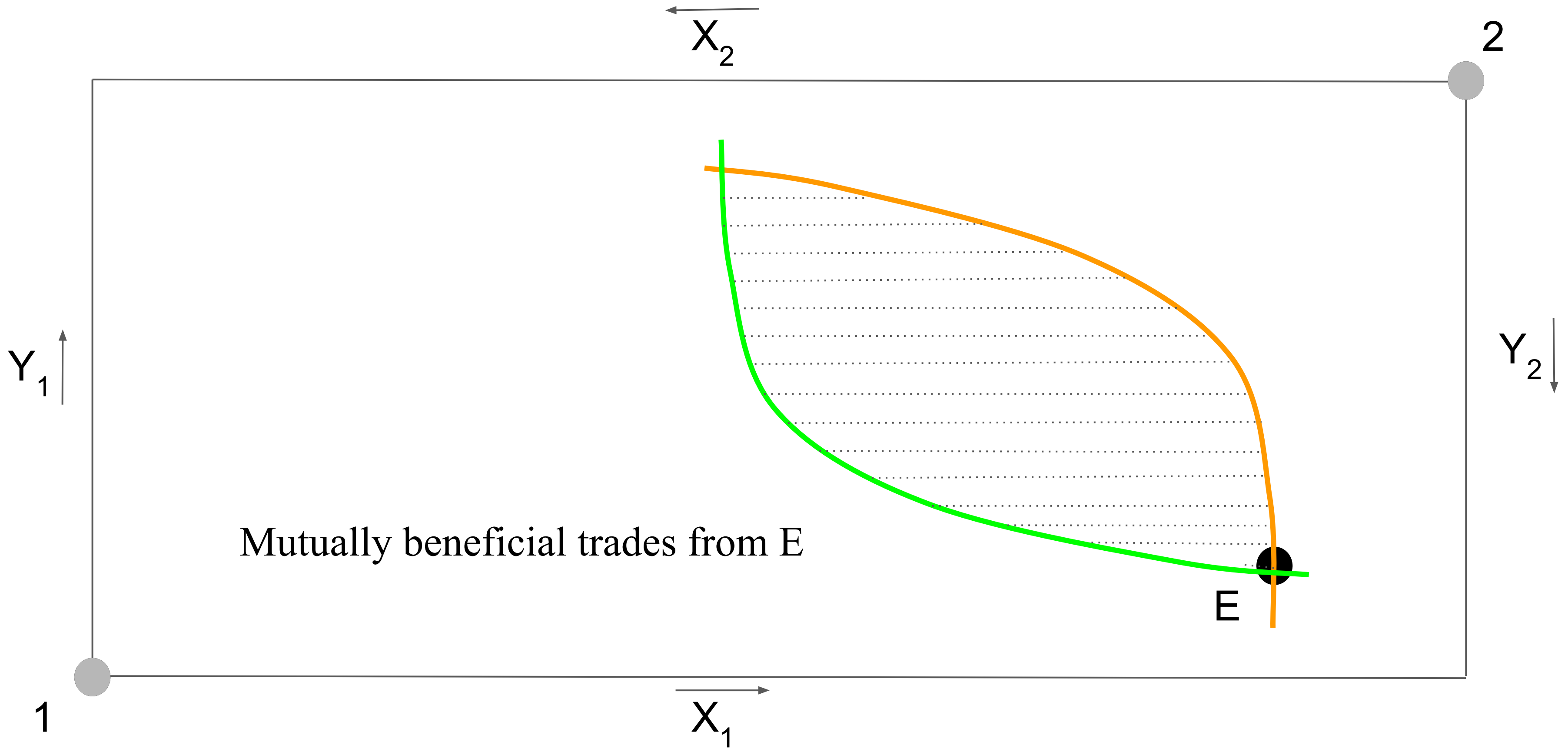


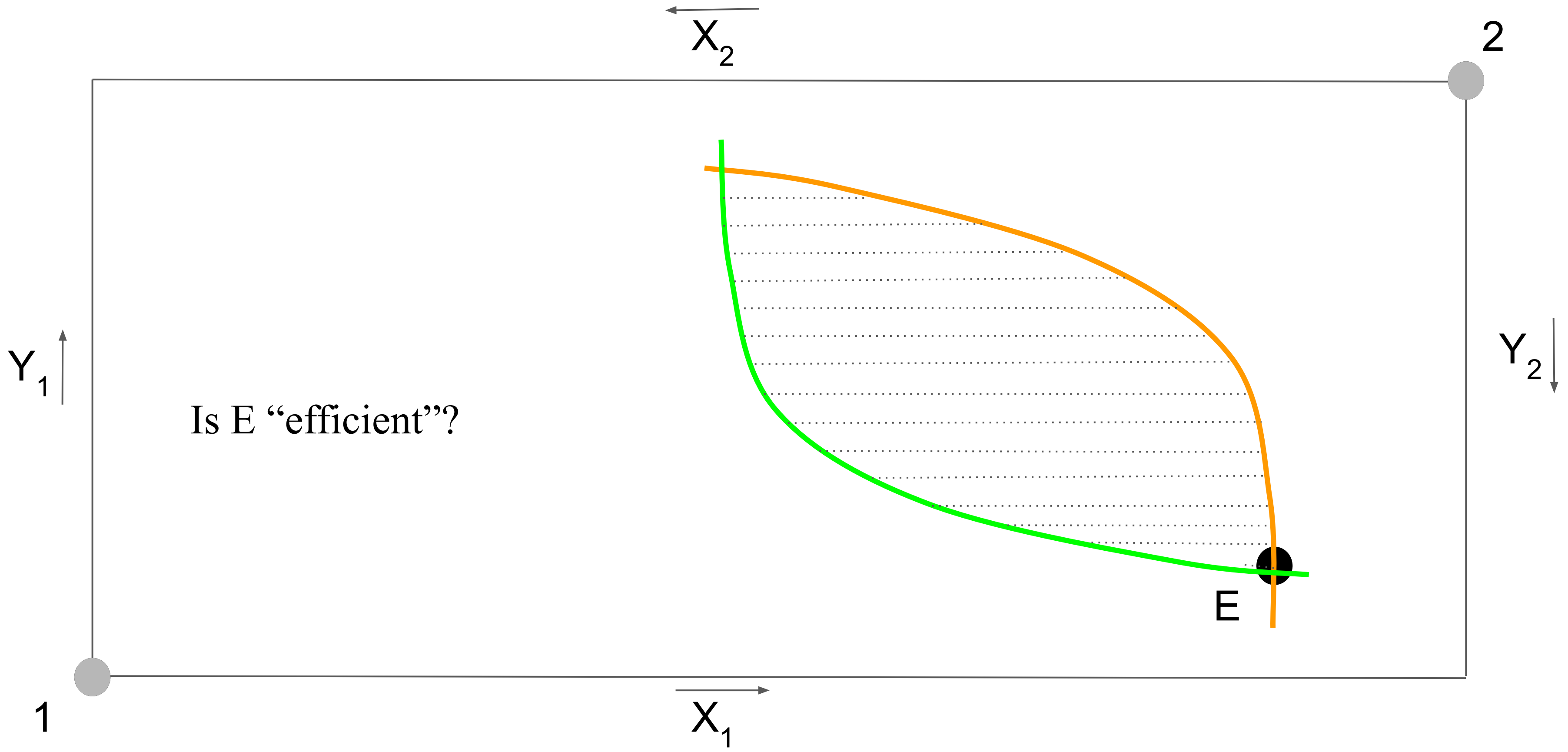

Both agents prefer B over E and A

# Edgeworth Box




Mutually beneficial trades from E

# Edgeworth Box



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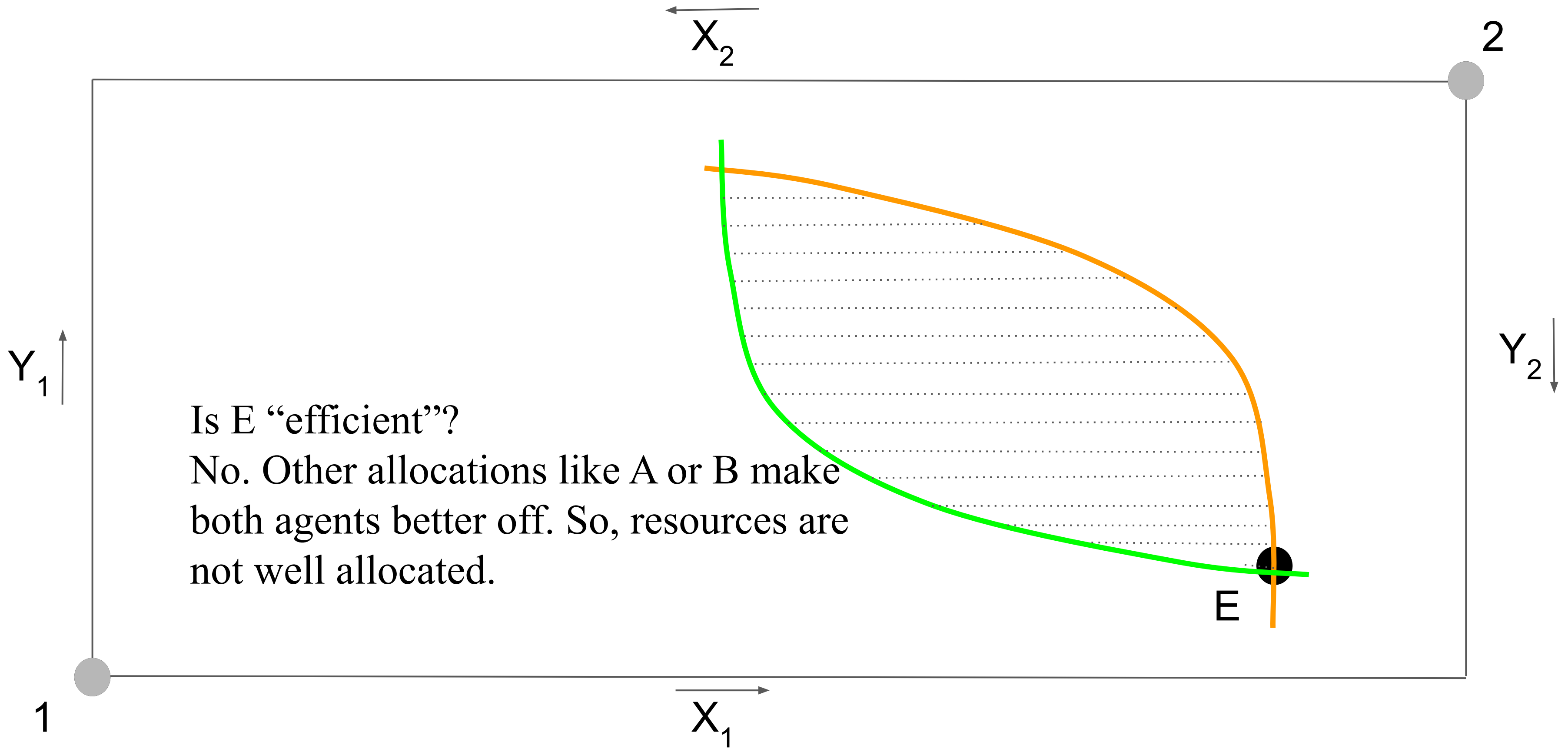
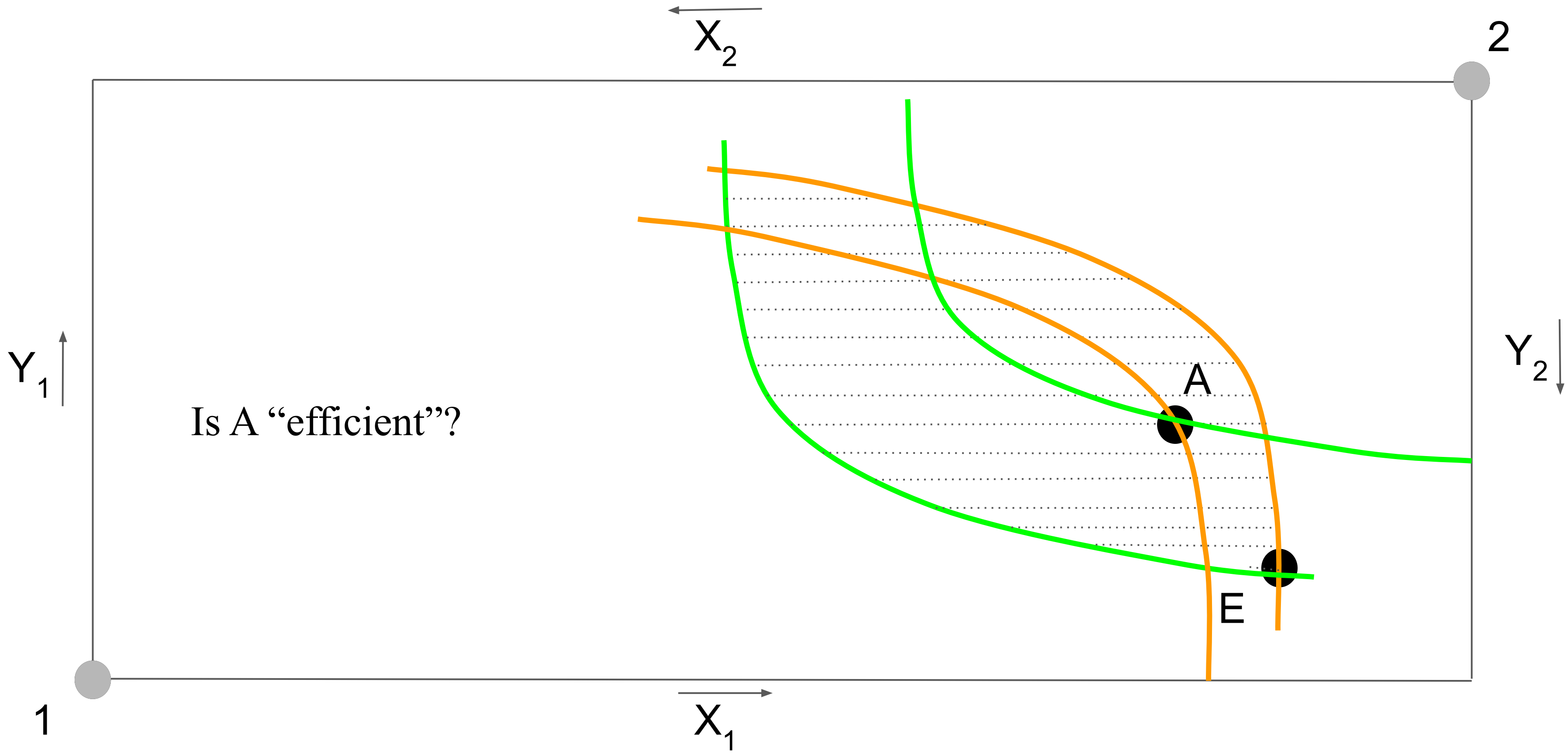


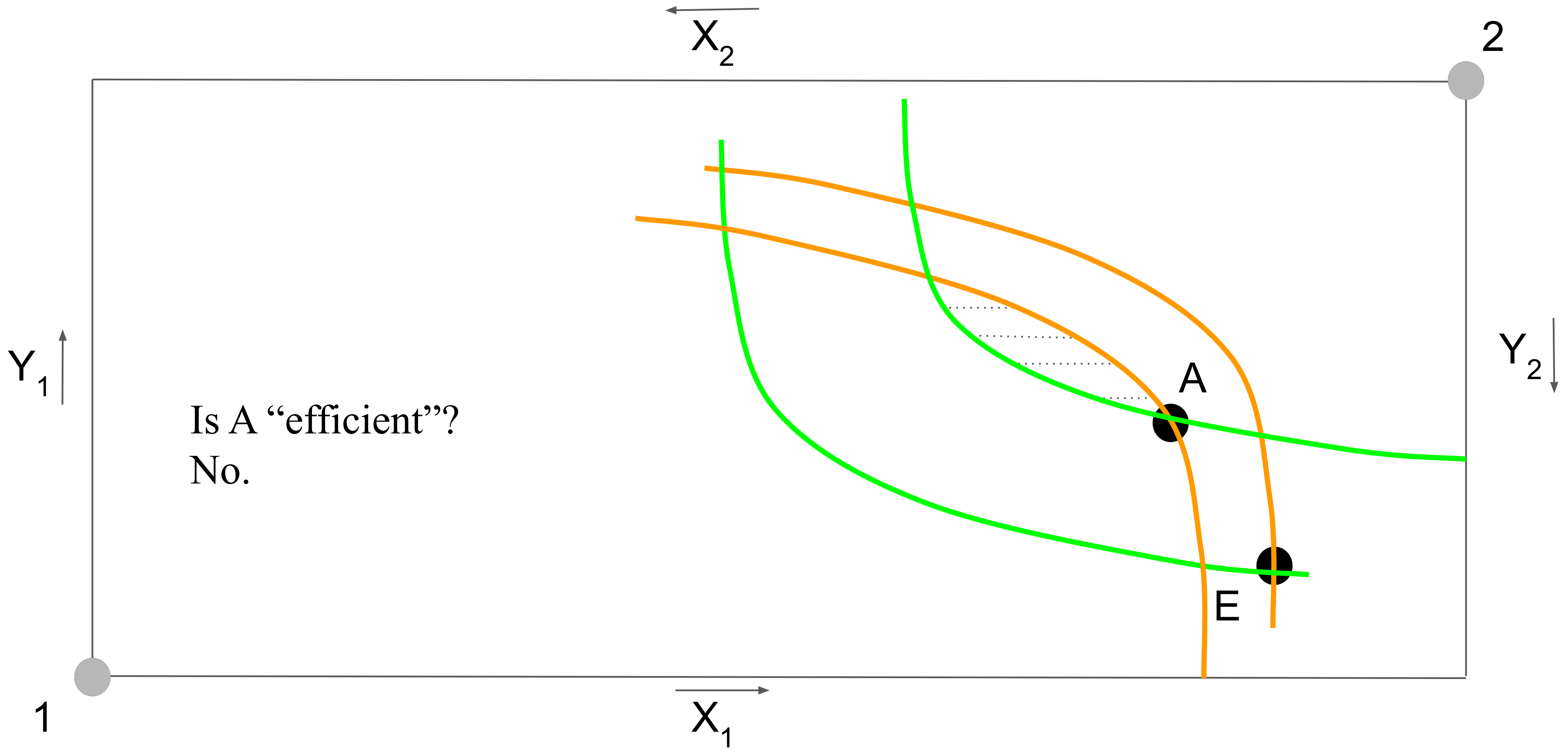

Is E “efficient”?
No. Other allocations like A or B make both agents better off. So, resources are not well allocated.

# Edgeworth Box



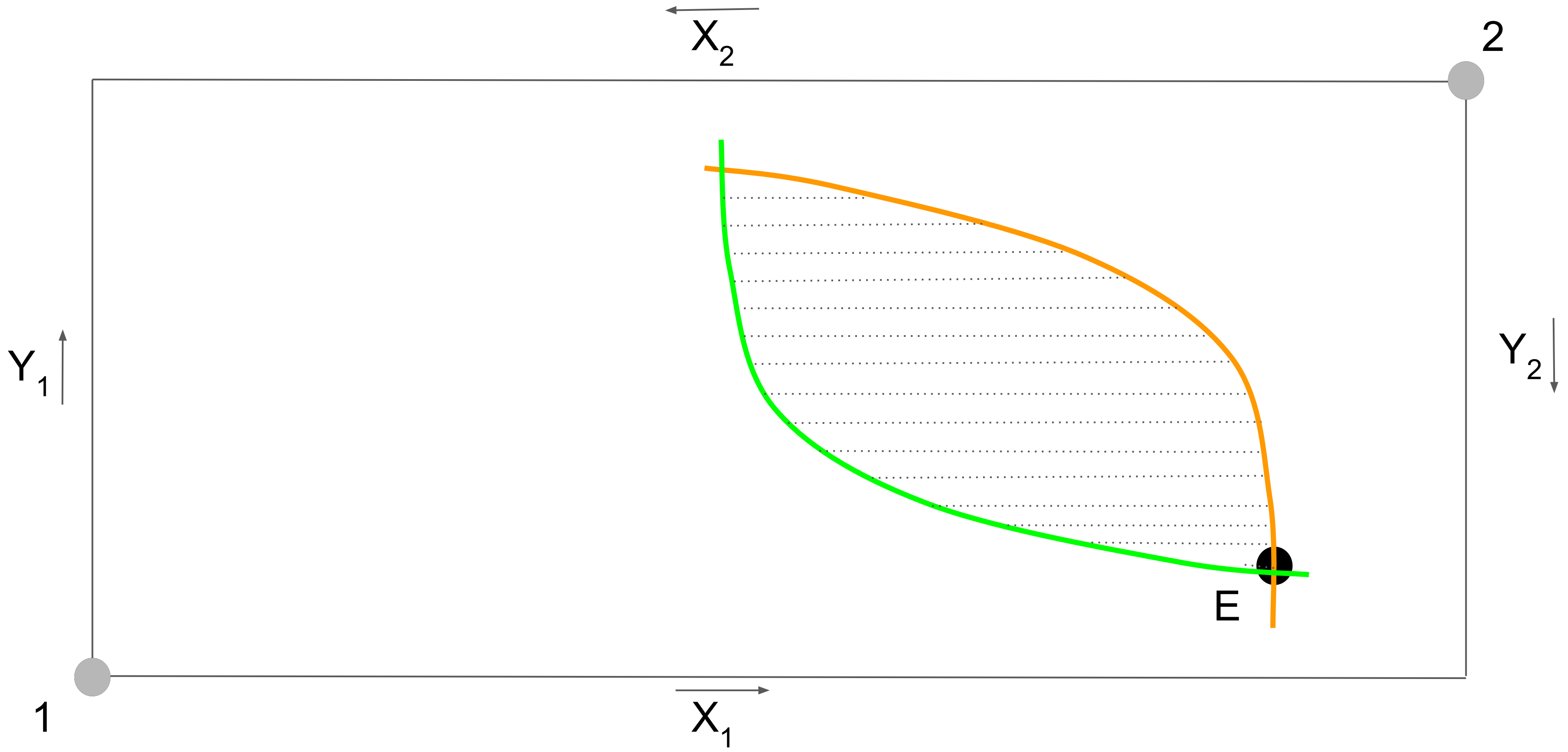

Is A “efficient”?

# Edgeworth Box



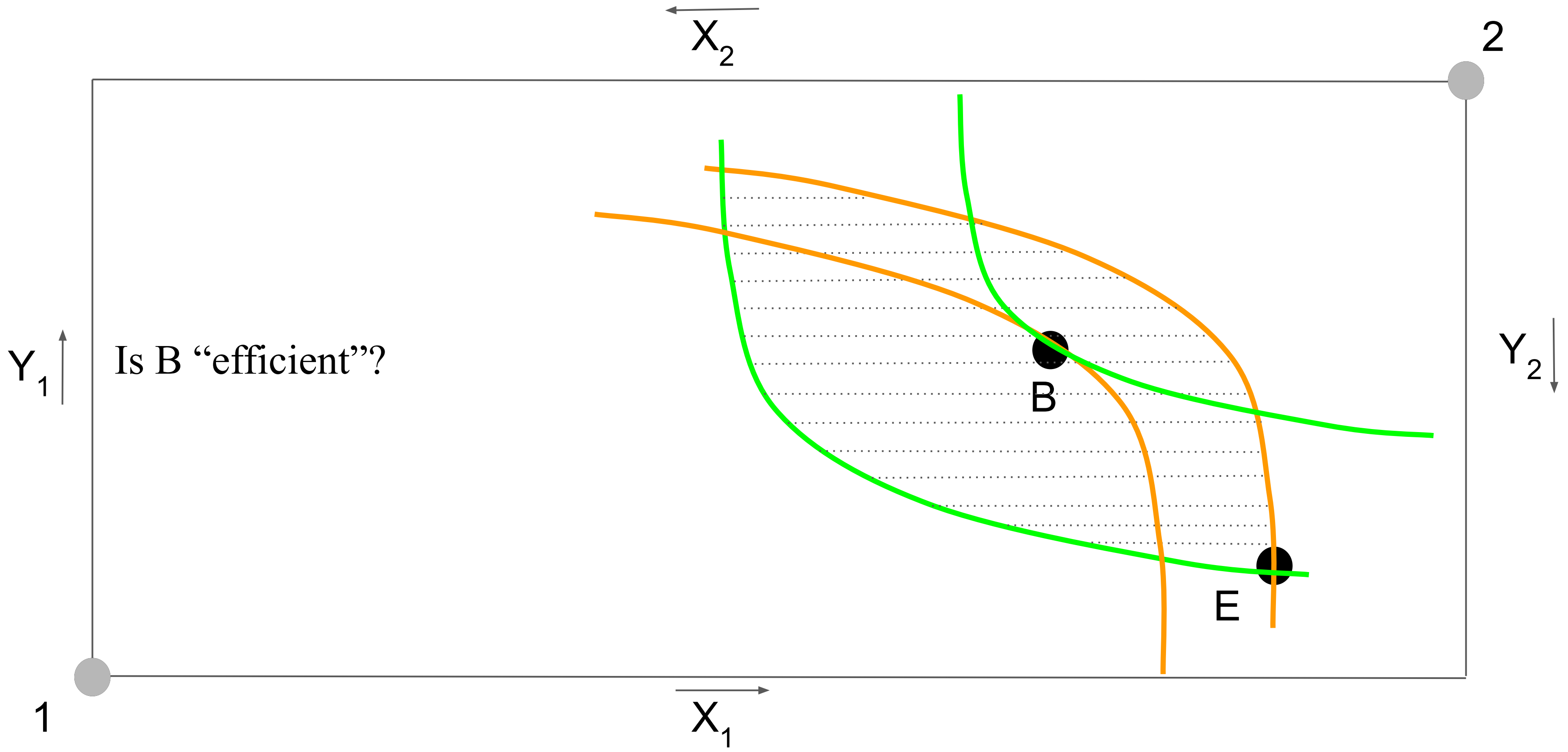

Is A "efficient"?
No.

# Edgeworth Box



# Edgeworth Box

Is B "efficient"?



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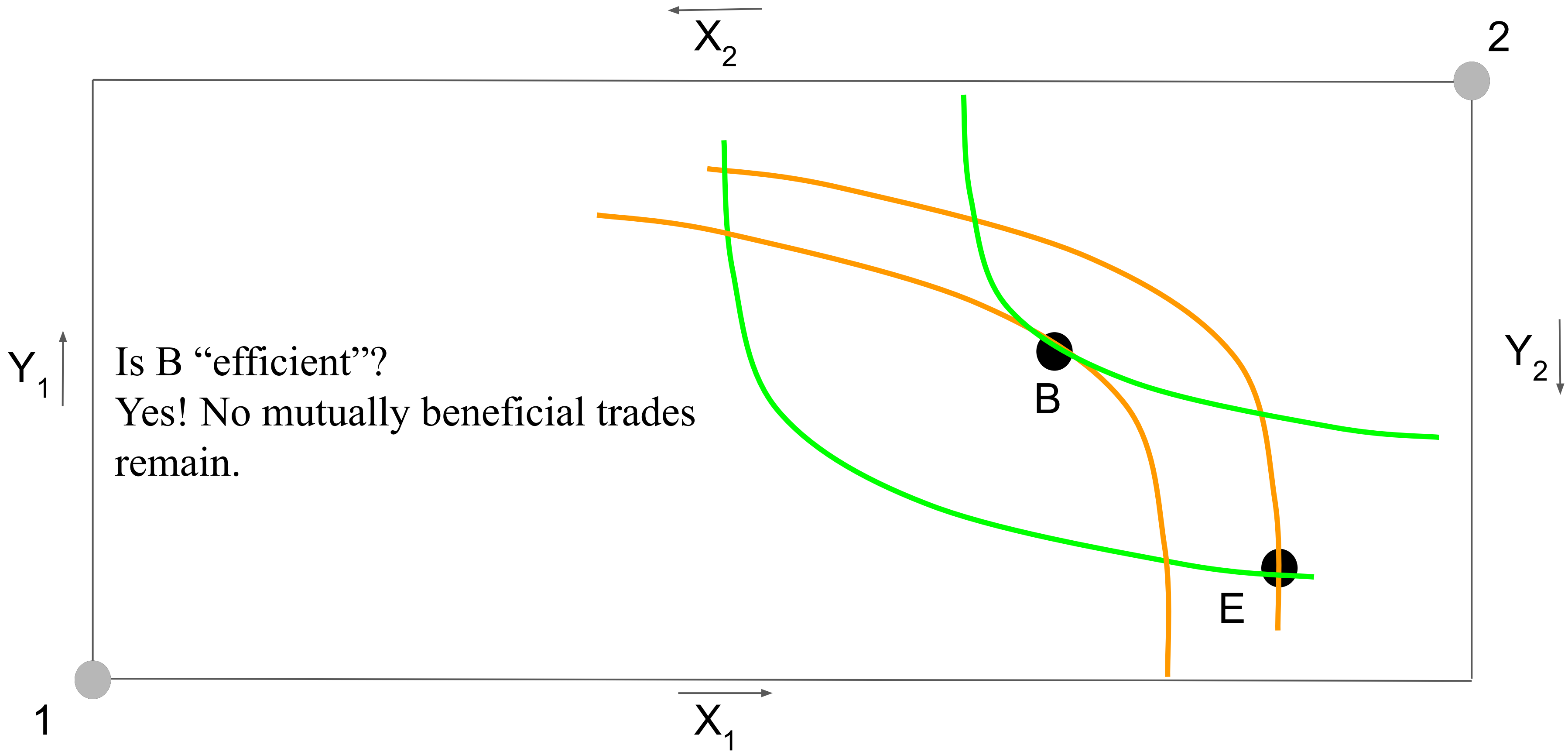
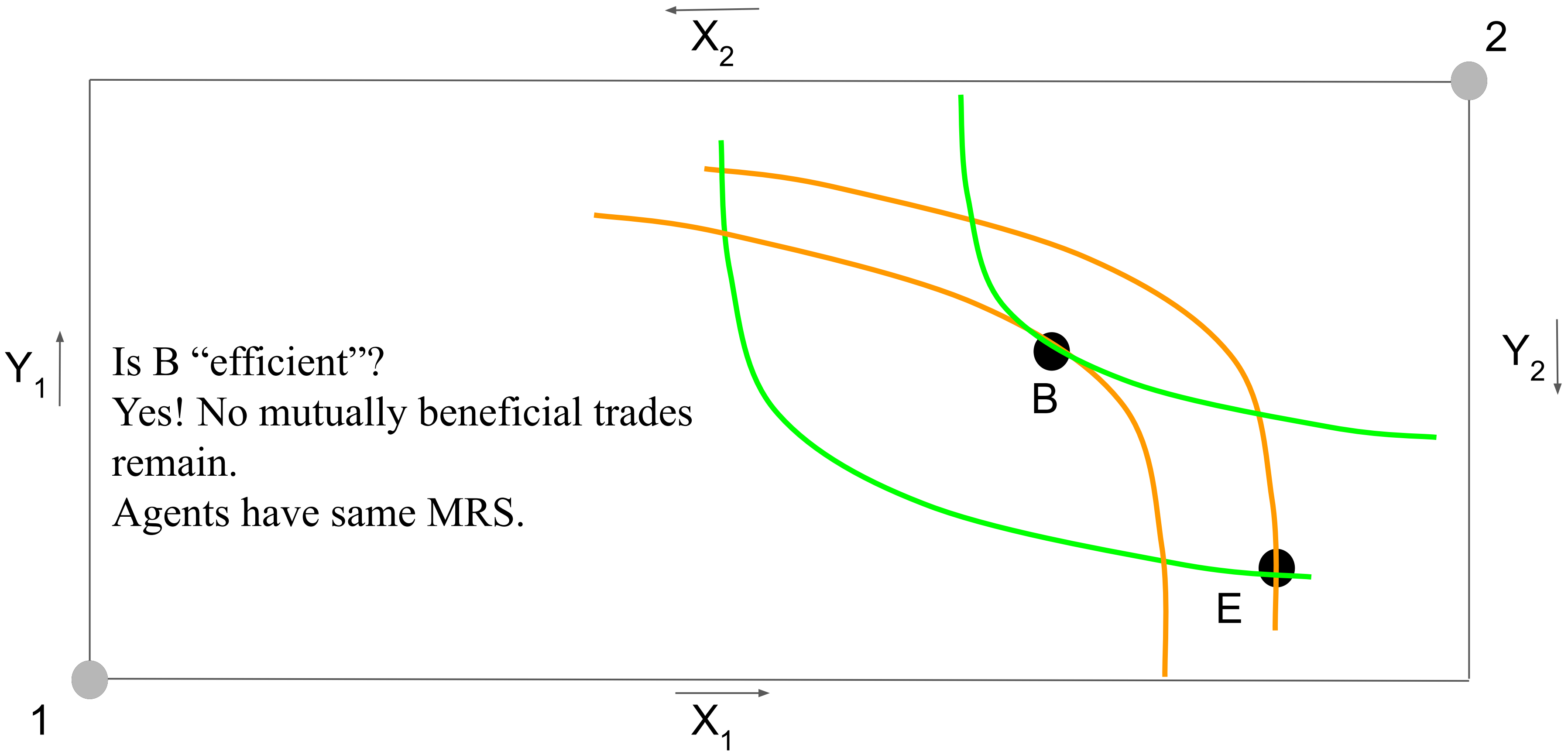



Is B "efficient"?
Yes! No mutually beneficial trades remain.

# Edgeworth Box

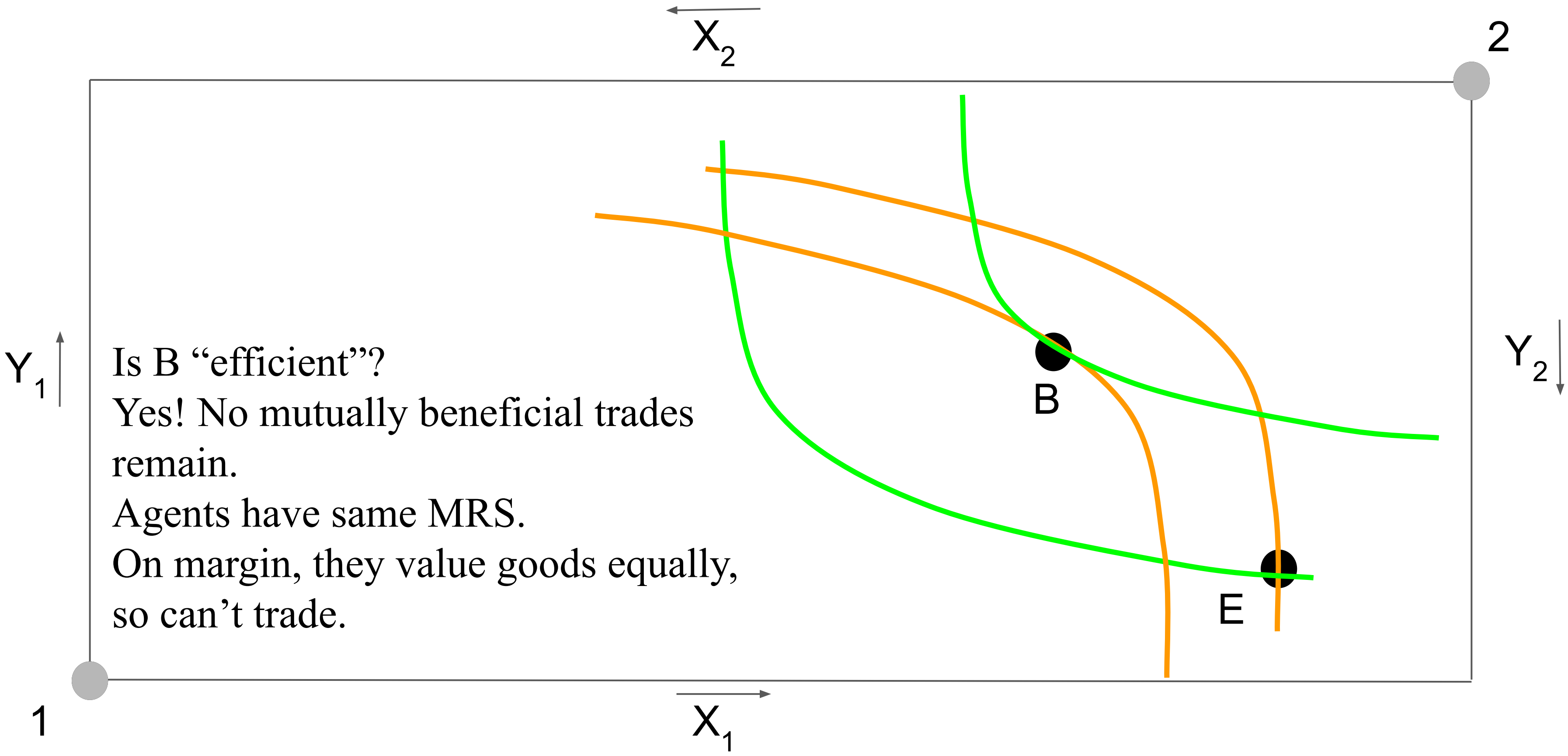



Is B “efficient”?

Yes! No mutually beneficial trades remain.

Agents have same MRS.

# Edgeworth Box



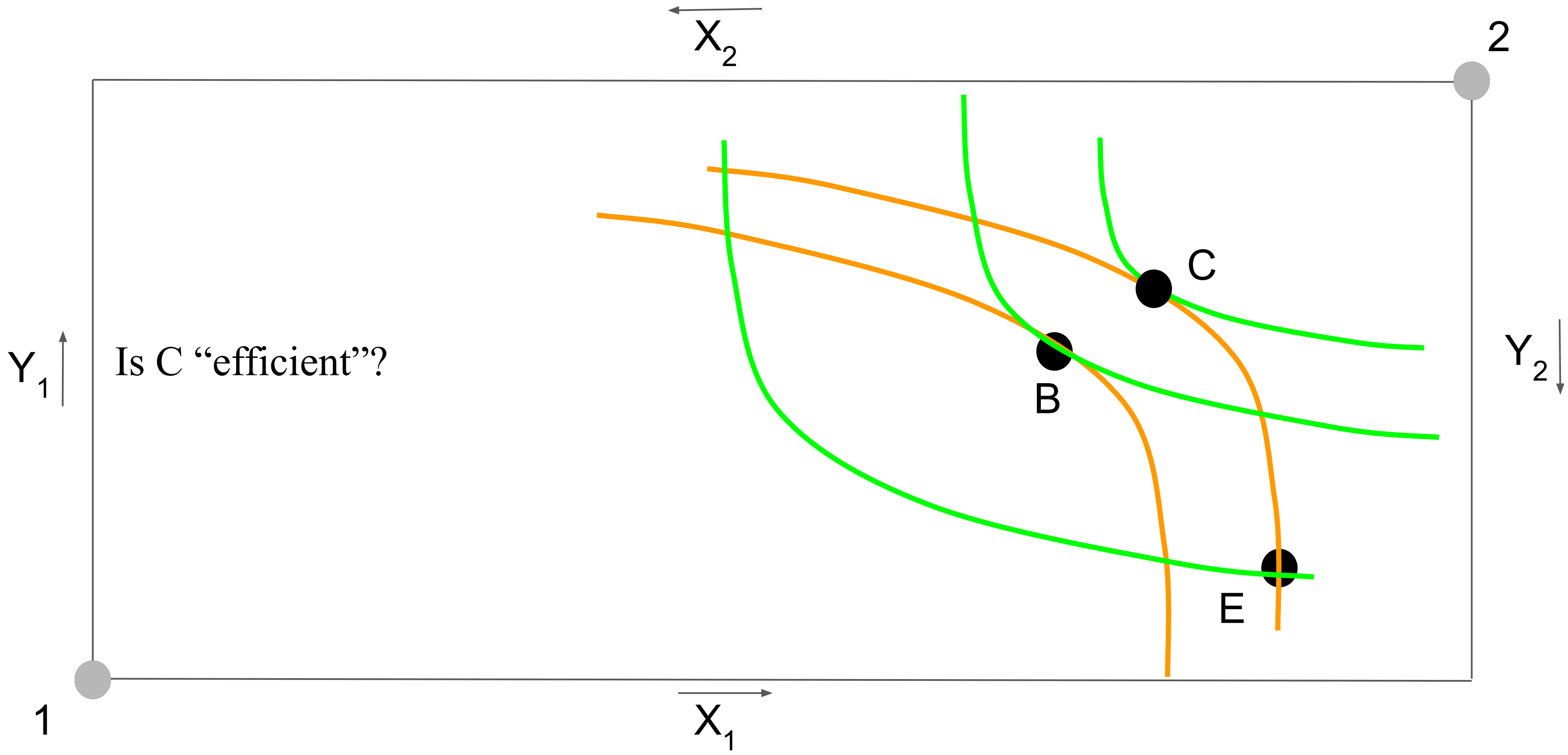

Is B “efficient”?

Yes! No mutually beneficial trades remain.

Agents have same MRS.

On margin, they value goods equally, so can’t trade.

# Edgeworth Box



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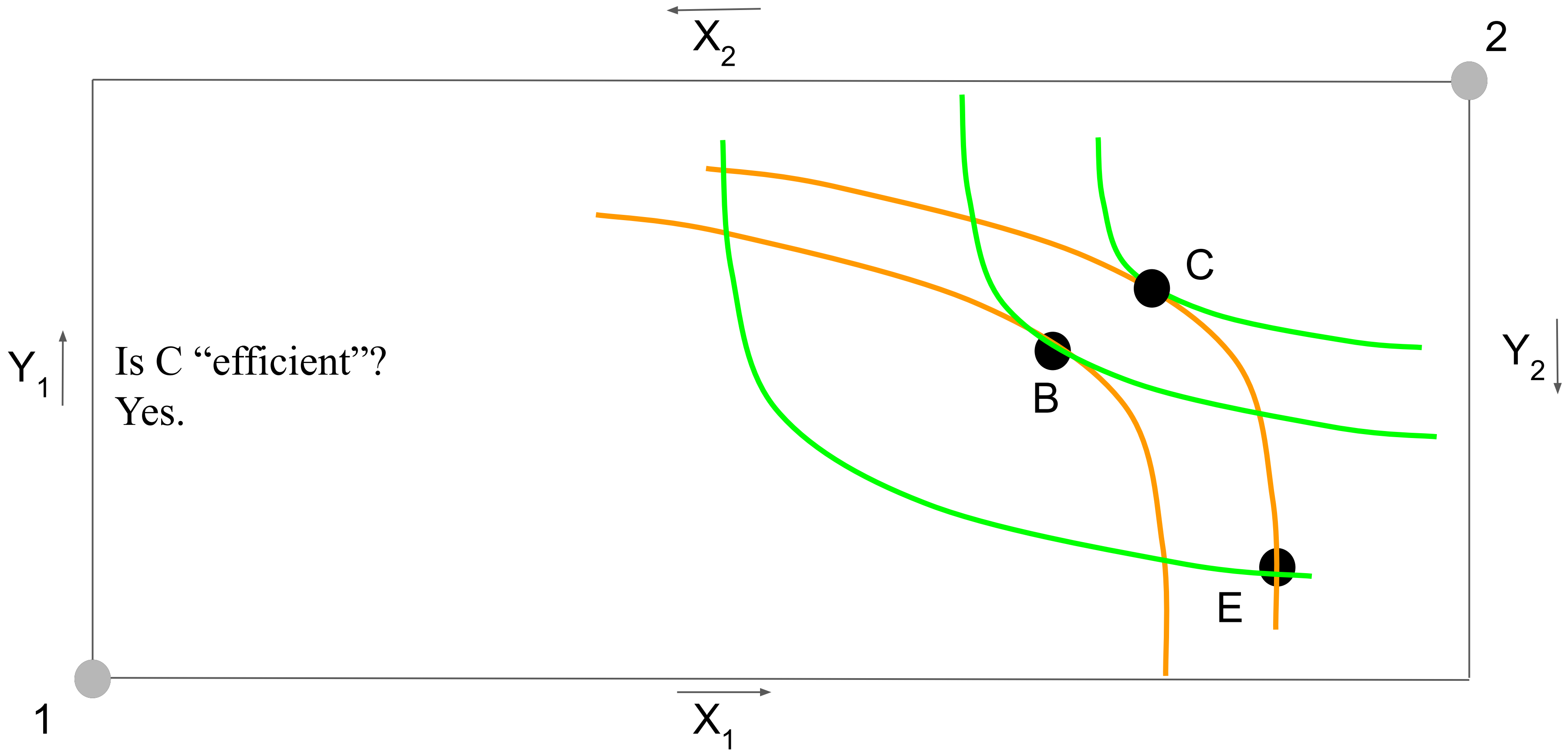
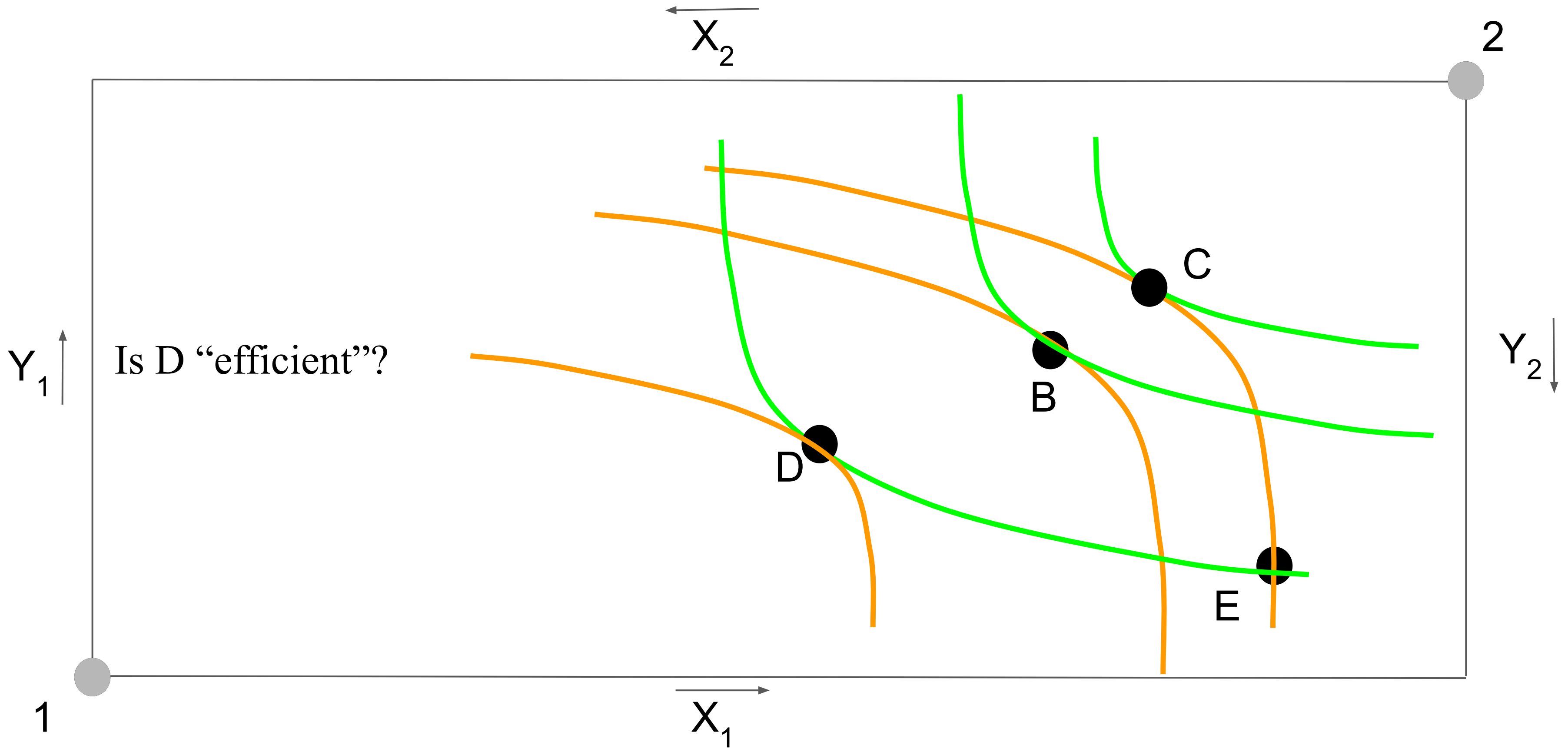



Is C “efficient”?
Yes.

# Edgeworth Box



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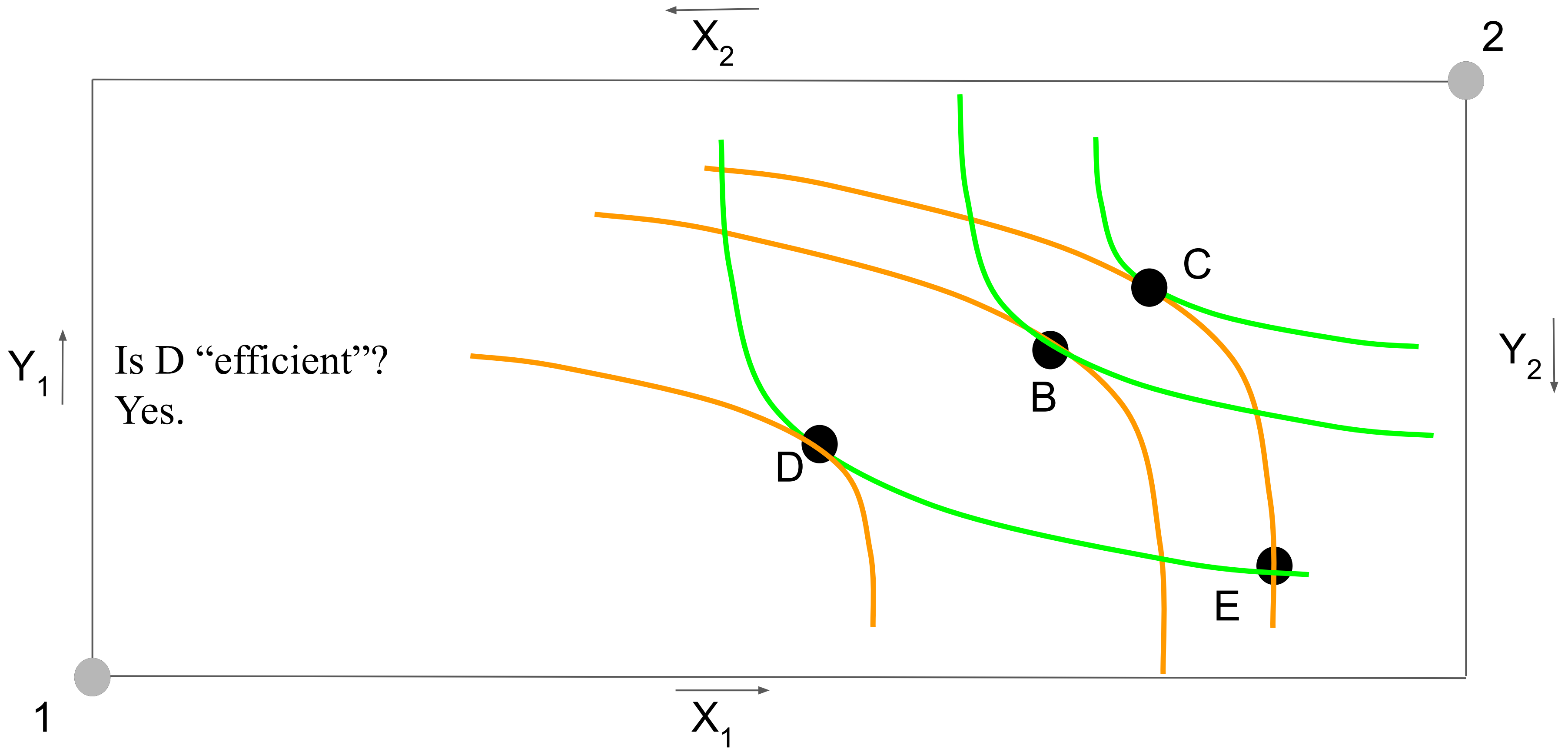

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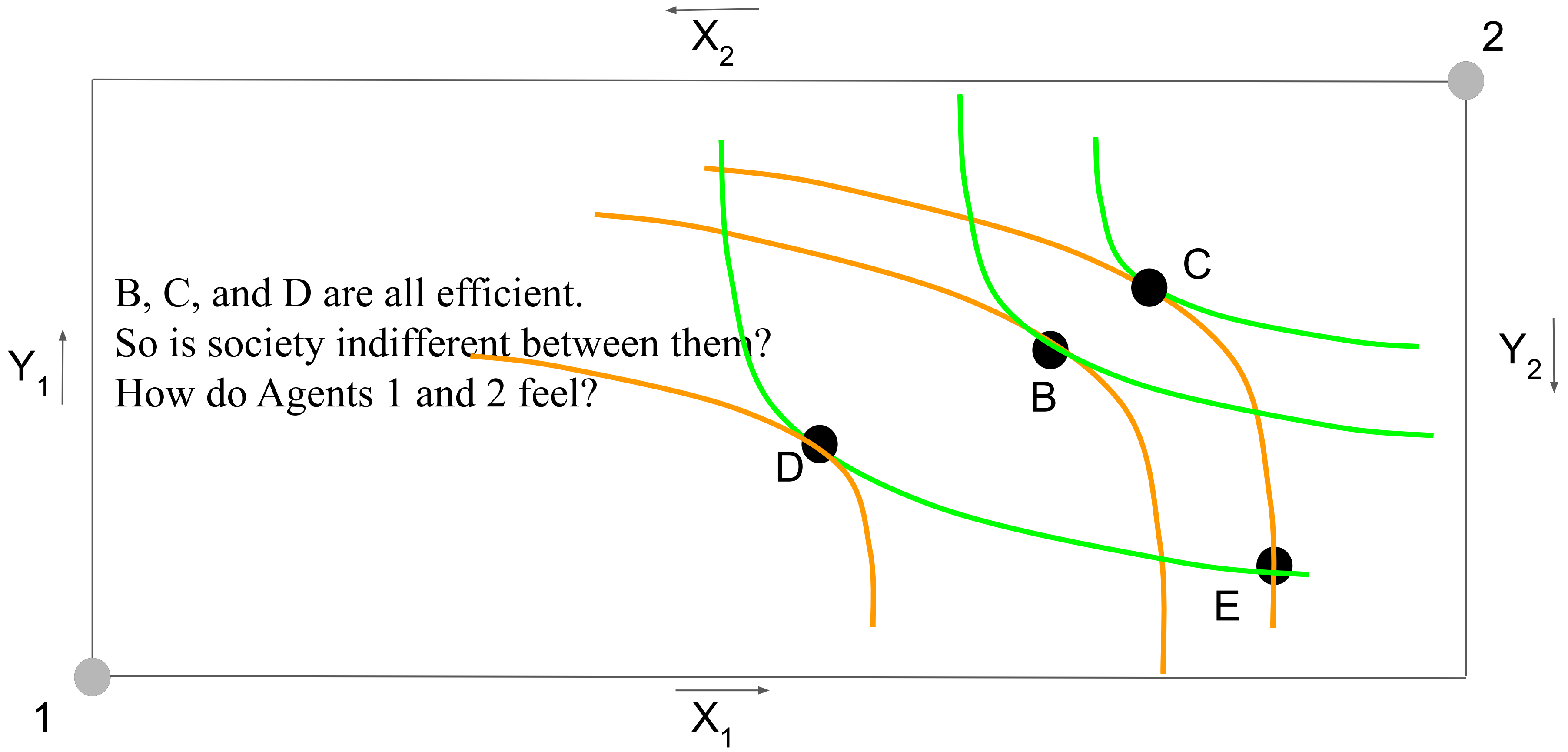
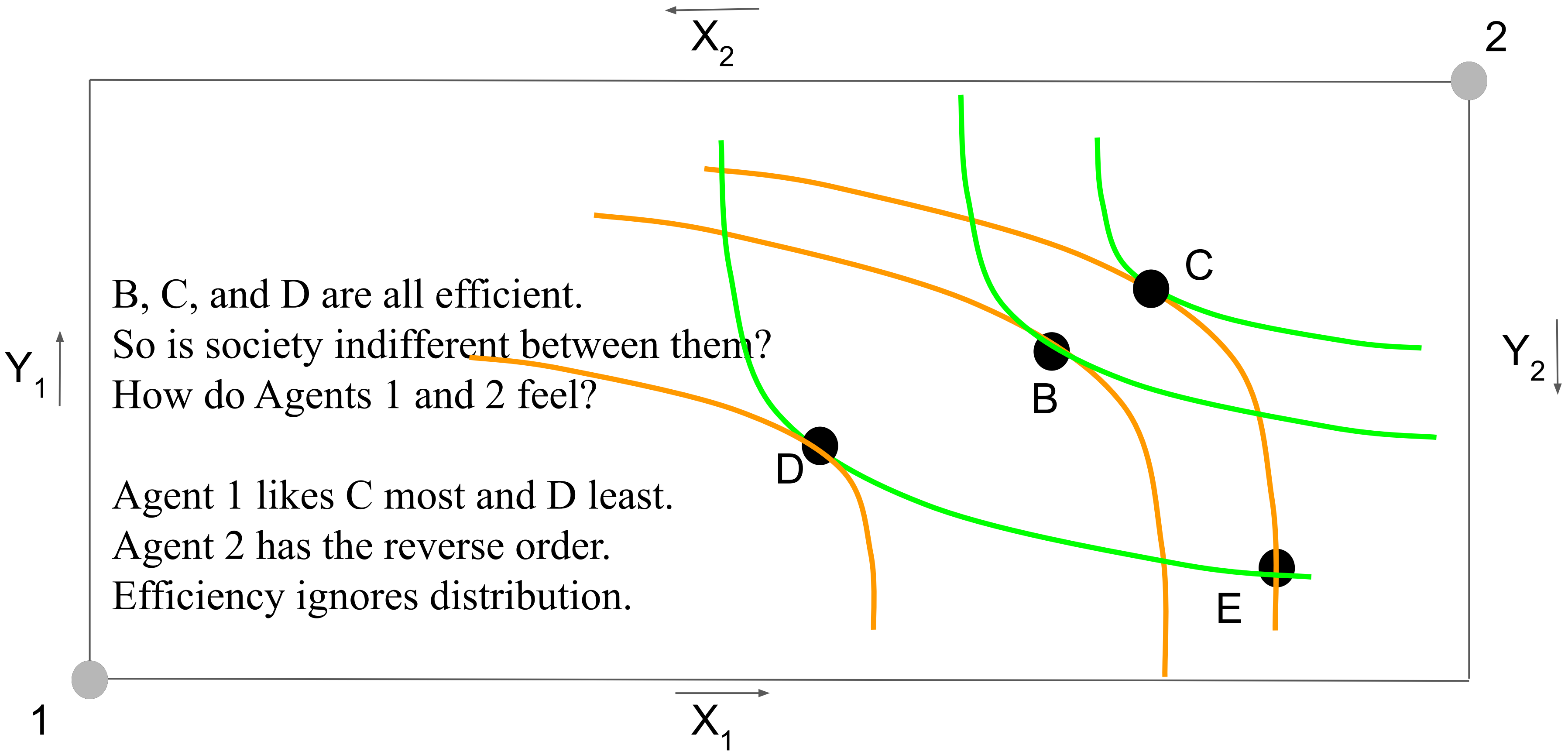



B, C, and D are all efficient.
So is society indifferent between them?
How do Agents 1 and 2 feel?

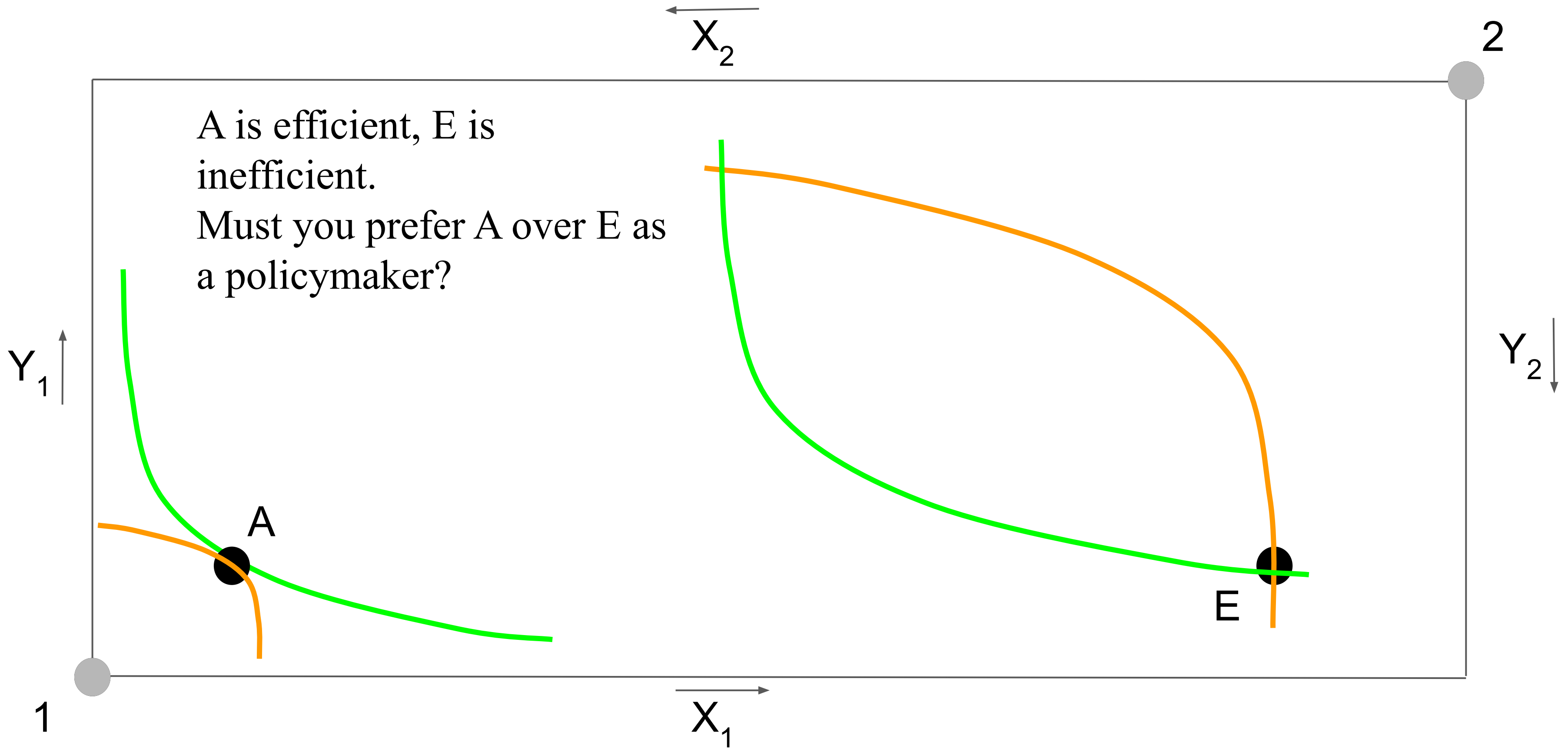
# Edgeworth Box



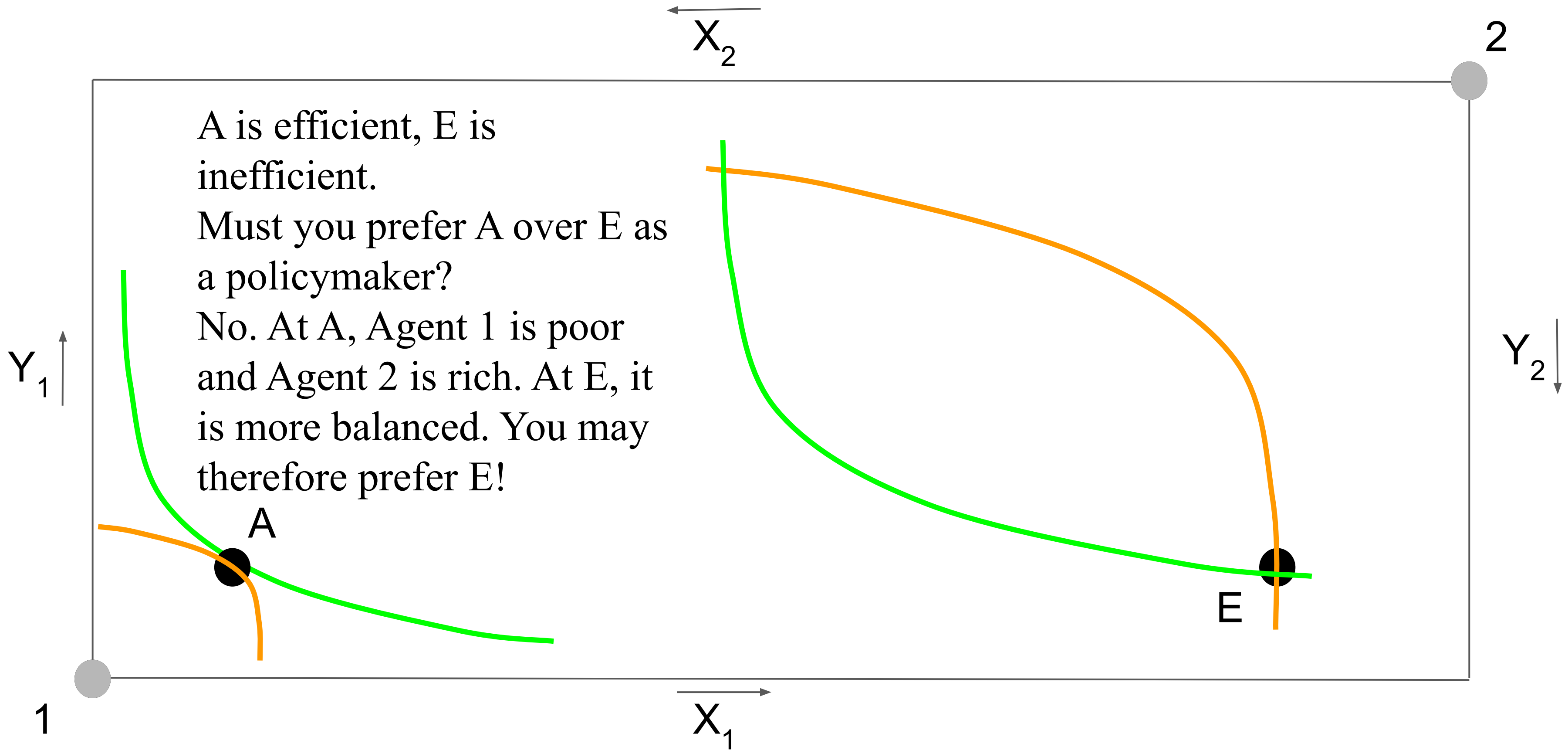

B, C, and D are all efficient.
So is society indifferent between them?
How do Agents 1 and 2 feel?

Agent 1 likes C most and D least.
Agent 2 has the reverse order.
Efficiency ignores distribution.

# Edgeworth Box



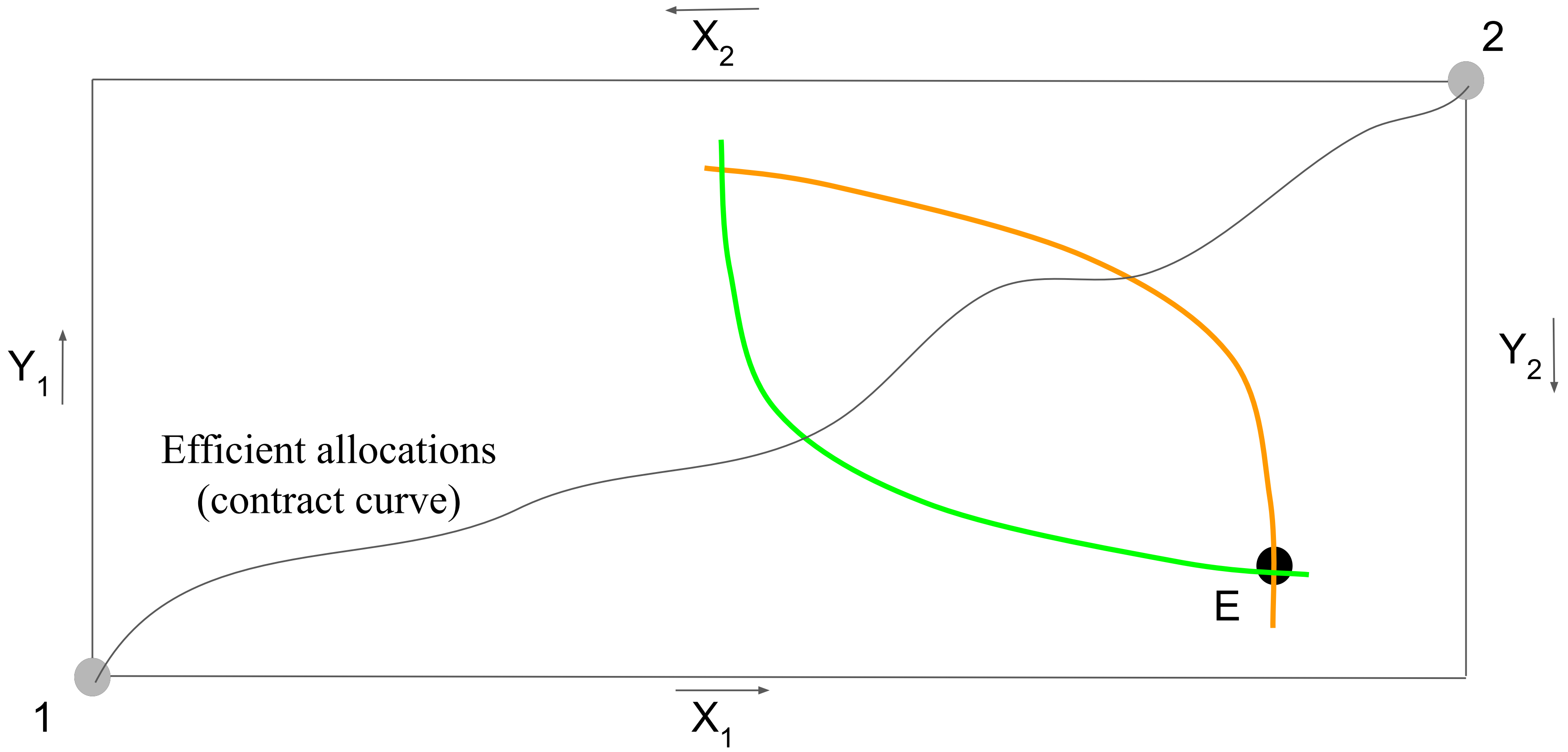
# Edgeworth Box



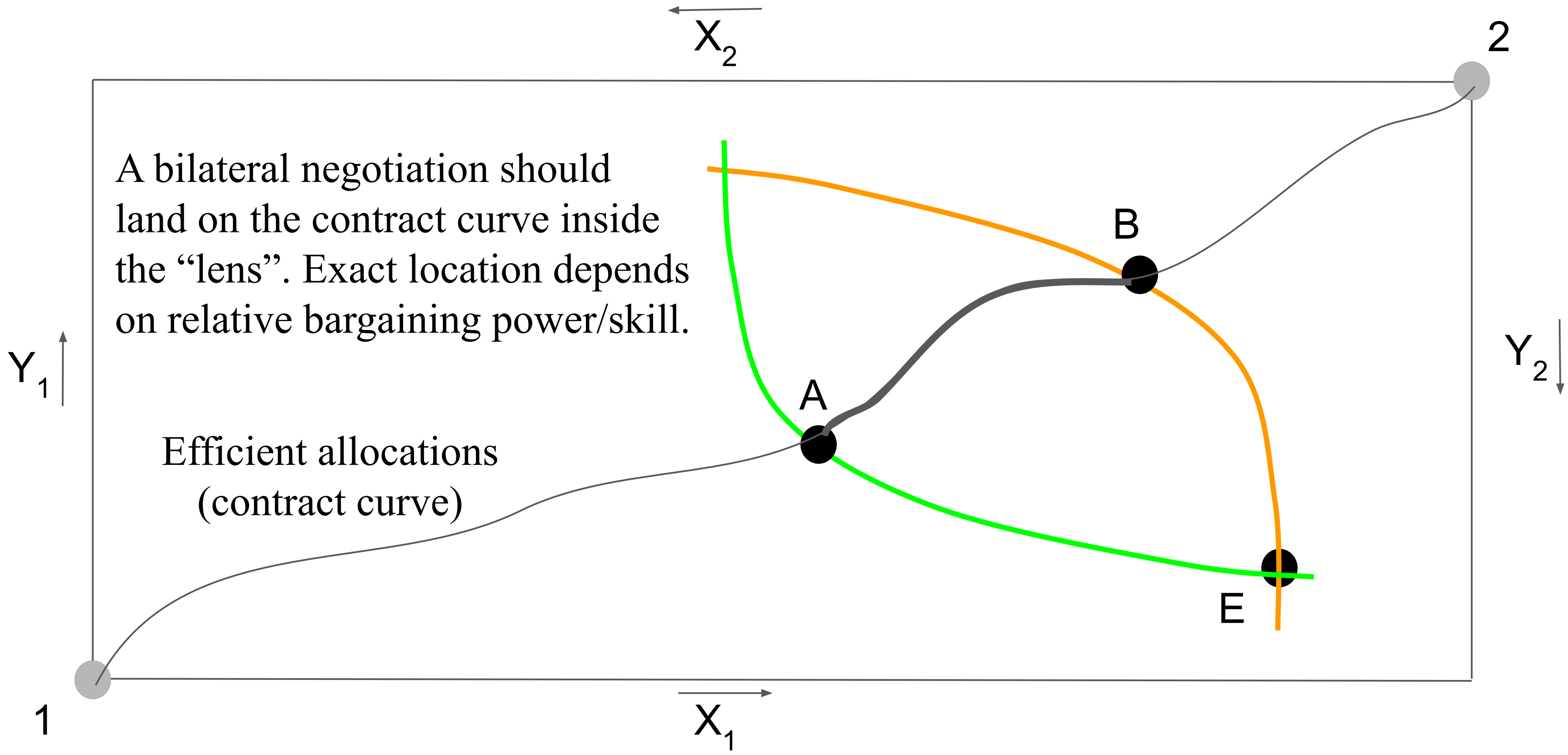
# Efficiency and equity

- Efficiency requires that we be unable to improve both Agents' utilities simultaneously
  - Allocation A is inefficient if there exists some allocation C such that both agents are better off than at A.
  - Allocation B is efficient if there is no allocation C such that both agents are better off.
- An efficient outcome may be undesirable for equity reasons
  - If one agent is very wealthy and the other is very poor, we may not be able to help the latter without harming the former (efficient), but it may be very undesirable
- So if an allocation is inefficient, there is some other allocation that is objectively better (and is efficient)
  - But not every efficient allocation is preferable to a particular inefficient one

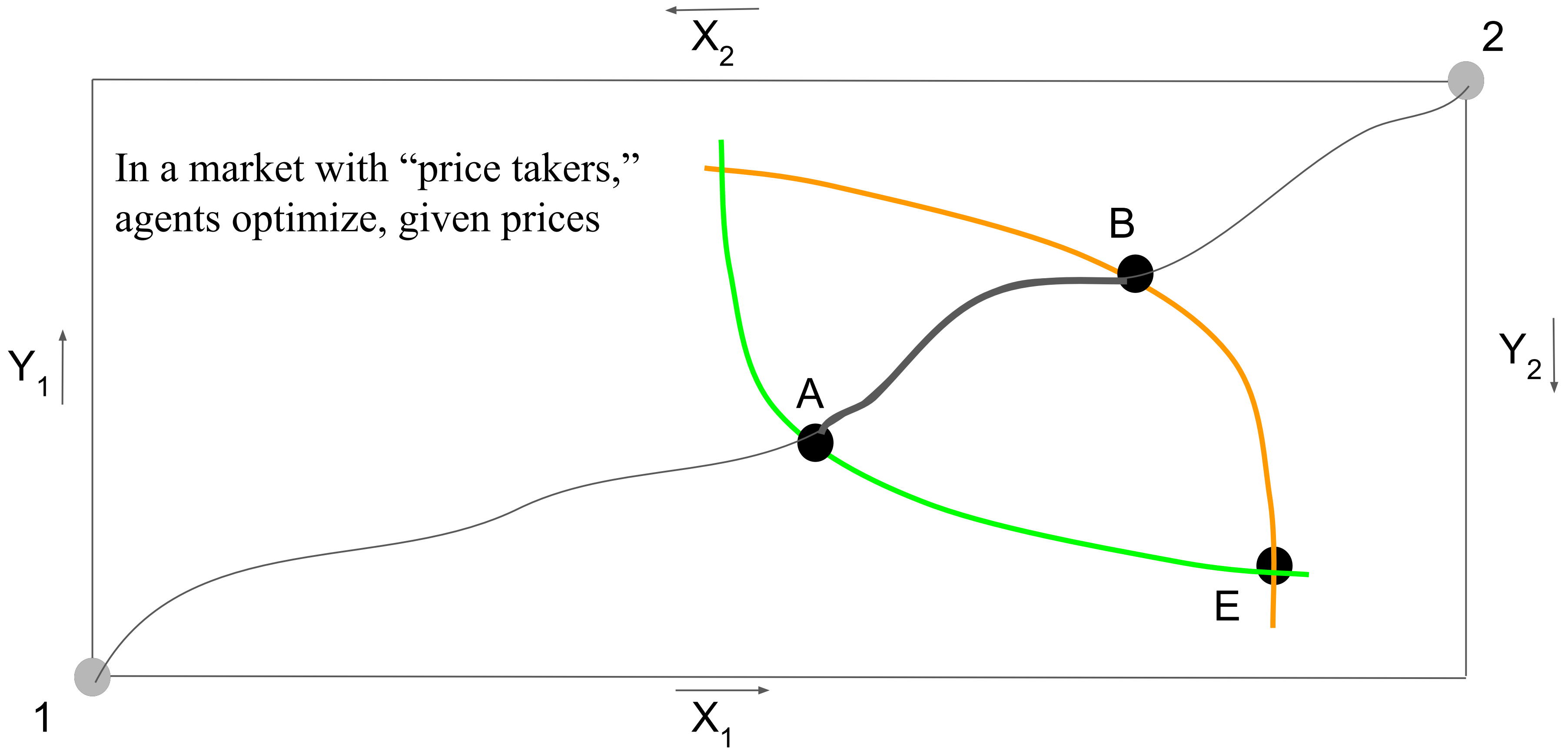
# Contract curve



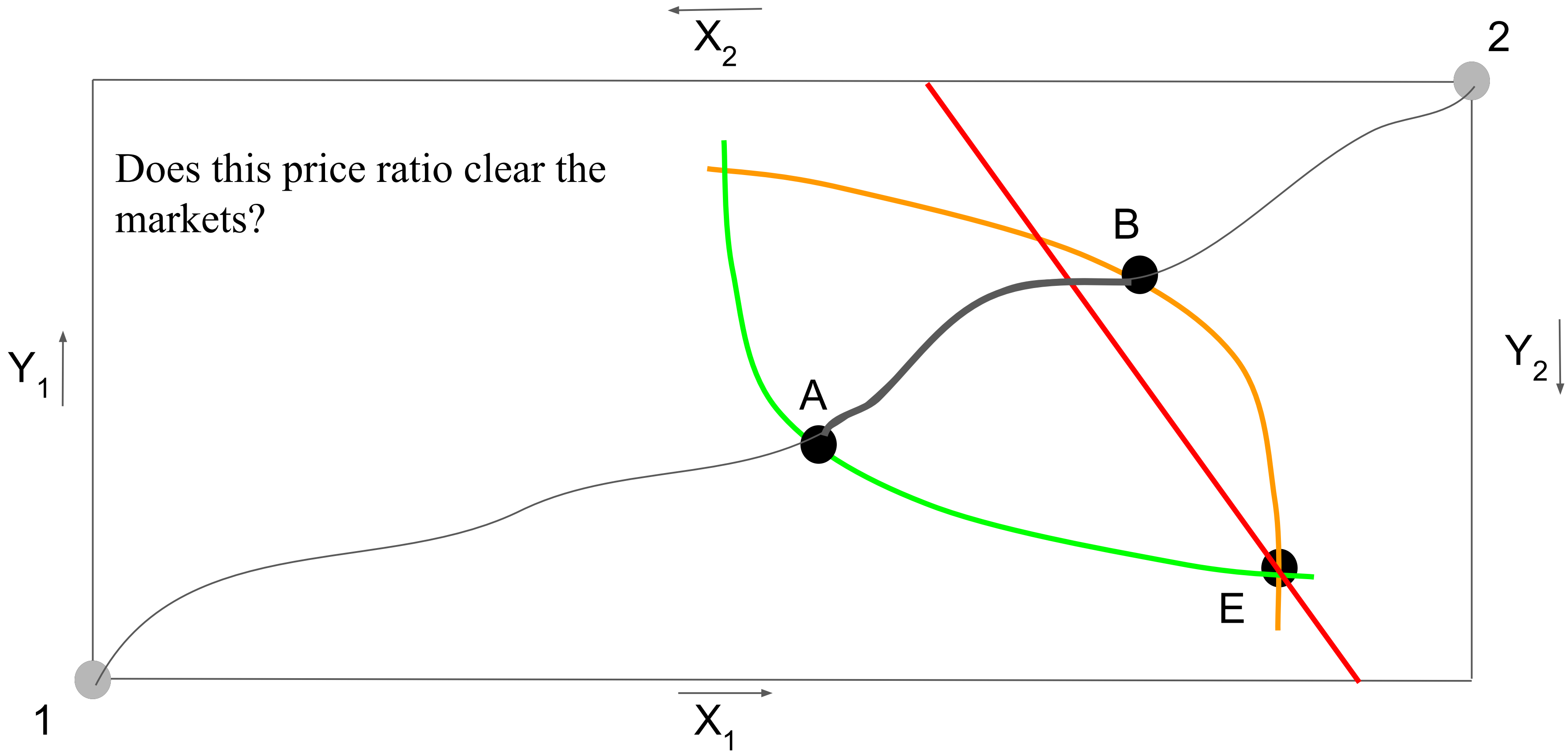
# Contract curve



# Market outcome

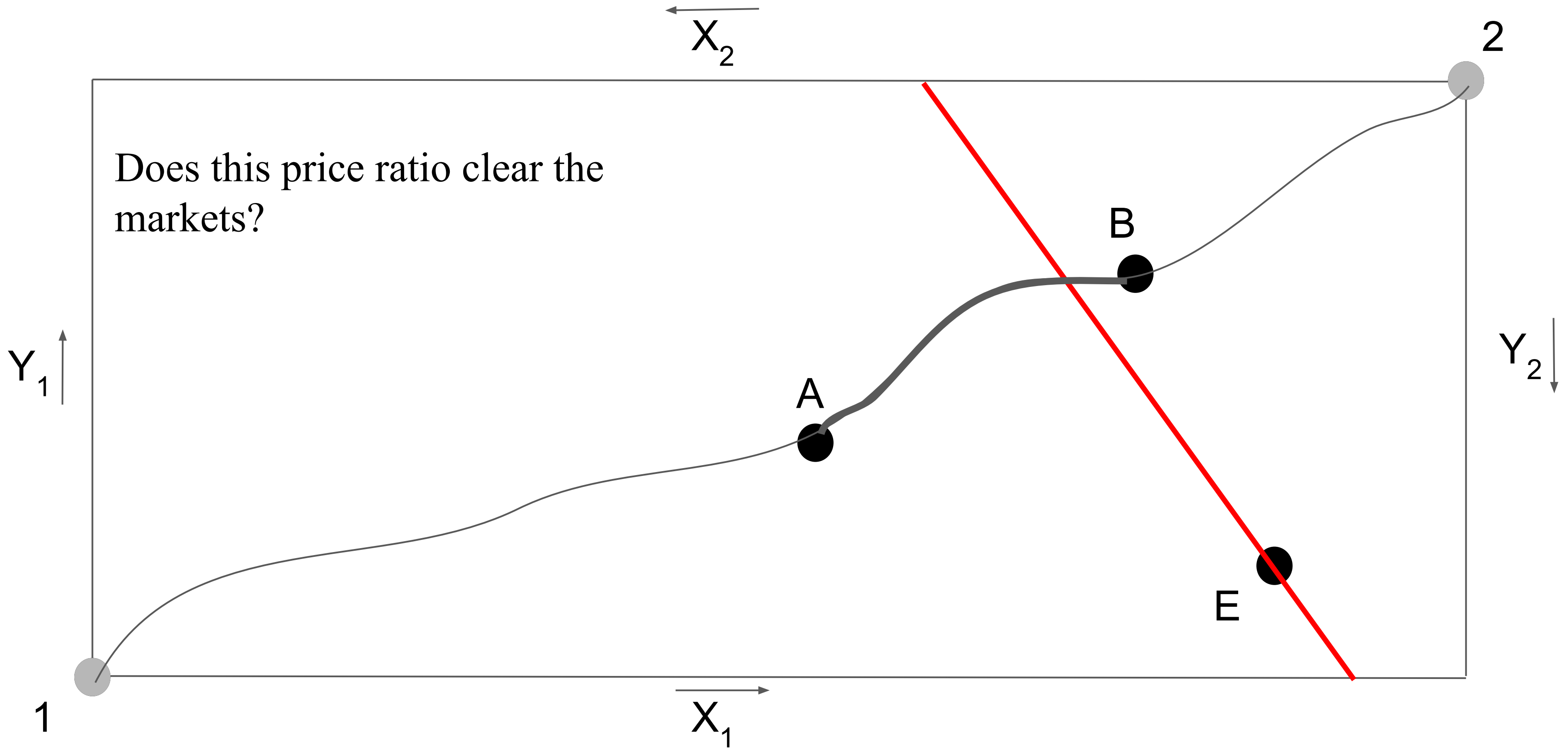


# Market outcome



# Market outcome

Does this price ratio clear the markets?



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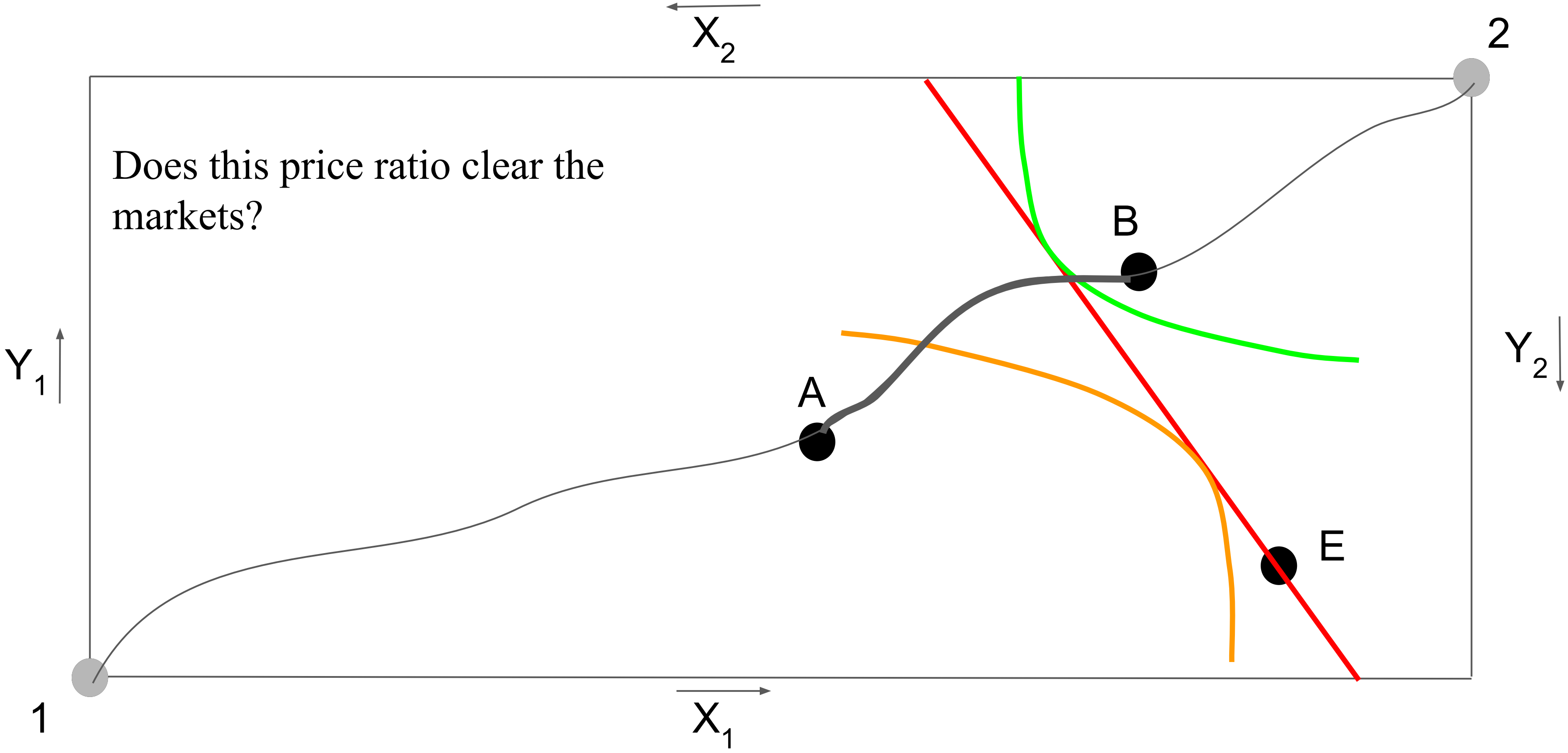

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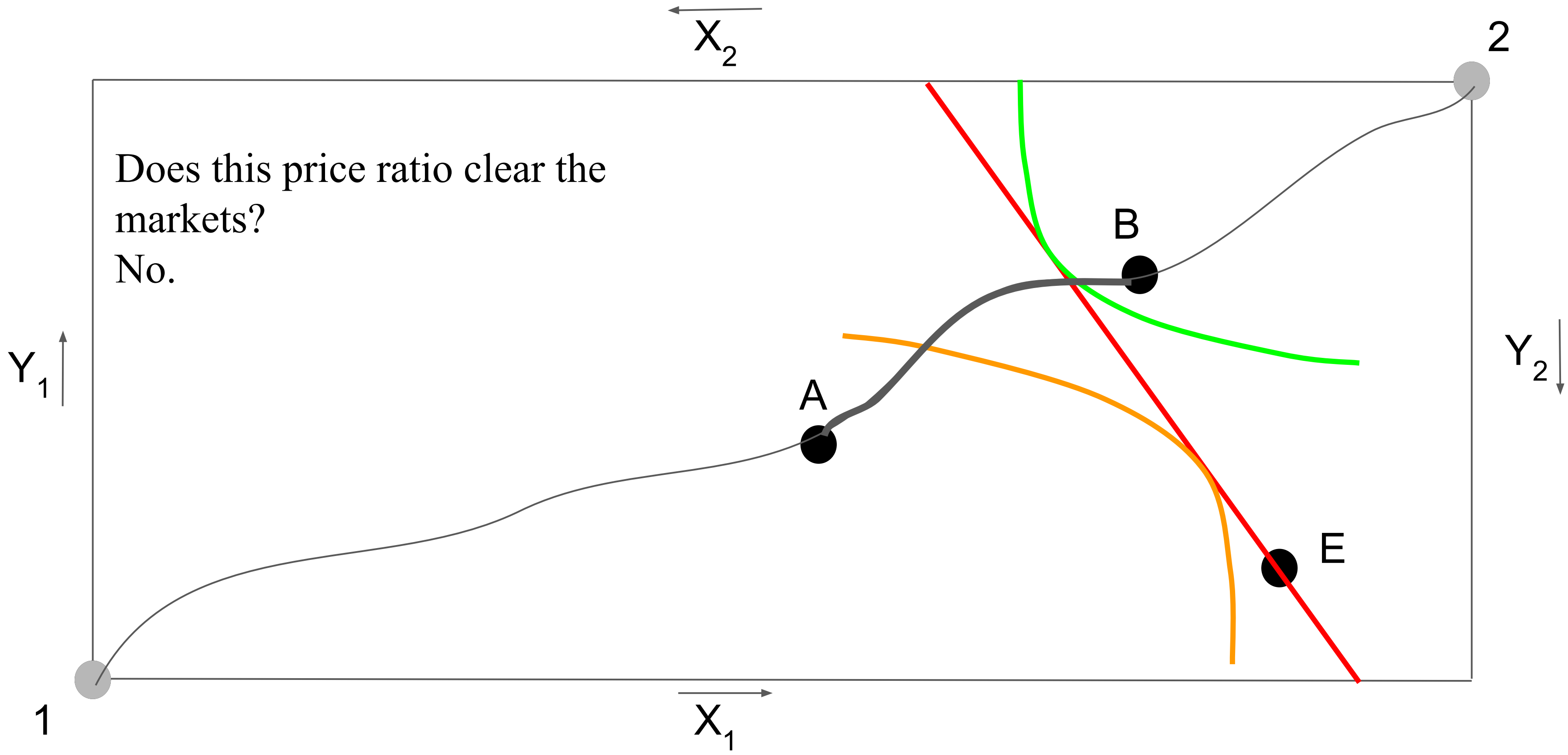
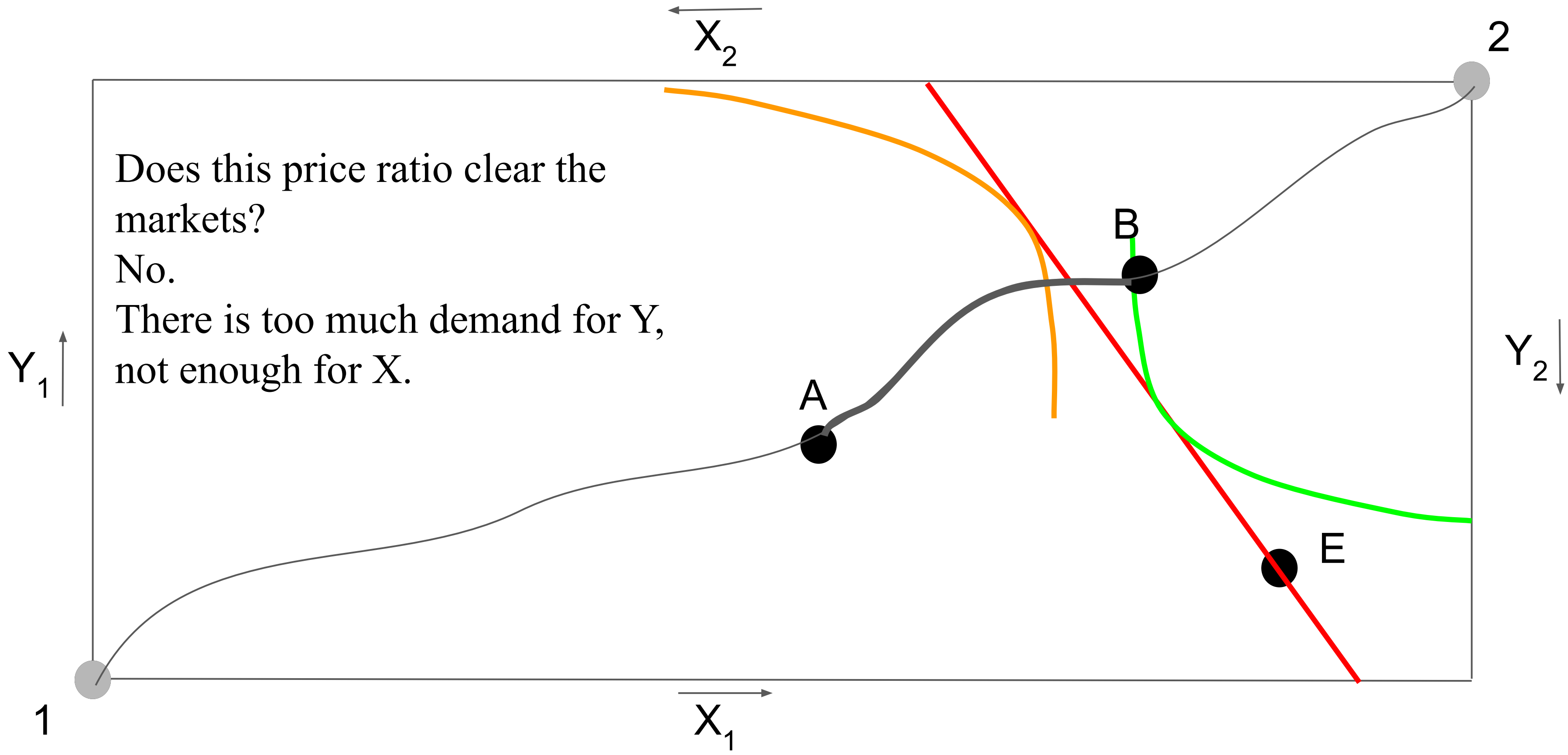
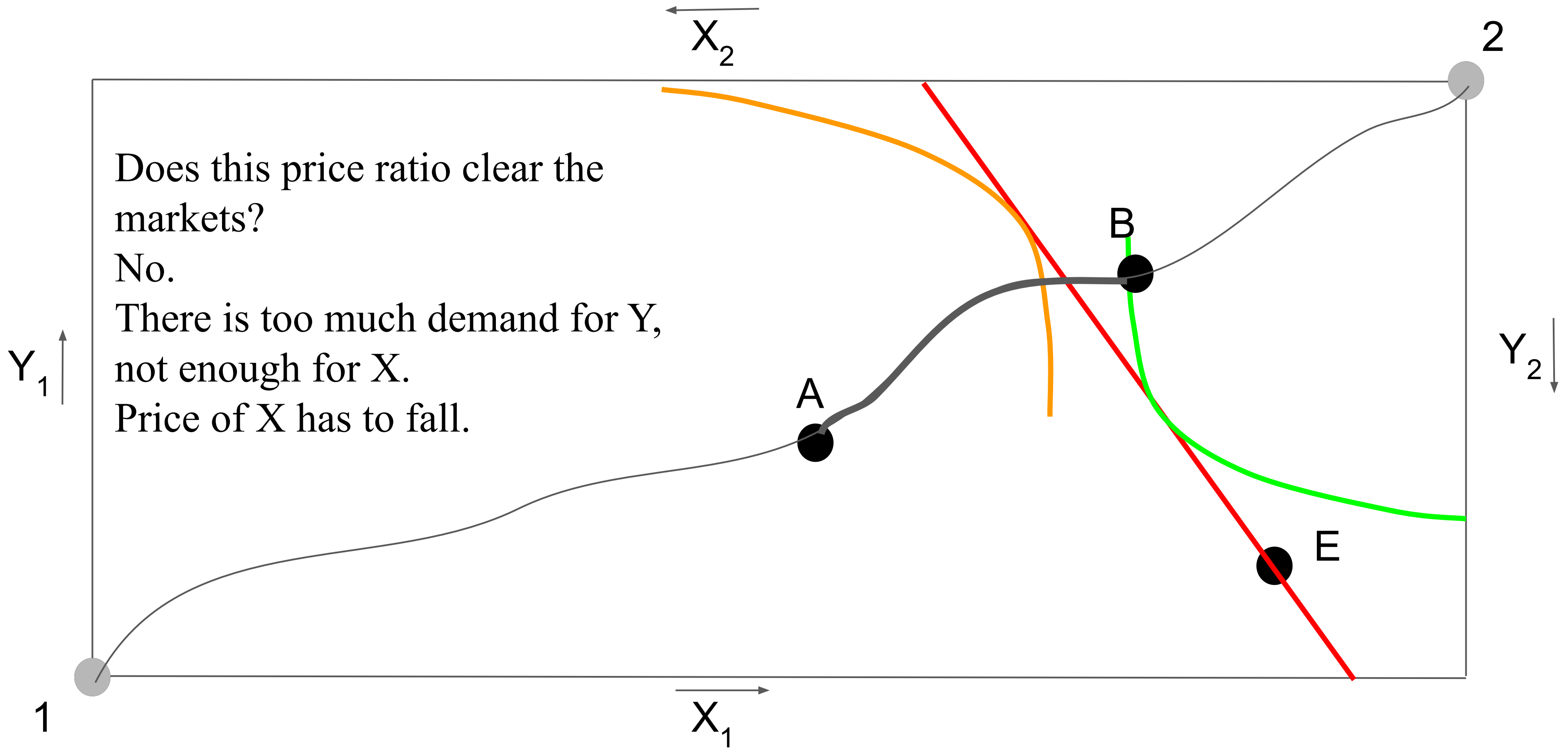


# Market outcome

Does this price ratio clear the markets?
No.
There is too much demand for Y, not enough for X.



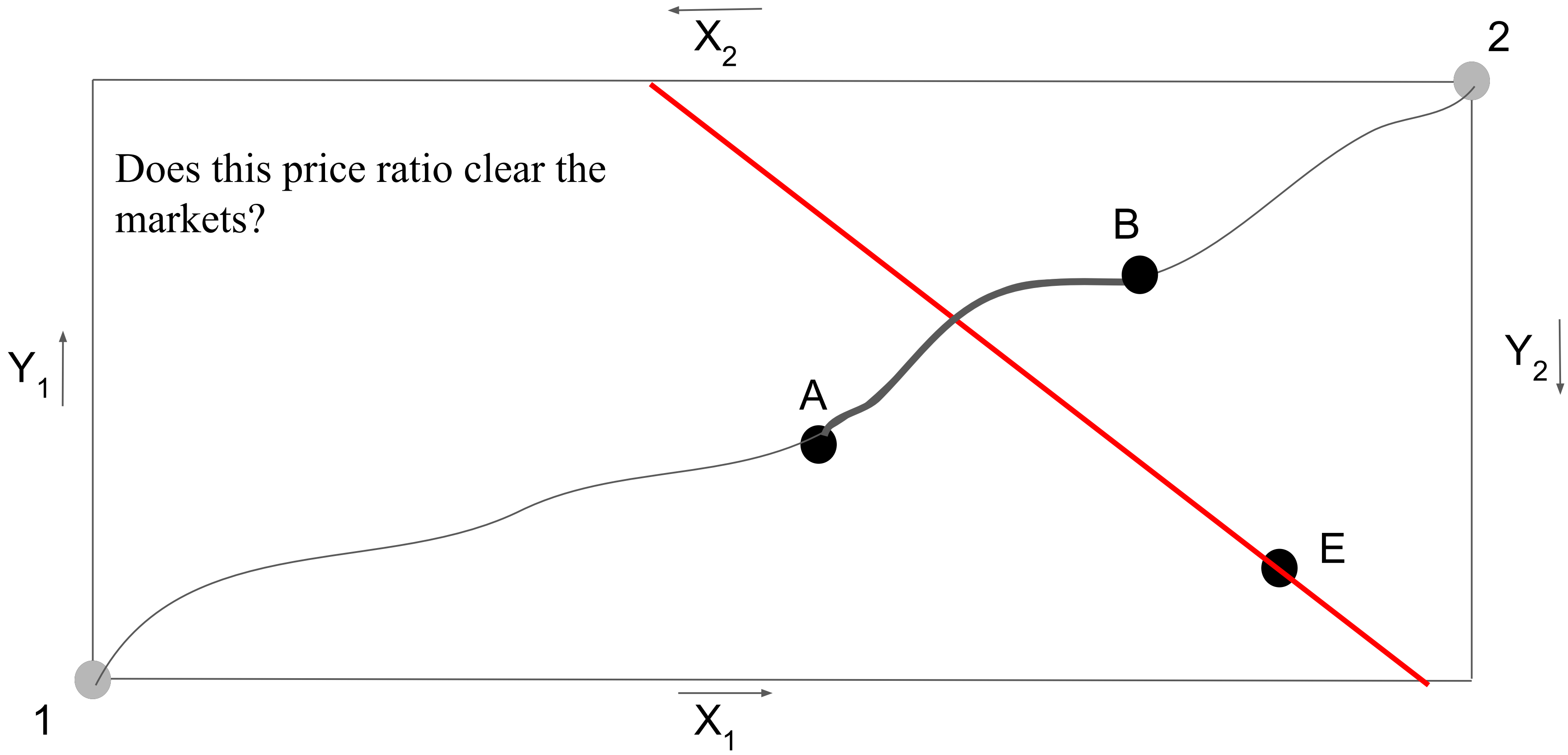
# Market outcome




Does this price ratio clear the markets?
No.
There is too much demand for Y, not enough for X.
Price of X has to fall.

# Market outcome

Does this price ratio clear the markets?



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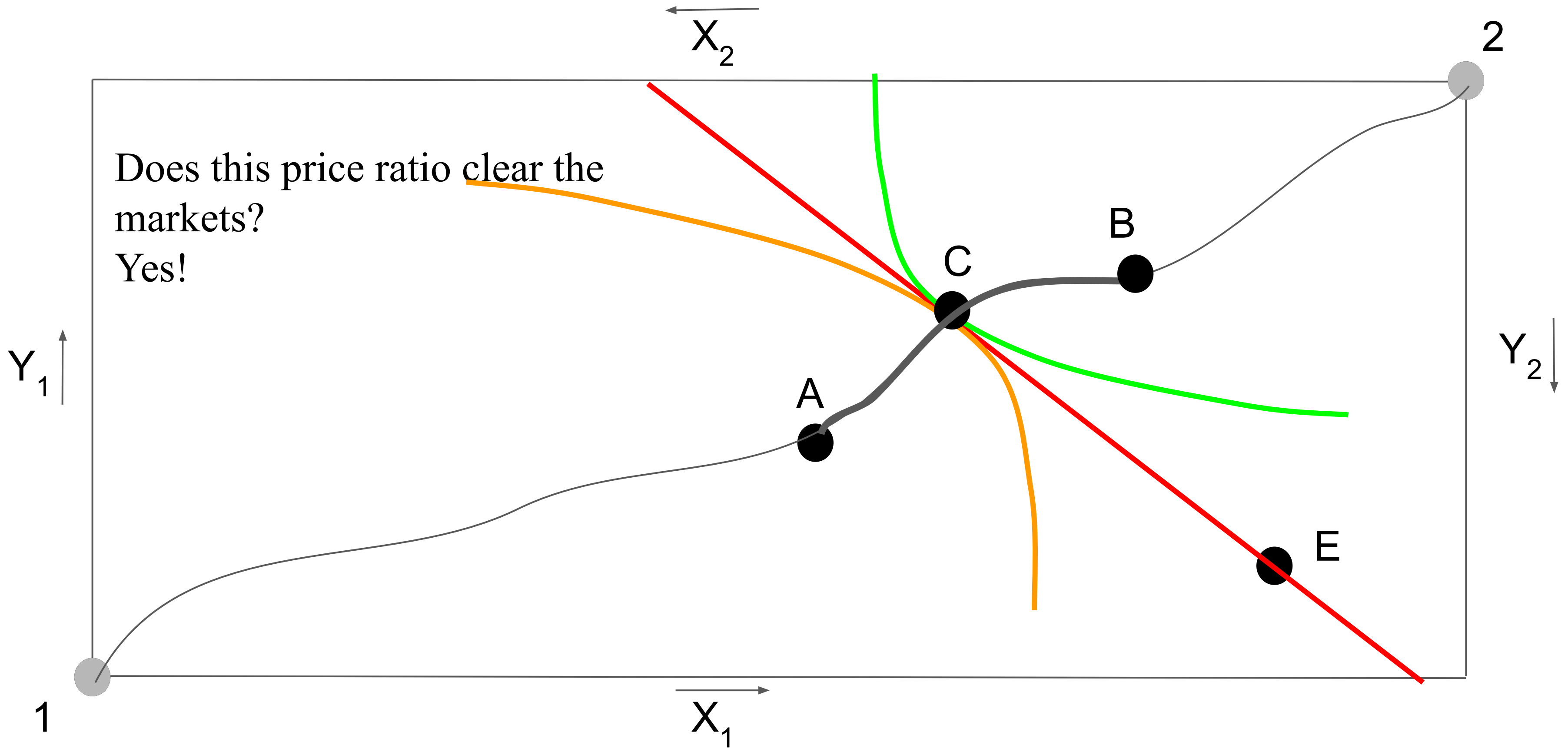

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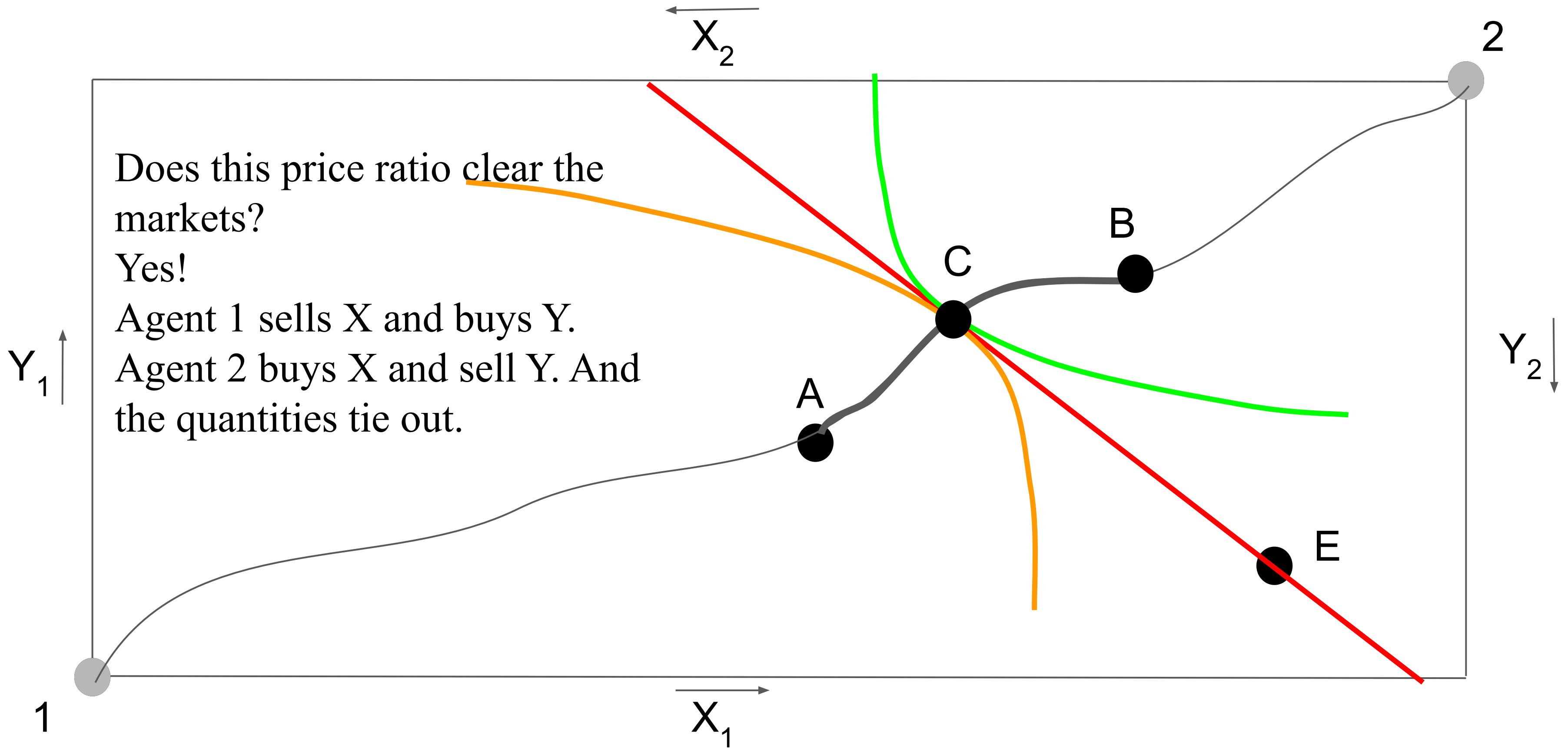
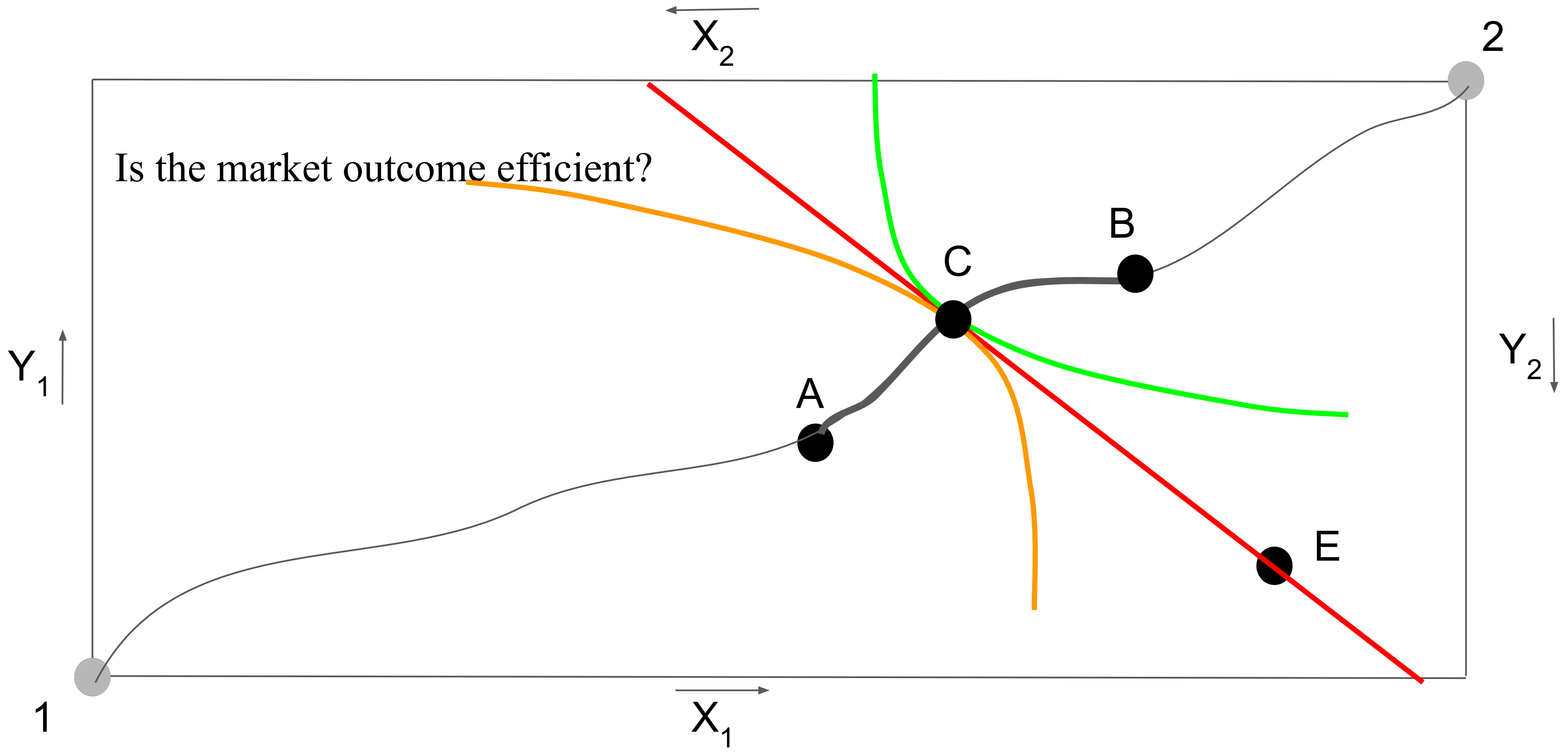



Does this price ratio clear the markets?
Yes!
Agent 1 sells X and buys Y.
Agent 2 buys X and sell Y. And the quantities tie out.

# Market outcome

Is the market outcome efficient?



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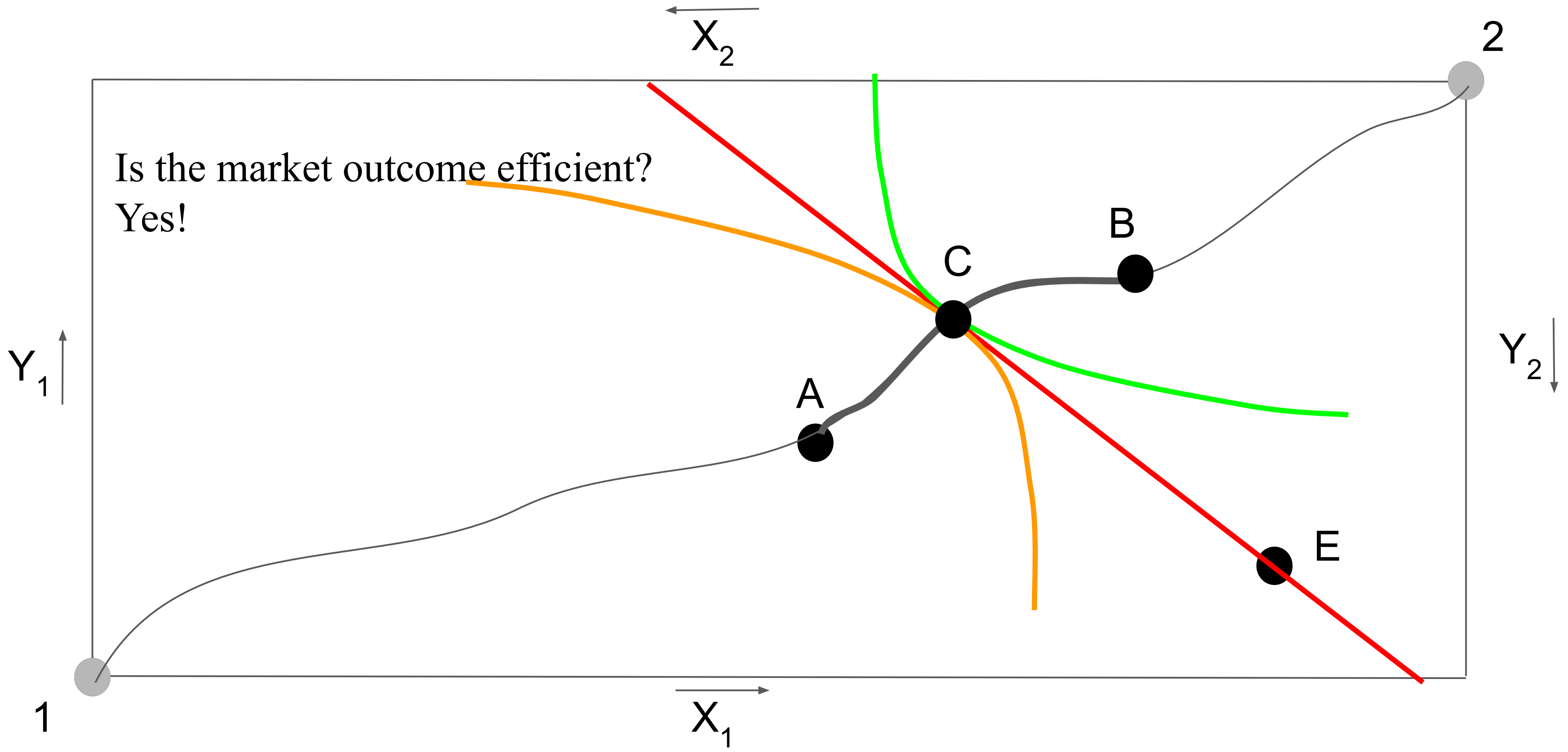


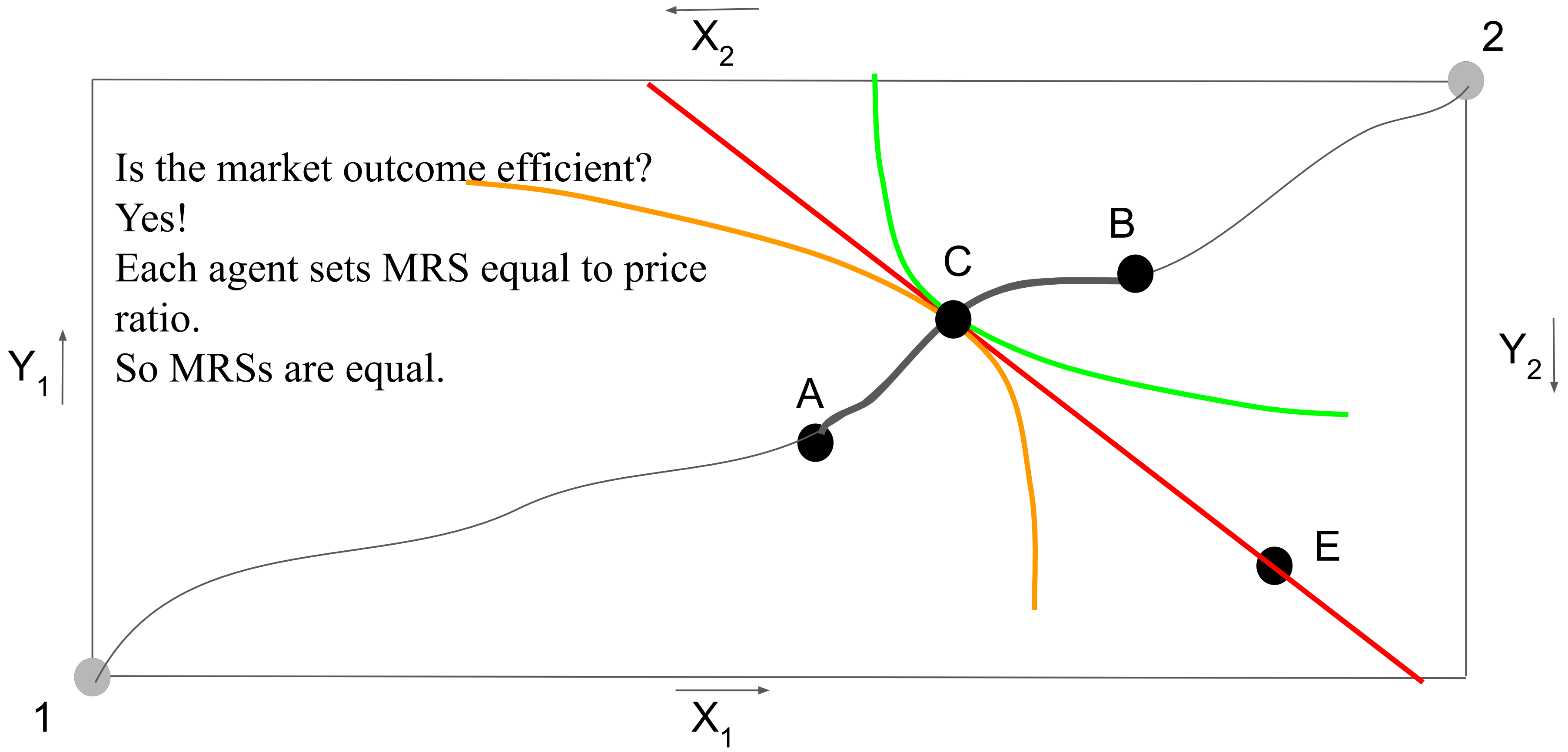
# Market outcome

Is the market outcome efficient?

Yes!

Each agent sets MRS equal to price ratio.

So MRSs are equal.



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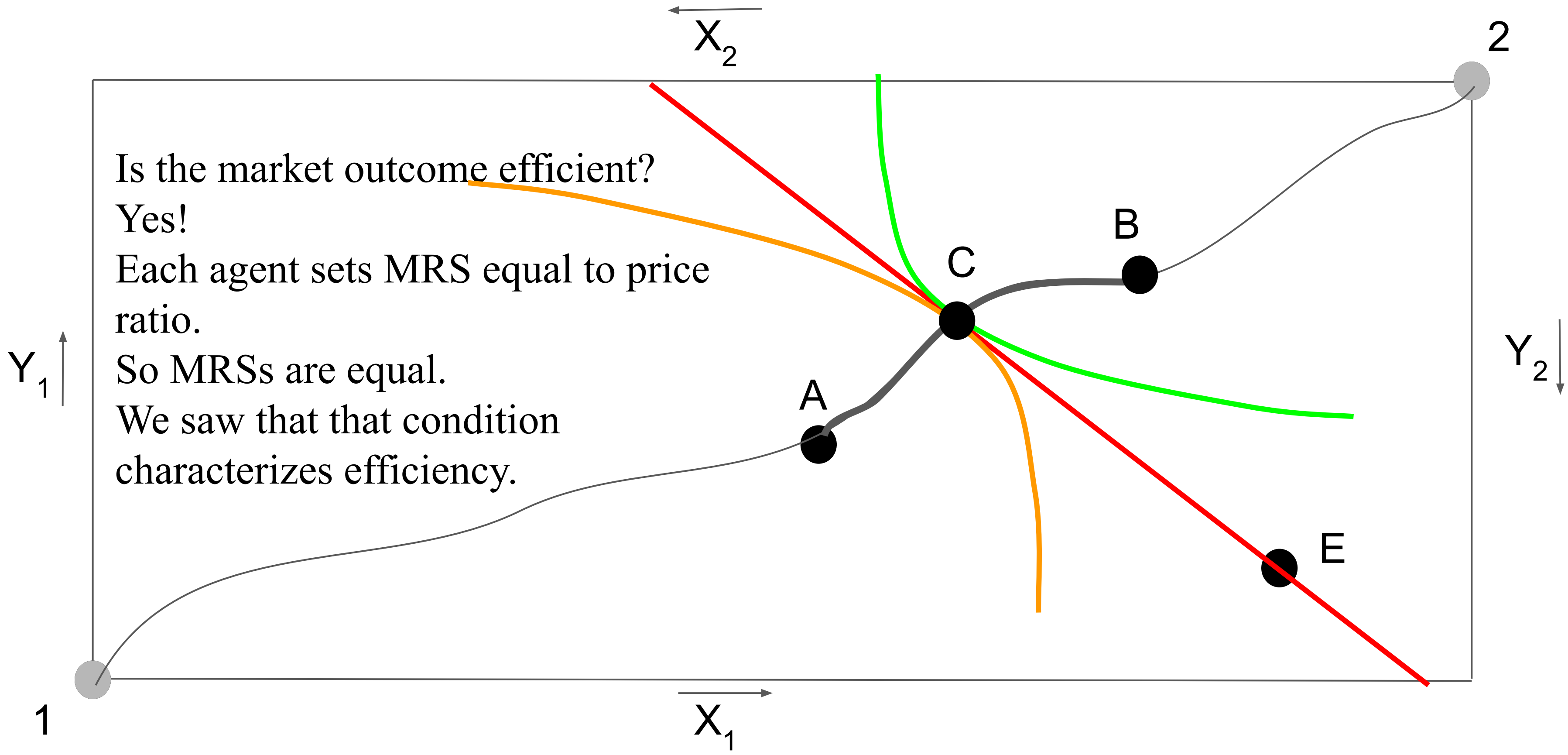


Is the market outcome efficient?

Yes!

Each agent sets MRS equal to price ratio.

So MRSs are equal.

We saw that that condition characterizes efficiency.

# First Fundamental Theorem of Welfare Economics

- If consumers are price takers and there are no externalities, the market outcome is efficient.
    - Basically just proved it – equality of MRSs means no mutually-beneficial trades can be made.

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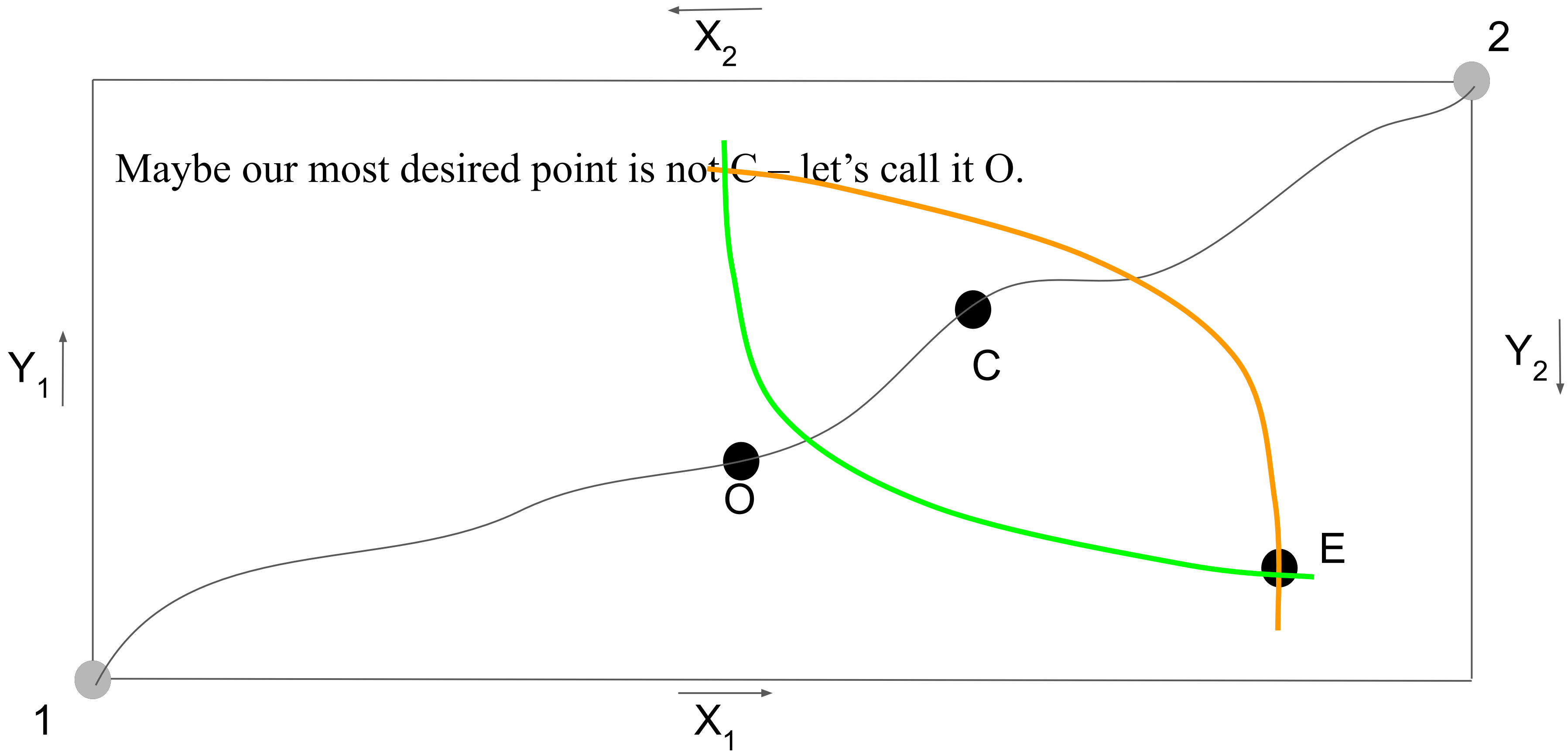
- If consumers are price takers and there are no externalities, the market outcome is efficient.
  - Basically just proved it – equality of MRSs means no mutually-beneficial trades can be made.
- This is more compelling than our supply-and-demand graph efficiency claim because this looks at all markets at once, and provides a rigorous foundation for where agents' "values" come from (utility)
- So under good conditions, markets do something very well – eliminate "waste"
- But that does not mean we have to like the outcome – maybe it's unfair

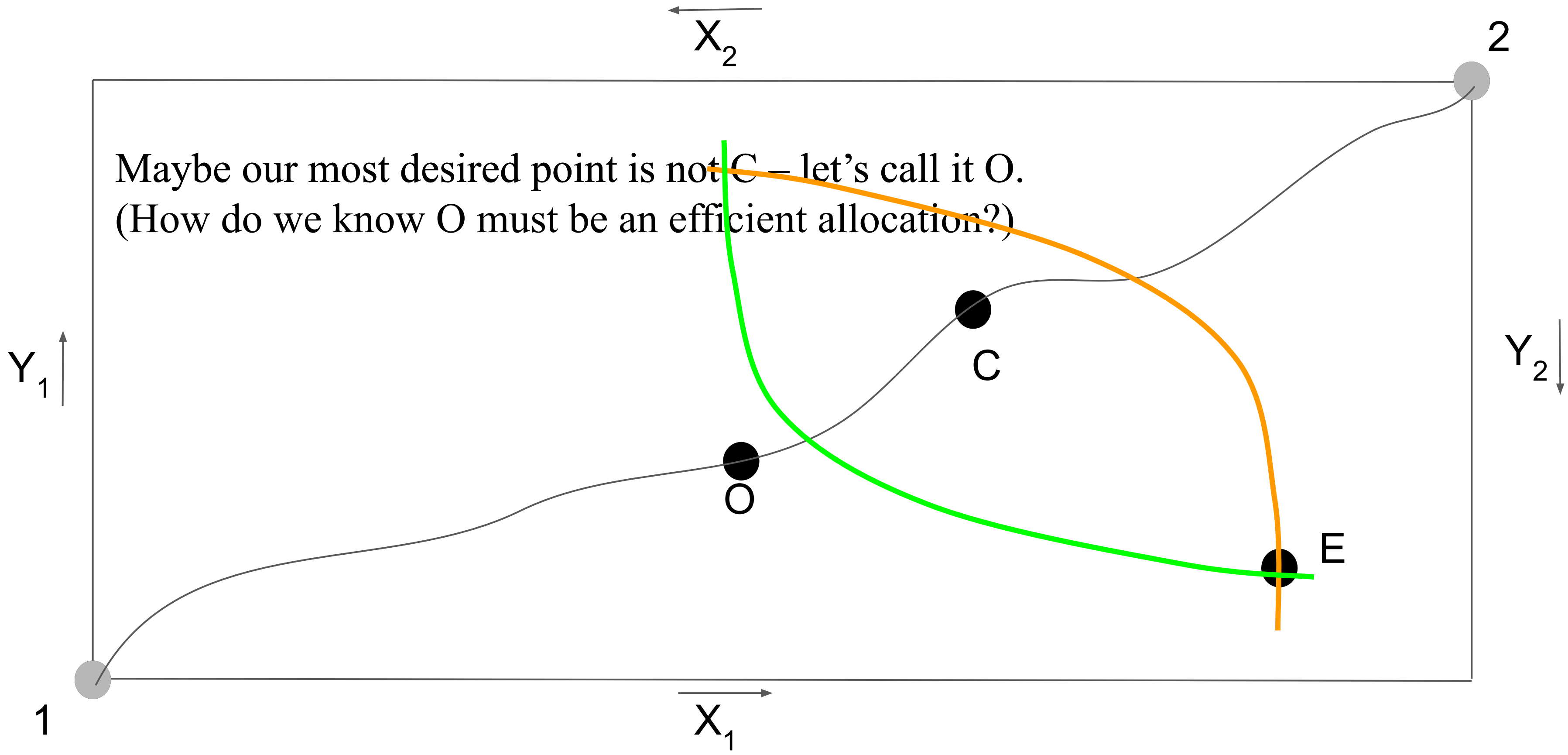
# Social optimum

Maybe our most desired point is not C – let's call it O.



# Social optimum

Maybe our most desired point is not C – let's call it O.
(How do we know O must be an efficient allocation?)



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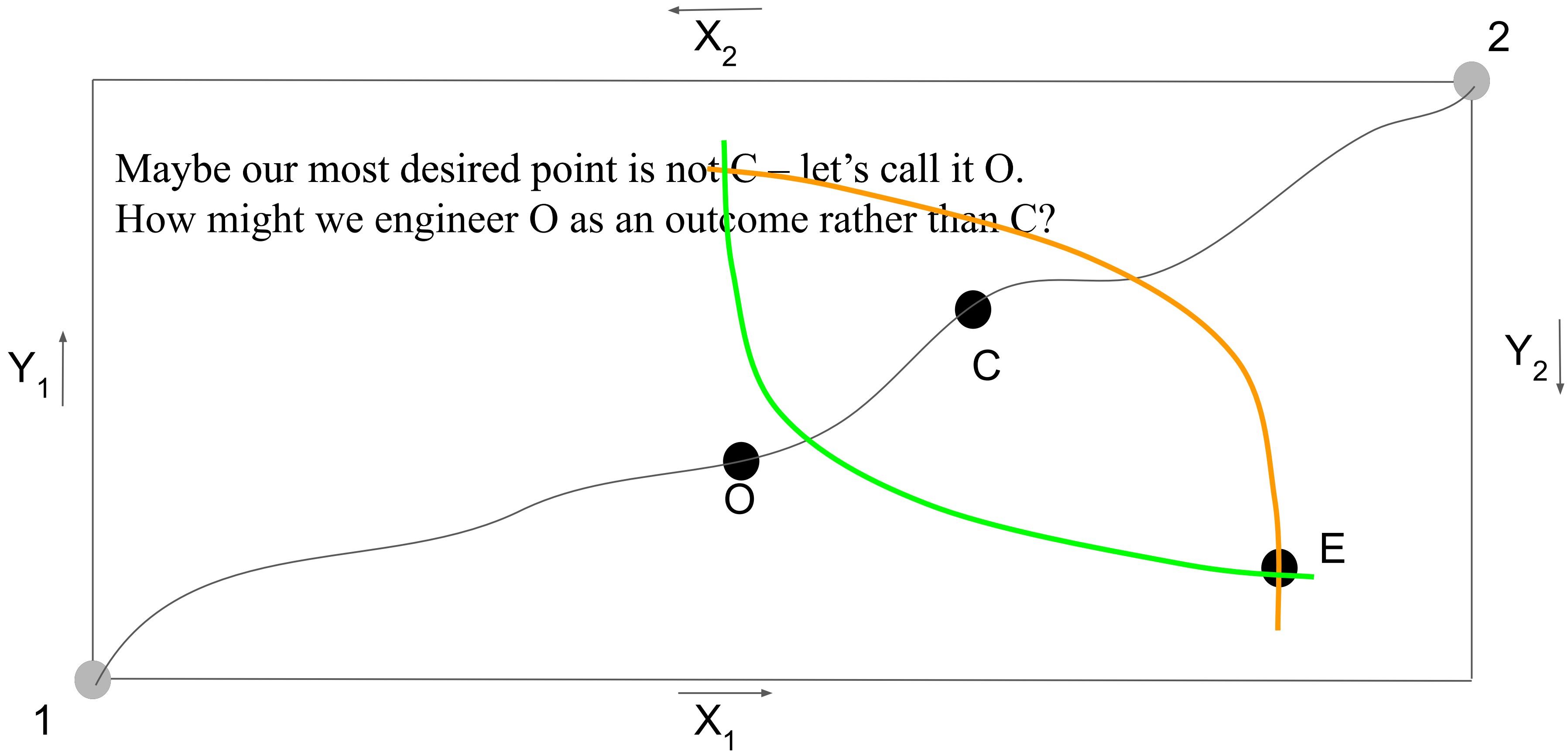
Maybe our most desired point is not C – let's call it O.
How might we engineer O as an outcome rather than C?



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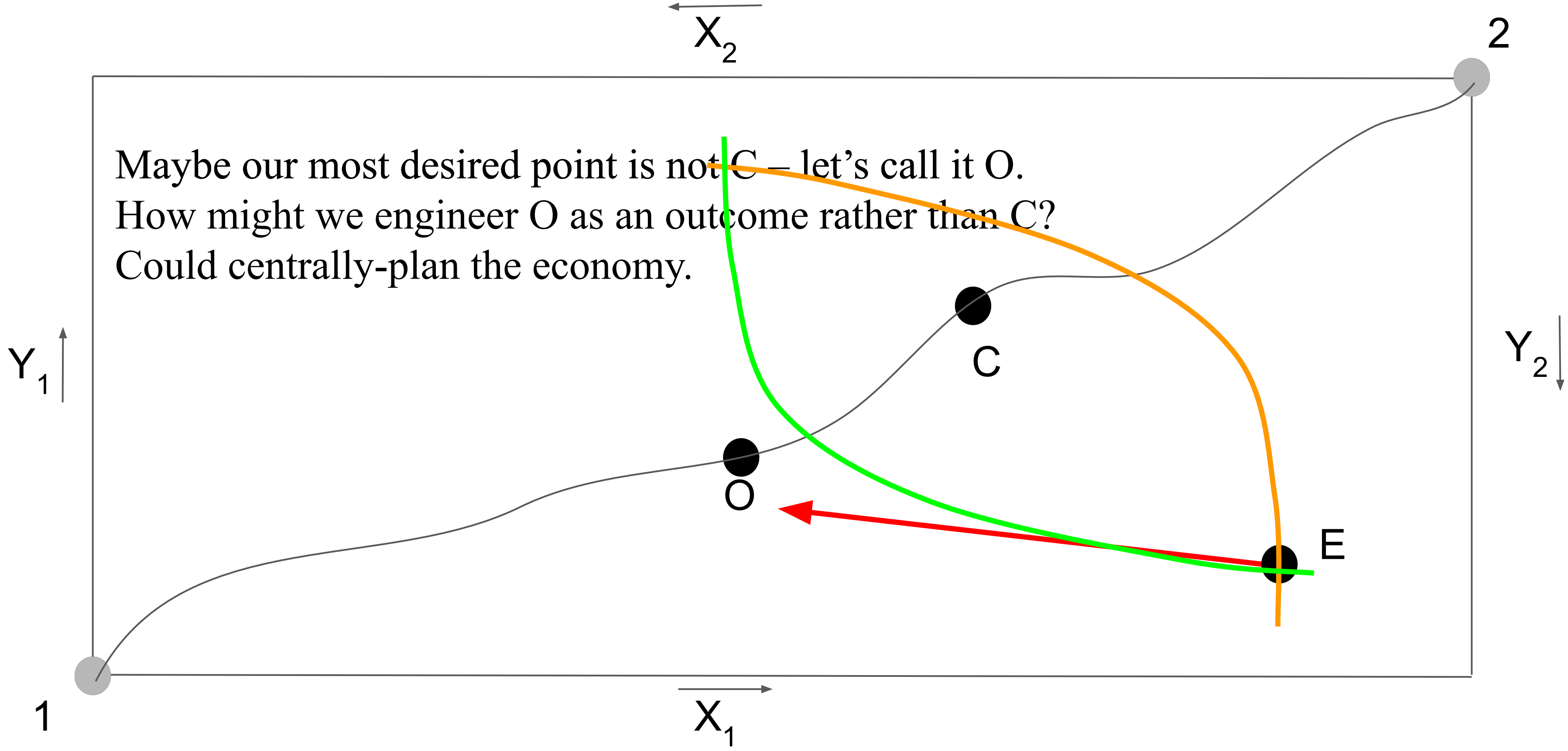
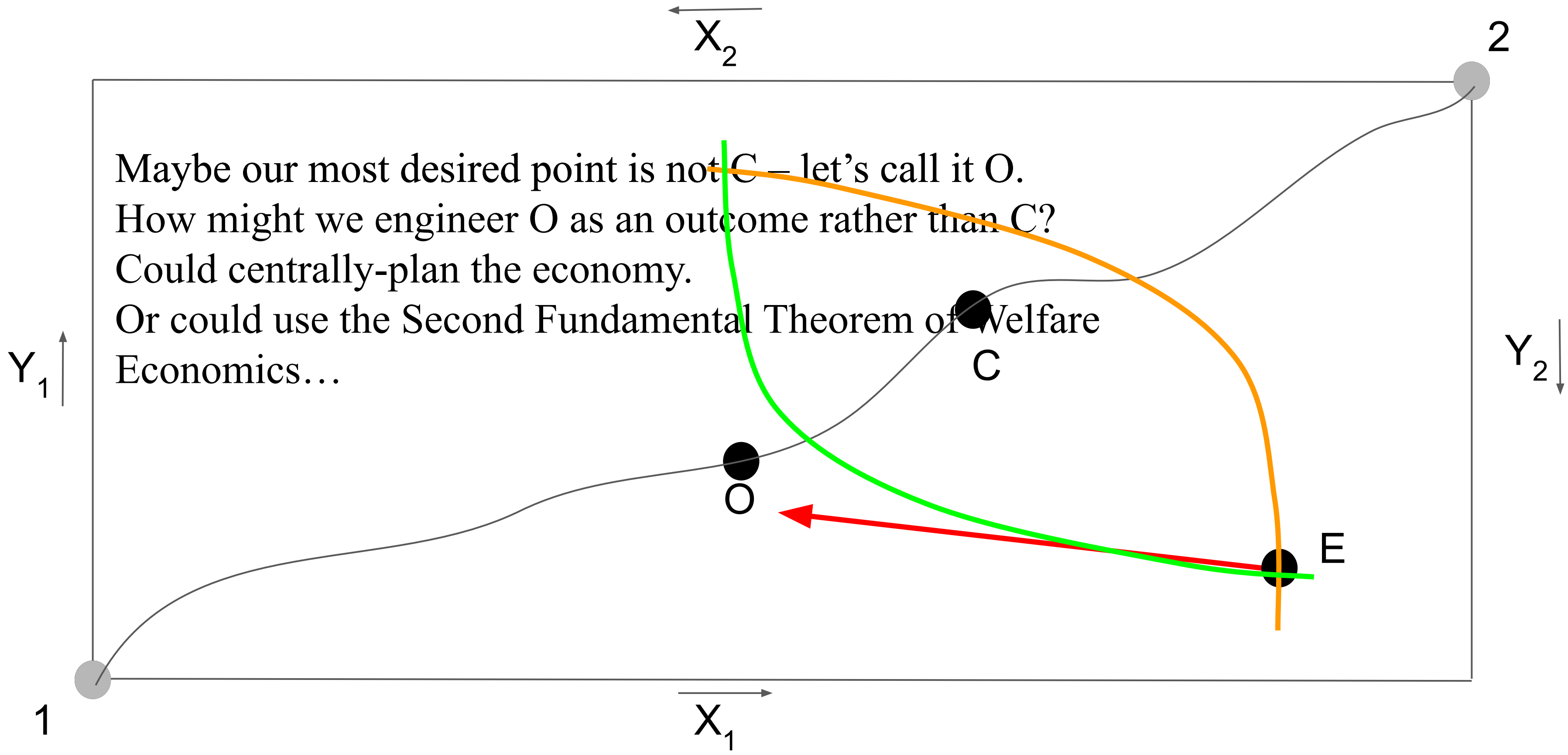
How might we engineer O as an outcome rather than C?
Could centrally-plan the economy.



# Social optimum

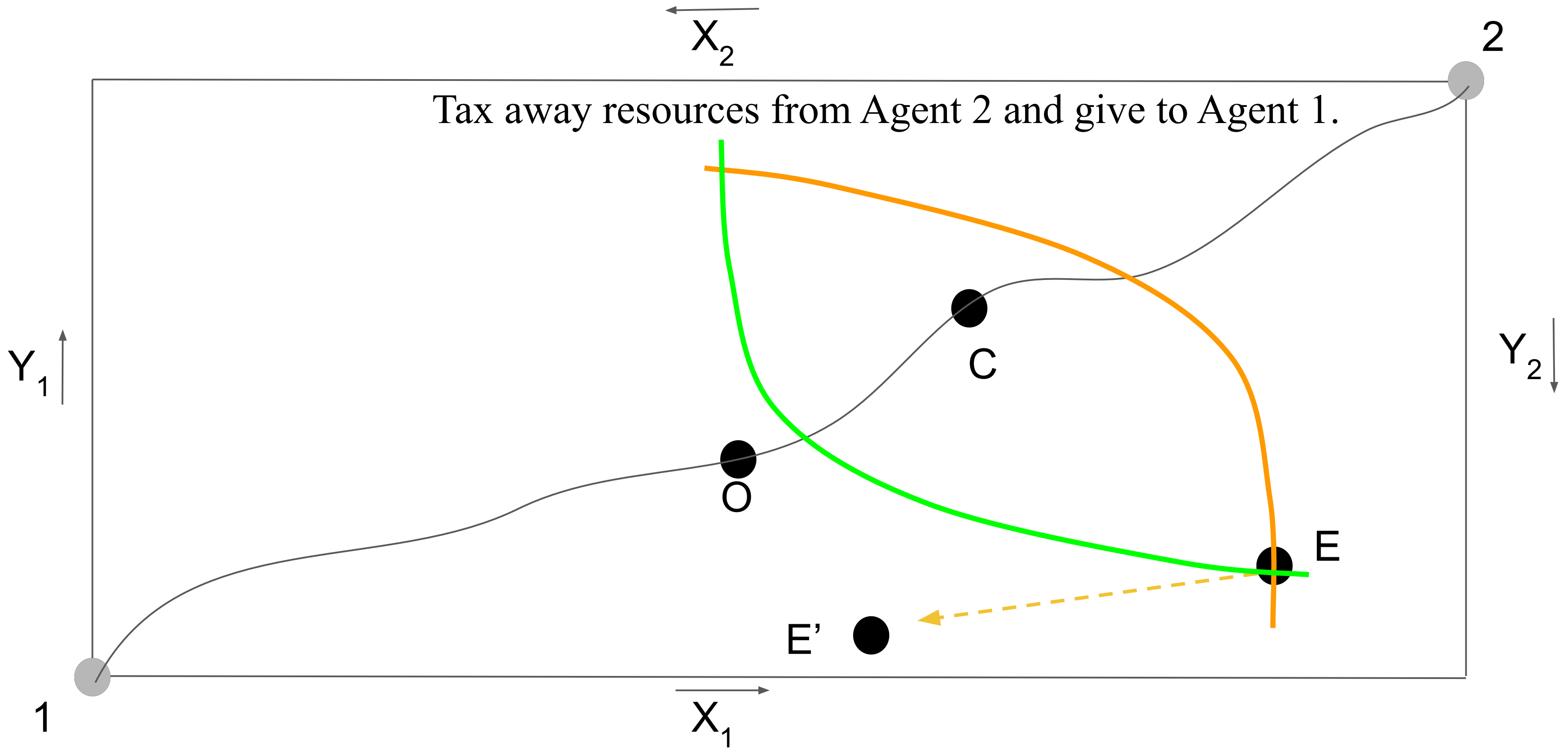
Maybe our most desired point is not C – let’s call it O.
How might we engineer O as an outcome rather than C?
Could centrally-plan the economy.
Or could use the Second Fundamental Theorem of Welfare Economics…



# Second Fundamental Theorem of Welfare Economics

- If consumers are price takers and there are no externalities, then any efficient outcome can be generated by markets, as long as we make the right pre-market tax/transfers.
  - Harder to prove, so we won't do so.

# Second Fundamental Theorem of Welfare Economics



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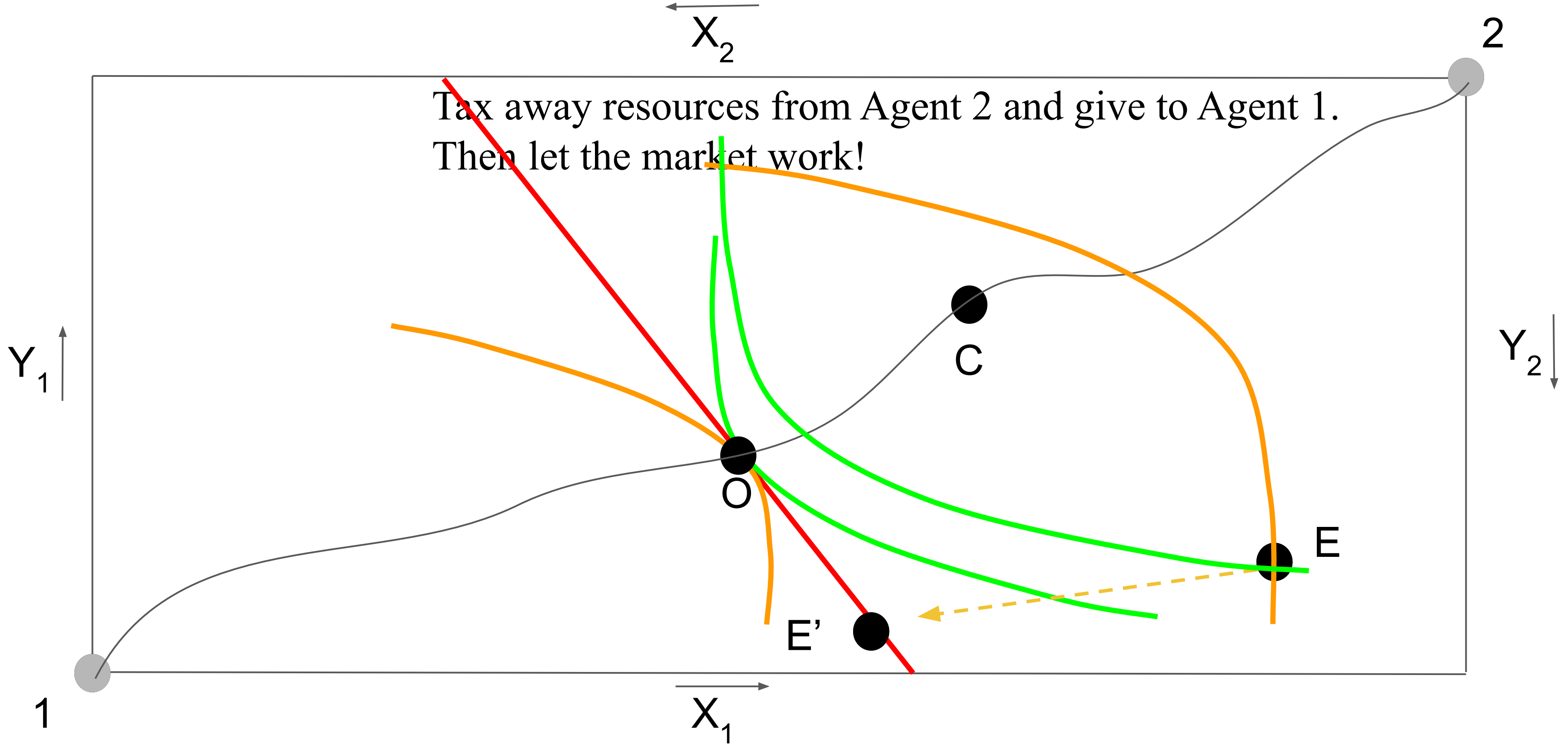

# Second Fundamental Theorem of Welfare Economics

- If consumers are price takers and there are no externalities, then any efficient outcome can be generated by markets, as long as we make the right pre-market tax/transfers.
  - Harder to prove, so we won’t do so.
- The market price mechanism does not create inequity, nor does it resolve it:
  - If you have inequitable endowments, the market will get you an inequitable (but efficient) equilibrium
- If you want greater equity, you don’t need to undo the market price mechanism
  - Assigning allocations to people would be super-hard and probably fail
  - Instead, transfer wealth and then just let them choose allocations with post-tax money
  - If you have equitable endowments, the market can generate equitable (and efficient) equilibrium
- Caveat: income-based tax/transfers create a distortion not considered here…

# Welfare Theorems and policy

- As abstract as the theory is, it has very practical consequences
- Main idea: resolve inequity by transferring resources/opportunity, not by intervening within product markets or determining outcomes
- Consider two inequality-reduction proposals:
  - Tax yachts and subsidize bus passes
  - Transfer money from rich to poor
  - Which does our analysis suggest is a better policy?

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- Consider two inequality-reduction proposals:
  - Tax yachts and subsidize bus passes
  - Transfer money from rich to poor
  - Which does our analysis suggest is a better policy?
- The latter – redistribute through income taxation!
  - This is quite a robust result that survives much more sophisticated models/analyses
- Taxation of individual goods should be reserved for dealing with non-inequality-related market failures, like externalities